Is APV Better than WACC for Non-Stationary Debt Ratio?

Howard Qi and Guangsen Han

Abstract

The WACC method is normally considered suitable for firms maintaining a constant debt ratio; while the APV method is more convenient when debt policy and tax rate are more complex. However, we show that this is incorrect in that the APV method actually requires knowledge about more variables (than the WACC method does) in order to implement accurately. On top of this, the central issue (with the APV method) regarding the discount rate for the tax shields is still an open question to a large extent, which makes the APV method even more unreliable.

The two-year example provided in this study is set up in a style as general as possible. This allows for an easy extension to more general and realistic situations. Therefore, we clarify a widespread yet mistaken notion about the WACC and the APV method. Not only does it correct a longstanding misconception in academics, but it also has useful implications in practice.

Introduction

It is a widespread belief that the WACC (weighted average cost of capital) valuation method is valid only when the firm continuously rebalances its capital structure to maintain a constant debt ratio, while the APV (adjusted present value) method, proposed by Myers (1974), is more robust and can be used more conveniently for firms not having a specific target debt ratio and/or firms in emerging economies where leverage decisions are a matter of opportunistic nature and the tax legislation can change frequently. For example, Sabal (2007) asserts that the APV method is more appropriate for firms in emerging markets because the tax shields can fluctuate due to a variety of uncertainties such as new tax legislations and opportunistic leverage decisions. By the same token, Sabal also claims that the WACC method is a good approximation for firms in industrialized economies where they tend to maintain a target leverage ratio. These views are fairly representative (see for example, Pereiro 2002). In this study, we analyze whether the claim of the APV's superiority (over the WACC method) is indeed theoretically sound and practically feasible.

The reason that the APV method appears to be more flexible than the WACC method is mostly based on two points. First, suppose the WACC method does require constant debt ratio and constant tax rate, then the tax shields cannot be arbitrary but a fixed proportion of the total firm value, which is a very restrictive condition. Second, if the tax shields over different periods of time become irregular due to changing tax rate and/or uncertainty in debt policy, the APV method is still applicable because it does not impose any restrictions on the tax shields. In other words, the APV method requires the correct forecasts of the future tax shields, while the WACC method requires certain pattern of the forecasted tax shields as well. However, a careful examination of these two points seems to cast serious doubts on their accuracy as well as the validity of some of their implications we usually encounter.

Howard Qi is an Assistant Professor of Finance at Michigan Technological University. Guangsen Han is a senior research scientist and engineer at the Institute of Agricultural Mechanical Engineering of Shanxi Province, China. Contact address: School of Business and Economics, Michigan Technological University. 1400 Townsend Drive, Houghton, MI 49931. Email: howardqi@mtu.edu; fax: 906-487-2994.

The first point about constant debt ratio as a necessary condition for the WACC method to be valid can be shown to be incorrect. For example, Johnson and Qi (2008) demonstrate that the WACC method at least is also applicable for the fixed-debt scenario where the leverage ratio may vary period by period. In addition, Qi (2010) argues that it is not a reliable belief that most firms in industrialized economies tend to maintain a constant debt ratio. On the contrary, it is normally harder for large firms to frequently change their debt level to meet the target they do have one, and there is ample evidence in support of this view. For example, Fischer, Heinkel and Zechner (1989) find that firms operate within a wide range of leverage ratio because they do not continuously rebalance their capital structure. Graham and Harvey (2001) collect evidence that a large number of managers do not rebalance their capital structure in response to equity market movements because the presence of adjustment costs prevents firms from rebalancing continuously. Hittle, Haddad and Gitman (1992) show that only a small percentage of the large US firms set a target leverage ratio. Alternative theories such as the pecking order theory has demonstrated to have considerable explanatory power as to why firms may not have a well defined target ratio (see e.g., Myers 1993; Liesz 2003; Byoun and Rhim 2003; and Titman and Wessels 1988). Study by Frank and Goyal (2003) finds that the pecking order theory works much better especially in the 1970s and 1980s.

