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Geometric or Arithmetic Mean: A Reevaluation

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Abstract

This study evaluates two competing forecasting models of rates of returns and recommends the preferable model for academicians and practitioners. In the first model, which was developed by Jacquier, Kane, and Marcus (2002), the forecast is a weighted mean between the geometric mean and the sum of the geometric mean and half the variance, where the weights are determined by the relative importance of the estimation period and the forecasting period. The second model, which is an adaption by Bodie, Kane, and Marcus (2008) of the first model, where the arithmetic mean is substituted for the sum of the geometric mean and half the variance. This substitution is not explained or justified in any way. The purpose of this paper is to explore the statistical significance and impact on forecasts of this substitution. In theory, these two models could be the same in large samples generated from normally distributed returns. However, the relative ability of these two competing models to forecast for small samples of actual returns is unknown. In this study, we use three approaches to compare these two models. First, we compare the inputs, the arithmetic mean and the sum of the geometric mean plus half the variance, of the two competing models. Next, we compare the forecasts of the two competing models. Last, we compare the forecasting errors of the two competing models. We find statistically significant differences in the inputs and the forecasts, but no meaningful difference in the models' performance of forecasts as indicated by forecasting errors. In light of these results, despite the statistical differences, we find no economic difference between the forecasting errors of the two models and recommend the simpler of the two models which uses the arithmetic mean.

I. Introduction

When academicians develop competing models without testing the differences between the models, then practitioners and other academicians are uncertain which model is better to use. This problem is all the more confusing when two of the authors are the same for both models. Jacquier, Kane, and Marcus (2002) develop the first model as a weighted mean between the geometric mean and the sum of the geometric mean and half the variance, where the weights are determined by the relative importance of the estimation period and the forecasting period. The second model is developed by Bodie, Kane, and Marcus (2008) is a similar model except the arithmetic mean is substituted for the sum of the geometric mean and half the variance. Although the second model references the first, no justification for the substitution is given. This situation raises the question of the statistical significance and economic impact of this substitution. The empirical exploration of this question in small samples of various short-term time horizons is the focus of this paper.

Theoretically, in large samples with normally distributed returns that are independently and identically distributed through time, the arithmetic mean is exactly equal to the sum of the geometric mean and half the variance. Hence, in theory, there should be no difference between these two forecasting models. However, in small samples of real data, the distribution rates of return can change over time and exhibit serial correlation through time. This leads to four

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questions. First, in small samples of actual rates of return, is the estimate of the arithmetic mean equal to the sum of the estimates of the geometric mean and half the variance? Second, are forecasts generated by these two competing models equal to each other? Third, if forecasts are generated from a variety of historical data, then are there economically significant differences in the forecasts of the two models? Finally, given the analysis of the first three questions, then which model is preferable? In this paper, we analyze the first three questions and then recommend the model of overall preference.

II. Literature Review

For decades there has been recurring interest in forecasts of long-term portfolio returns. Should the geometric or the arithmetic mean of past returns be used to forecast future returns of individual investments and portfolios? The existence of significant differences between the two measures, as some authors suggest, may have important implications on the valuation of assets, and the extent of the equity –bond premium (long-run difference return advantage of stocks over government bonds). The debate over arithmetic and geometric means started with the birth of portfolio theory. Markowitz (1952) first developed portfolio theory in terms of mean/variance optimization which assumed higher moments were zero. This symmetrical distribution is consistent with normally distributed rates of return, not lognormally distributed rates of return. The mean used by Markowitz was the arithmetic mean.

However, Latane (1959) showed that, if investors want to select the portfolio with highest terminal wealth, they would select the portfolio with the highest geometric mean return. Elton and Gruber (1974 a) derive optimal portfolio theory for lognormally distributed returns. Then in Elton and Gruber (1974 b), the authors show that if returns are lognormally distributed, then maximizing the geometric mean maximizes expected utility.