The second point about the superiority of the APV method (compared with the WACC method) because it can handle arbitrary tax shields seems to be a plausible notion. It is normally understood that the main difficulty with the APV method is to determine the appropriate discount rate for the tax shields in order to calculate the present value of the tax shields. Sabal (2007) demonstrates with his well-designed numerical example that this issue has been resolved by Fernández (2004) by defining the present value of the tax shields as the difference between the value of the taxes paid by the levered firm and by the unlevered firm, respectively. This, if true, would finally make the APV method practically feasible and theoretically correct, and thereby validate the notion of APV's superiority over the WACC method. However, Qi (2010) shows that this claim is not warranted because some internal inconsistencies and errors hidden in the example offered in Sabal (2007) necessarily invalidate it as a logical sequitur. What Qi (2010) shows is that APV's superiority, however perhaps true, is not really proved in studies such as Sabal (2007) and Pereiro (2002). This leaves the APV's superiority still an open question.

In this investigation, we fill the gap by directly showing that the notion of APV superiority over the WACC method is a fallacy even though it does appear to be quite appealing. For more details in this line of research, see for example, Cooper and Nyborg (2006), Brealey et al (2005), Johnson and Qi (2008), Qi (2010), Sabal (2007), Pereiro (2002), Booth (2002, 2007), Ruback (2002), and Modigliani and Miller (1958, 1963). This study is organized as follows. Section I reviews the literature and explains the current status of the research about this topic, which leads to the question we set out to answer. Section II uses an example of two-year project to illustrate algebraically (in a very general sense) the basics about the APV and the WACC methods, and the issues facing their implementation. Section III analyzes and compares the two methods. This section categorically refutes the APV's superiority over the WACC method. Section IV concludes.

II. Project Valuation – A Case of Two-Year Project

Suppose we have a two-year business project whose free cash flows (*FCF*s)[24] are

24 Free cash flow is defined as the cash amount available for distribution to all investors of the business if no debt

forecasted to be FCF_1 and FCF_2 in the first and the second year after the business is started. In other words, we simplify the situation by making two assumptions – (1) the first profit comes exactly 1 year after the business starts; (2) The business is scrapped after two years with a net scrapping value of zero.

At first glance, these two assumptions seem to be quite restrictive. Nevertheless, they do not hamper us from generalizing the result for more common projects and firms. The simplicity actually helps to highlight the underlying principles. It is quite easy to extend it to multiple-year projects, say n years. For example, we can still assume the scrapping value to be zero and let FCF_2 absorb the sale price of the business; or we can let the scrapping value be the business value at time $t = 2$, i.e., V_2 which represents the value of cash flows from year 3 to year n, and keep FCF_2 intact. We find the former approach to be technically a bit easier and cleaner, thus we always assume a zero scrapping value in this study. For a corporation whose business life is unlimited, we may simply make a further yet trivial extension by letting $n \to \infty$. To put it differently, our case of the 2-year business allows us to make some generalized conclusions about more realistic and normal situations. Table 1 shows the relevant cash flow streams.

Table 1. Cash flows from a two-year project with zero scrapping value

	Year 0	Year 1	Year 2
Free cash flows (*FCF*)	0	FCF_1	FCF_2
Tax shield (*TS*)	0	$R_{D1}D_1T_{C1}$	$R_{D2}D_2T_{C2}$
Total available cash flows (*C*)	0	$FCF_1 + R_{D1}D_1T_{C1}$	$FCF_2 + R_{D2}D_2T_{C2}$
Value of the business project (*V*)	V_0	V_1	V_2

It is worth clarifying that our two-year setup provides technical simplicity without harming theoretical generalization. Another way to see this point is to note that our framework links three points in time by finding present values for future cash flows in both year 1 when the firm is running and year 2 when the project is scrapped. This careful setup can establish a recursive valuation procedure for projects of a life of any number of periods.[25]

Here we assume in year 1 and 2, respectively, cost of debt is R_{D1} and R_{D2}, corporate tax rate is T_{C1} and T_{C2}, and debt amount adopted by the firm is D_1 and D_2. We note that this setup is very general since we do not impose any restrictions on these variables. In other words, the tax shields are as flexible as they can be.[26] Also notice that for year 0, we neglect any relevant

were used. The true cash flow available for a levered firm's investors is equal to the free cash flow plus the tax shield.

25 We thank the anonymous referee for raising the importance of the feasibility of extending the two-year scenario to the general situations where the project has an arbitrary length of life. Indeed, without this critical extendibility, our two-year scenario would offer little value. Therefore, we emphasize once again that in this study we are actually using a device of technical simplicity to address a profound and quite general topic with full validity. Rigorous mathematical proof of this claim is readily available upon request.