Damodarian (2002) states "Conventional wisdom argues for the use of the arithmetic mean. In fact, if annual returns are uncorrelated over time, and our objective was to estimate the risk premium for the next year, the arithmetic mean is the best unbiased estimate of the premium. In reality, however, there are strong arguments that can be made for the use of geometric means. First, empirical studies seem to indicate that returns on stocks are negatively correlated over time. (See Fama and French 1988). Consequently, the arithmetic mean return is likely to overstate the premium. Second, while asset pricing models may be single-period models, the use of these models to get expected returns over long periods (such as 5 or 10 years) suggests that the single period model may be much longer than a year. In this context, the argument for geometric mean premiums becomes even stronger."

In contrast, when considering which is the superior measure of investment performance, the arithmetic mean or the geometric mean, Bodie, Kane, and Marcus (2002) state the following. "The geometric average has considerable appeal because it represents the constant rate of return we would have needed to earn in each year to match actual performance over some past investment period. It is an excellent measure of *past* performance. However, if our focus is on future performance, then the arithmetic average is the statistic of interest because it is an unbiased estimate of the portfolio's expected future return (assuming, of course, that the expected return does not change over time). In contrast, because the geometric return over a sample period is always less than the arithmetic mean, it constitutes a downward-biased estimator of the stock's expected return in any future year." Their example uses returns that are independent over time. This statement does not consider any possible bias in the forecasting of terminal portfolio value that was first described by Blume (1974). Although Blume considered this bias, his assumption of normally distributed returns did not result in a measure of this bias.

Jacquier, Kane, and Marcus (2002) start with lognormal distributed stock price returns and state the simple mathematical fact that if the distribution of returns is known for certain, then the expected value of the distribution is the arithmetic mean. However, if the true distribution of returns is not known, then sampling from the lognormal distribution with a right-hand skew introduces a bias that varies with the ratio of the length of the forecasting period and the length of the estimation period. They propose a compound growth rate that provides unbiased estimates of future portfolio values as the following:

$$G\,(F/E) + (G + 1/2\sigma^2)(1 - (F/E)) \qquad (1)$$

where G is the historical geometric mean of stock price returns, F is the forecast horizon, and E is the estimation period.

Based on this research article, Bodie, Kane, and Marcus (2008), in their popular MBA investment textbook, have the forecast of cumulative returns equals:

$$G\,(F/E) + \mu\,(1 - (F/E)) \qquad (2)$$

where μ is the historical arithmetic mean of stock price returns. In the above expression, the authors Bodie, Kane, and Marcus substituted the arithmetic mean for the geometric mean plus half the variance. Jacquier, Kane, and Marcus state that for more volatile investments the difference in the arithmetic and geometric mean is larger than half the variance. This calls into question the substitution of the arithmetic mean into the forecasting model in the textbook by Bodie, Kane, and Marcus. What is the impact of this substitution on the forecast of long term returns? Additionally, Jacquier, Kane, and Marcus present only long term forecasts of large samples. Practitioners are also interested in the accuracy of both forecasting models with short-term forecasts in small samples. Therefore, this paper proposes to compare the equivalence of these two models and the comparative accuracy of each model in a variety of settings.

III. Data

Our objective is to explore differences in sampling distributions characteristics of the forecasting models in a variety of settings. Therefore, we perform similar analysis on three different data sets. The first set uses rolling samples of the monthly returns of 10 randomly selected companies representing the different sectors in the economy from 1995 through 2007 covering financial services, manufacturing, and technology sectors. The companies returns used in this study with their ticker symbol in parentheses are: General Electric (GE), International Business Machines Corp. (IBM), Bank of America Corp (BAC), AT & T Inc (T), Texas

Instruments, Inc (TXN), The Boeing Co (BA), Dell Inc. (DELL), Walt Disney (DIS), American International Group Inc. (AIG), and Exxon Mobil Corp. (XOM).

The second set consists of rolling samples of the rates of return on five asset classes (Treasury bills, intermediate–term Treasury bonds, long-term Treasury bonds, large cap stocks, and small cap stocks) from 1926 through 1995 given in Bodie, Kane, and Marcus (2002). The first estimation period is 15 years covering 1926-1940 and subsequent15-year samples are repeated until 1995 resulting in 56 samples.