26 However, we note that for risky business cash flows, the realized tax shields may be less because of various reasons leading to full or partial loss of tax shield, such as alternative minimum taxes, the existence of non-debt tax shields, etc. These would reduce the realized tax shields by a certain proportion and we can accommodate this effect by adopting an effective corporate tax rate, which does not really affect our analysis in this study. See for example

cash flows such as the initial investment outlay. This is because these cash flows are already in present value. If we want to calculate the net present value, we simply subtract the initial outlay; otherwise, we may ignore it if we just wish to price the business project rather than its NPV. The total cash flow $C_t = FCF_t + R_{Dt} D_t T_{Ct}$ is more commonly called capital cash flow (CCF) as in Ruback (2002)[27] which represents the after-tax cash amount available for distribution to all the equityholders and bondholders of the project or the firm. In other words, we treat the free cash flow (*FCF*) as the baseline and adjust it by adding the tax shield $R_{Dt} D_t T_{Ct}$. Given this setup, the goal of pricing this business project is to determine its value at time $t = 0$, i.e., V_0.

The way to determine V_0 is to work backwards from V_2 to V_0. Notice at $t = 2$, the project's value is exactly the total final cash flow it can generate, that is

$$V_2 = FCF_2 + R_{D2} D_2 T_{C2} \tag{1}$$

Next, we determine V_0 through two approaches, the WACC and the APV methods.

III.1. The WACC Method

At time $t = 1$, the project's value is then given by discounting the weighted average cost of capital over the second year, i.e., $WACC_2$ defined as

$$WACC_2 = \frac{D_2}{V_2} \times R_{D2} \times \left(1 - T_{C2}\right) + \frac{V_2 - D_2}{V_2} \times r_{E2} \tag{2}$$

where r_{E2} is the return on equity over the second time period. This is the standard definition except it recognizes that all the variables do not have to be constant over time. Then the business value at $t = 1$ is

$$V_1 = \frac{FCF_2 + R_{D2} D_2 T_{C2}}{\left(1 + WACC_2\right)} + \left[FCF_1 + R_{D1} D_1 T_{C1}\right] \tag{3}$$

Thus, by the same token, we have

$$V_0 = \frac{\dfrac{FCF_2 + R_{D2} D_2 T_{C2}}{\left(1 + WACC_2\right)} + \left[FCF_1 + R_{D1} D_1 T_{C1}\right]}{\left(1 + WACC_1\right)} \tag{4}$$

where $WACC_1$ is defined similarly as in (2) except that the time index is changed from 2 to 1. Combining (1) and (2), one can easily see that in order to compute V_0, one must have information about FCF_t, D_t, R_{Dt}, T_{Ct}, as well as r_{Et} for both time periods. In theory, precise

---

Qi, Liu and Johnson (2010) and Graham (2000) for the loss of tax shields.

27 Ruback (2002) proposes an alternative valuation method called the CCF approach which is a variation of the APV method. In this study we do not address issues related to the CCF method. For details about the CCF method, see for example, Ruback (2002), Qi (2010), Johnson and Qi (2008), and Booth (2002, 2007).

predictions of them may not be possible. In reality, one may make some practice assumptions to simplify the issue. For example, if the firm is known to keep the same debt, then $D_2 = D_1 = D_0$ and D_0 is observable; if cost of debt does not vary too much, then we can approximate $R_{D2} = R_{D1} = R_{D0}$ and R_{D0} is observable. Cost of equity r_{Et} can be extracted from market data as an approximated forecast. FCF_t may be predicted using the pro forma method. On the other hand, if the firm constantly rebalances its debt ratio to a constant target, then we may have a different approximation, i.e., FCF_t and D_t are proportional to each other. After all, without some additional information to allow us to make some simplifications and sensible approximations, the WACC method would be basically infeasible.

III.2. The APV Method

The APV method, proposed by Myers (1974), separates the total cash flow into two streams – free cash flows and tax shields – and then discounts them with appropriate discount rates as follows

$$V_1 = \left\{ \frac{FCF_2}{\left(1+R_{A2}\right)} + \frac{R_{D2}D_2T_{C2}}{\left(1+R_{TS2}\right)} \right\} + \left[FCF_1 + R_{D1}D_1T_{C1} \right] \tag{5}$$

The first two terms represent the discounted cash flows from t = 2, where R_{A2} and R_{TS2} are asset return[28] and the proper discount rate for the tax shield in time period 2. However, to get V_0, we need to maintain the separation of the cash flow streams from t = 2 to t = 0 shown below

$$V_0 = \left\{ \frac{FCF_1}{\left(1+R_{A1}\right)} + \frac{FCF_2}{\left(1+R_{A2}\right)\left(1+R_{A1}\right)} \right\} + \left\{ \frac{R_{D2}D_2T_{C2}}{\left(1+R_{TS2}\right)\left(1+R_{TS1}\right)} + \frac{R_{D1}D_1T_{C1}}{\left(1+R_{TS1}\right)} \right\} \tag{6}$$

where R_{A1} (R_{A2}) and R_{TS1} (R_{TS2}) are asset return and the proper discount rate for the tax shield in time period t = 1 (t = 2). The first part of Eq. (6) represents the present value of the business project if it were all-equity financed. The second part is the present value of the tax shields. These discount rates may be considered as foreword rates because they cover future time periods. Of course, we may also use the spot rates rather than forward rates to discount these cash flows. In order to arrive at V_0, we need information about all the variables appearing on the right-hand side of (6). Again, as with the earlier WACC case, theoretically it is impossible to correctly foresee these variable's future values. Practically, one may impose more restrictions if we have extra information. This is similar to what happens with the WACC method. For example, if the tax shields can be guaranteed (based on how the contract is designed) as risky as the corporate loan (bond), then the proper discount rate for the tax shields should be equal to cost of debt, i.e., $R_{Dt} = R_{TSt}$. Normally, if debt D_t and the corporate tax rate T_{Ct} do not change over

---

28 It is also called the opportunity cost of capital which is the after-tax return generated by the firm's assets when the firm is all-equity financed. This discount rate is used to discount the firm's *FCF*s.

time for sure, then the proper discount rate for tax shields may be the riskfree rate r_f.[29]

To correctly determine the discount rate R_{TSt} for the tax shields is not a trivial issue. Here we do not get into the specifics concerning what value R_{TSt} should take. Instead, we point out that this issue is still open to debate and far from closure as claimed in Sabal (2007), Fernández (2004) as well as most recently in Liu (2009) and Qi (2011). For example, studies by Harris and Pringle (1985), Ehrhardt and Daves (2002), Ehrhardt (2005) believe R_{TSt} should be equal to asset return R_{At} regardless of the debt policy. Miles and Ezzell (1980) modify the this approach by changing R_{A1} to r_f for the first tax shield in period 1 on the basis that this tax shield is known at $t = 0$. MM (1958, 1963) and Brealey et al (2005) use $R_{TSt} = R_{Dt}$, while Cooper and Nyborg (2006) argue these choices are all correct depending on whether the firm chooses fixed-debt or constant-leverage capital structure policy.

Furthermore, not only is the discount rate for the tax shields still an open question, but also asset return R_{At} is not observable. Normally, R_{At} is assumed to be constant (for no reason except for simplicity) and backed out by using the simplest MM's Debt-Irrelevance Theorem,[30] which is subject to many real-world flaws. Taken together, the APV method provides a conceptually different framework to price a business project. However, it does require quite an amount of information, perhaps more than it appears to need behind its seeming simplicity.

IV. Compare the WACC and the APV Methods

The two valuation methods price the same business project from different perspectives. They are theoretically equivalent. This equivalence relationship is vividly described by Booth (2002, 2007) as "you can always get there from here".

However, in practice, there is considerable amount of differences which originate from how much knowledge we must have about the variables appearing on the right-hand-side of equations (4) and (6). Still using our two-year project to carry out the comparison, we specifically list those variables that must be correctly assessed (or forecasted) before the WACC or the APV method can be practically implemented. Table 2 lists these must-know variables for each method.