The third data set starts with the second data set to generate rolling samples of mean-variance optimized portfolios for given levels of risk aversion for each five year sample with ending date in 1935 through 1995. The mean variance framework of the third data set requires knowledge of expected returns. The implementation uses historical returns. Mean-variance optimization framework efficiently allocates wealth to the five asset classes: small cap stocks, large cap stocks, long–term Treasury bonds, intermediate-term Treasury bonds, and treasury bills for different levels of investor preferences for high expected returns. The procedure entails first computing the means and covariance matrix from actual sample of historical returns of the five asset classes. The sample size is set to be 10 years. The optimization is performed for five different levels of risk preferences: (1) minimum, (2) conservative, (3) moderate, (4) aggressive, and (5) maximum. We calculate minimum variance subject to maximum return, which is the highest mean return of asset class for the given period, by changing the weight of portfolio. Then we calculate returns on conservative, moderate, and aggressive portfolio plans. We set the risk aversion coefficient (A) as 1, 5, and 10 and keeping them constant we calculate expected returns, where A=1 is aggressive risk aversion coefficient and A=10 as conservative. We simultaneously calculate the utility ($U= E(R) - 1/2\sigma^2A$) and expected returns.

The use of rolling samples from these diverse data sets is intended to reveal the sampling distribution characteristics of these two models in small samples with different short-term forecasting horizons. The issue of possible differences in sampling distribution properties between the estimates of the arithmetic mean and the sum of the geometric mean plus half the variance is the underlying reason for questioning the substitution of the first statistic for the second statistic by Bodie, Kane, and Marcus. Additionally, the variation in the impact on forecasting errors from this substitution becomes apparent in the contrasting data sets.

IV. Methodology

To analyze the differences in the sampling distribution characteristics, we perform three types of tests on each of the three data sets. First we test if the estimate of the arithmetic mean is equal to the estimate of the sum of the geometric mean plus half the variance.

Sampling the monthly stock returns of 10 randomly selected companies from February 1995- December 2007, we estimate the arithmetic mean, geometric mean and the variance using 96 rolling samples of 60 months. Using the average of the statistics of the 96 rolling samples, we calculate the difference between the estimate of the arithmetic mean and the estimate of the sum of the geometric mean and half the variance. We perform similar test on the second sample covering five asset classes (Treasury bills, intermediate–term Treasury bonds, long-term

Treasury bonds, large cap stocks, and small cap stocks). The estimation period is 15 rolling years starting with the first sample covering 1926-1940. We estimate the arithmetic mean, geometric mean and the variance using rolling samples which gives us 56 estimates. We conduct similar test on the third data set of returns generated from optimized portfolios over 46 rolling samples from 1936-1995.

Next, we forecast future returns using Jacquier, Kane, and Marcus (2003) proposed weighted average and Bodie, Kane and Marcus (2008) with the substitution of the sum of the geometric mean and half the variance for the arithmetic mean. For the first data set, we calculate monthly forecast returns over 1 year and 7 year horizons, based on 5-year estimation periods for 10 randomly selected individual company stocks returns. Two forecast periods are chosen to see the influence of the difference between the forecast and estimation periods on the forecasted returns as the models by Jacquier, Kane, and Marcus (2003) and Bodie, Kane, and Marcus (2008) are weighted averages of geometric and arithmetic means with the weights measured by the relative importance of the forecast and estimation periods. For the second data set, using 15 year-estimation periods with annual returns of the five asset classes, we forecast future returns over a short term horizon of 10 years, and a long term horizon of 20 years. By using 15- year estimation periods, we end up with 56 rolling samples for the 10-year forecasts and the 20-year forecasts. Similar estimation period short term (10 years) and long term (20 years) forecast horizons are used for the third data set comprising optimized portfolio annual rates of returns. We have 36 rolling samples for the short-term and the long term forecasts. We calculate the difference of the sample mean forecasts of the two models and perform a t-test for statistical significance in the difference. This test allows us to determine whether both models yield similar forecasts.

For the third hypothesis of the economic significance of any difference in the forecasts of the two models, our criterion is that one model has low forecasting error when the other has high forecasting error. The procedure for testing this hypothesis is to first estimate the forecasting errors using Jacquier, Kane and Marcus (2003) formula and actual returns and forecasting error using Bodie, Kane, and Marcus (2008) formula and actual returns. We apply this procedure to all data sets. We test for economic significance in the difference in forecasts over long-term and short-term time horizons. We want to explore small sample properties in different time horizons.

V. Results

Table 1 reports the results pertaining to the first hypothesis whether it is appropriate to substitute the arithmetic mean for the sum of the geometric mean plus half the variance. We test this hypothesis with a series of t-tests of the difference between the estimate of the arithmetic mean of rates of return and the estimates of the sum sample geometric mean plus half the variance. With the first data set covering monthly returns of ten randomly selected individual stocks, nine out of the ten there was statistically significant difference in the estimates. With the second data set, we find statistically significant difference between the estimates using annual rates of return of five asset classes. With the third data set, we find statistically significant difference between the estimates using annual rates of return generated from five optimized portfolios with varying degrees of risk aversion. Therefore, based on these statistically significant differences in three different samples, we conclude that the arithmetic mean is not equivalent to the sum of the geometric mean plus half the variance in small samples.

Table 2 shows the results of the second hypothesis whether the forecasts generated from the two models are the same. We test this hypothesis with a series of t-tests in the mean difference in the forecasts. With the first data set, eighteen out of twenty differences in forecasts were statistically different. With the second and the third data set, all the differences in the forecasts, short-term and long-term, are statistically significant. Therefore, based on these statistically significant differences in three different samples, we conclude that the two models yield different forecasts.

Table 3 reports the results for the hypothesis of the economic significance of any difference in the forecasts of the two models. For economically significant difference in the forecasting models, our criterion is that one model has low forecasting error when the other has high forecasting error. For data set one, the estimation period is 5 years and the forecast periods are 1 and 7 years. The two forecasting errors are strikingly similar. Over the short-term forecast horizon, the difference between the actual return and forecast is statistically significant for five individual stocks. However, over the longer horizon forecast period, 7 years, nine out of ten individual stock returns forecast error is statistically significant and negative (Table 3a). For data set two, using five asset classes annual rates of return, the forecasting errors are negative and statistically significant for the five asset classes for the short-term forecast (10 years). Similar results are found for the long-term forecasts (20 years) except that the forecasting error for small stock is no longer statistically significant (Table 3b). The third data set, using returns from optimized portfolios, the forecasting errors are statistically significant for three out of the five optimized portfolios. Overall, by the criterion stated above, the two models forecasting ability has no economically significant difference.

V. Conclusion

In conclusion, we evaluated the differences between two competing forecasting models of rates of return. The first model by Jacquer, Kane, and Marcus (2002) is a weighted mean between the geometric mean and the sum of the geometric mean and half the variance, where the weights are determined by the relative importance of the estimation period and the forecasting period. The second model by Bodie, Kane, and Marcus (2008) is a similar model except the arithmetic mean is substituted for the sum of the geometric mean and half the variance. Academics and practitioners are interested in choosing between these two competing models.

Theoretically, in large samples with normally distributed returns, the arithmetic mean is exactly equal to the sum of the geometric mean and half the variance. So, in theory, there should be no difference between these two forecasting models. However, when we used small samples of actual rates of returns from three different data sets, (ten individual stocks, five asset classes, and five optimized portfolios), our analysis finds there is a statistically significant difference between estimates of the arithmetic mean and estimates of the sum of the geometric mean and half the variance. This difference between these estimates results in a statistically significant difference in forecasts generated by the two models. Looking at forecasting errors of the two models in a variety of data, we find that the forecasting errors are very similar, and that generally when one model works well, so does the other. So while there are statistically significant differences in forecasts of these two models, there is no economically significant difference in their forecasting errors. As the second model is more compact, simpler, and performs as well as the first, it is the preferable forecasting model to use.