Table 2. The must-know variables for the WACC and the APV method.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Panel (A): WACC method requires knowledge of:										
FCF_1	FCF_2	R_{D1}	R_{D2}	T_{C1}	T_{C2}	D_1	D_2	r_{E1}	r_{E2}	
Panel (B): APV method requires knowledge of:										

29 Cooper and Nyborg (2006) specifically argues that in this case, $R_{TSt} = R_{Dt} = r_r$

30 See for example Brealey et al (2005).

FCF_1	FCF_2	R_{D1}	R_{D2}	T_{C1}	T_{C2}	D_1	D_2	R_{A1}	R_{A2}	R_{TS1}	R_{TS2}

Panel (A) of Table 2 shows that in order to implement the WACC method successfully, one need to forecast future values for 9 variables. Panel (B) shows a reliable implementation of the APV method requires knowledge of 11 variables. The two methods share 7 common variables (see columns 1 to 7). Correctly forecasting these variables is very difficult. In addition, the WACC method also requires future equity returns r_{E1} and r_{E2} (in columns 8 and 9) which are not easy to forecast, and the APV method requires future asset return and discount rates for the future tax shields, R_{A1}, R_{A2}, R_{TS1} and R_{TS2} (in columns 8-9), respectively. Based on the number of must-know variables, the WACC has clear superiority over the APV method.

If we focus on the difficulty of forecasting the values in Table 2 instead of counting the number of the must-know variables, we find the WACC method may still be superior over the APV method. This is because if the firm is expected to continue the business as usual, one may approximate future equity returns r_{E1} and r_{E2} as today's and recent historical (observable) equity return. In comparison, the APV method also hinges on the correct estimation of (unobservable) R_{A1}, R_{A2}, R_{TS1} and R_{TS2}. Even extracting current asset return R_{A0} from the market data can be unreliable since it must assume some underlying theory (such as the MM Irrelevance Capital Structure Theorem and the correct tax shields' value), let alone future asset returns. As to discount rates for the tax shields, R_{TS1} and R_{TS2}, it involves even more ambiguities because it is still unclear what value they should take given so much on-going debate. The issue is far from being settled as we explained earlier. Therefore, the WACC method may have a slight edge over the APV method since at least the market can provide accurate information of the current and historical equity return r_E.

In sum, both the WACC method and APV method are subject to a variety of uncertainty sources. However, from the number of must-know variables in order to reliably implement a method, the WACC method clearly is more convenient since it only requires to forecast for 9 variables, and the APV method, 11 variables. From the difficulty of the forecasting, the WACC method still has a slight edge over the APV method. Taken together, it is unambiguously that it is incorrect to assert APV's superiority over the WACC method (see e.g., Sabal 2007; Pereiro 2002). Our investigation shows that it is most likely the other way around.

V. Conclusions

It is widely believed that the WACC method is suitable for firms maintaining a constant debt ratio which is the case for most firms in industrialized economies; while the APV method is more convenient and suited for valuing firms going through significant capital structural changes as well as firms in emerging markets where tax legislation is more uncertain and firms choose the debt ratio on an opportunistic basis. However, we use a project of two-year horizon to (1) illustrate the two methodologies; (2) show that "the above believes" are not only incorrect, but also it turns out that the APV method is not more convenient than the WACC method. Using a direct one-by-one comparison, we make this point clear that the APV method requires knowledge about more variables (than the WACC method does) in order to implement accurately. On top of this, what is even more troublesome is that the central issue (with the APV method) regarding the discount rate for the tax shields is still an open question to a large extent, which makes the APV method even more unreliable.

The two-year example provided in this study is set up in a style as general as possible. This allows for an easy extension to more general and realistic situations. Therefore, we clarify a widespread yet mistaken notion that the WACC method only applies to the constant leverage scenario while the APV method is more convenient and accurate than the WACC method and can apply to situations where tax rate and debt policy are not fixed.

VI. Acknowledgement

We are very grateful to an anonymous referee and the Editor Monzurul Hoque for helpful comments and guidance. We also thank Ling Zhang, Julia Qi and participants of the SBE Research Seminar at Michigan Tech and the participants of the 2010 Annual Academy of Finance conference for useful comments. All remaining errors in this paper are our own.

References

Booth, Laurence, 2002, "Finding value where none exists: pitfalls in using adjusted present value", *Journal of Applied Corporate Finance*, 15, 8-17.
Booth, Laurence, 2007, "Capital cash flows, APV and Valuation", *European Financial Management*, 13, 29-48.
Brealey, R. A., S. C. Myers, and F. Allen, 2005, *Principles of Corporate Finance*, 8th Edition, McGraw-Hill/Irwin Series in Finance, Insurance, and Real Estate, New York.
Byoun, S., and J. C. Rhim, 2003, "Tests of the pecking order theory and the tradeoff theory of optimal capital structure", *Proceedings of the Midwest Business Economics Association*.
Cooper, I. A., and K. G. Nyborg, 2006, "The value of tax shields is equal to the present value of tax shields", *Journal of Financial Economics*, 81, 215-225.
Ehrhardt, M., 2005, "Incorporating Competition into the APV Technique for Valuing Leveraged Transactions", *Journal of Applied Corporate Finance* 17, 79-88.
Ehrhardt, M., and P. R. Daves, 2002, "Corporate Valuation: The Combined Impact of Growth and the tax Shield of Debt on the Cost of Capital and Systematic Risk", *Journal of Applied Finance* 12, No. 2, 31-38.
Fernández, P., 2004, "The value of tax shields is not equal to the present value of tax shields", *Journal of Financial Economics*, 73, 145–165.
Fischer, E., R. Heinkel, and J. Zechner, 1989, "Dynamic capital structure choice: Theory and tests", *Journal of Finance* 44, 19-40.
Frank, M. Z. and V. K. Goyal, 2003, "Testing the pecking order theory of capital structure", *Journal of Financial Economics* 67, 217-248
Graham, J. R. and C. Harvey, 2001, "The theory and practice of corporate finance: Evidence from the field", *Journal of Financial Economics* 60, 187-243.
Harris R. S., and J. J. Pringle, 1985, "Risk-Adjusted Discount Rates Extensions from the Average Risk Case", *Journal of Financial Research* 8, 237-244.
Hittle, L. C., K. Haddad, and L. J. Gitman, 1992, "Over-the-counter firms, asymmetric information and financing preferences", *Review of Financial Economics*, 81-92.
Johnson, D. and H. Qi, 2008, "WACC misunderstandings", *Journal of Academy of Finance* 6, No. 1, 32-40.
Liesz, T. J., 2003, "Why pecking order theory should be included in introductory finance courses", working paper, School of Business and Professional Studies, Mesa State College, Grand Junction, Colorado.
Liu, Y.-C., 2009, "The slicing approach to valuing tax shields", *Journal of Banking and Finance* 33, 1069-1078.
Miles, J., and J. R. Ezzell, 1980, "The weighted average cost of capital, perfect capital markets, and project life: a clarification", *Journal of Financial and Quantitative Analysis* 15, 719–730.
Modigliani, F. and M. H. Miller, 1958, "The cost of capital, corporation finance and the theory of investment", *American Economic Review* 48, 261-297.
Modigliani, F. and M. H. Miller, 1963, "Corporate income taxes and the cost of capital", *American Economic Review* 53, 433-443.
Myers, S., 1974, "Interactions of corporate financing and investment decisions – implications for capital budgeting", *Journal of Finance* 29, 1-25.
Myers, S. C., 1993,"Still searching for optimal capital structure", *Journal of Applied Corporate Finance* 39, 4-14.

Pereiro, L. E., 2002, *Valuation of Companies in Emerging Markets, A Practical Approach*, John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ, USA.
Qi, H., S. Liu, and D. Johnson, 2010, “A model for risky cash flows and tax shields”, *Journal of Economics and Finance* (23 November 2010), 1-14.
Qi, H., 2010, “Valuation methodologies and emerging markets”, *Journal of Business Valuation and Economic Loss* 5 (1), Article 2.
Qi, H., 2011, “Value and Capacity of Tax Shields: An Analysis of the Slicing Approach”, *Journal of Banking and Finance* 35, Issue 1, 166-173. (ISSN: 0378-4266)
Ruback, S. Richard, 2002, “Capital cash flows: a simple approach to valuing risky cash flows”, *Financial Management*, 31, no. 2, 5-30.
Sabal, J., 2007, “WACC or APV?”, *Journal of Business Valuation and Economic Loss Analysis* 2, Iss. 2, Article 1.
Titman, S., and R. Wessels, 1988, “The determinants of capital structure choice”, *Journal of Finance* 43, 1-19.