Table I

Test for the difference between the sum of the geometric mean plus half the variance and arithmetic mean of the rates of return, (*t* test statistics with significance in italics).

$$DIFFERENCE = (G + 1/2\sigma^2) - \mu$$

Data Set One

For ten individual companies, the mean difference in monthly percent return.

AIG	T	BA	BAC	DELL	DIS	GE	IBM	TXN	XOM
0.01 *15.10****	0.013 *26.17****	-0.013 *-10.3****	-0.001 *-3.36****	0.060 *14.35****	0.001 *1.23*	0.01 *10.99****	0.023 *21.8****	0.037 *8.95****	0.01 *30.26****

Data Set Two

For five asset classes, the mean difference in annual percent return.

Treasury Bills	Intermediate-Term Treasury Bonds	Long-Term Treasury Bonds	Large Stock	Small Stock
0.0023 *6.27****	0.0263 *5.86****	0.0298 *6.95****	0.2245 *17.36****	1.9810 *8.10****

Data Set Three

For five optimized portfolios, the mean difference in annual percent returns.

Minimum	Conservative	Moderate	Aggressive	Maximum
0.002 *5.81****	0.015 *18.05****	0.018 *13.26****	0.041 *15.0****	0.038 *16.2****

Table II

Test for the difference between forecasted return by the original forecasting model by Jackier, Kane, and Marcus minus the simplified forecasting model by Bodie, Kane, and Marcus, (*t* test statistics with significance in italics).

$$DIFFERENCE = [\ G\ (F/E) + (\ G + 1/2\sigma^2\ )(\ 1 - (F/E)\ )\ ] - [\ G\ (F/E) + \ \mu\ \ (\ 1 - (F/E)\ )\ ]$$

Data Set One

For ten individual companies, the difference of forecasts based on monthly percent return.

Short-term forecast

AIG	T	BA	BAC	DELL	DIS	GE	IBM	TXN	XOM
0.005 *15.1****	0.011 *31.57****	-0.013 *-10.3****	-0.002 *-4.06****	0.046 *14.35****	0.001 *-1.23*	0.006 *13.25****	0.020 *26.28****	0.030 *10.8****	0.005 *30.26****

Long-term forecast

AIG	T	BA	BAC	DELL	DIS	GE	IBM	TXN	XOM
-0.0025 *-15.1****	-0.0054 *-31.5****	0.0052 *10.27****	0.001 *4.06****	-0.043 *-14.3****	-0.0004 *1.23*	-0.0028 *-13.2****	-0.010 *-26.3****	-0.0148 *-10.8****	-0.002 *-30.2****

Data Set Two

For five asset classes, the difference of forecasts of annual percent return.

Short-term forecast

Treasury Bills	Intermediate-Term Treasury Bonds	Long-Term Treasury Bonds	Large Stock	Small Stock
0.0008 *6.27****	0.0088 *5.86****	0.0099 *6.95****	0.0748 *17.36****	0.6603 *8.10****

Long-term forecast

Treasury Bills	Intermediate-Term Treasury Bonds	Long-Term Treasury Bonds	Large Stock	Small Stock
-0.0008 *-6.27****	-0.0088 *-5.86****	-0.0099 *-6.95****	-0.0748 *-17.36****	-0.6603 *-8.10****

Data Set Three

For five optimized portfolios, the difference of forecasts of annual percent return.

Short-term forecast

Minimum	Conservative	Moderate	Aggressive	Maximum
0.0003 *5.08****	0.0044 *17.8****	0.0047 *16.0****	0.0122 *12.9****	0.0111 *13.1****

Long-term forecast

Minimum	Conservative	Moderate	Aggressive	Maximum
-0.0003 *-10.5****	-0.0049 *-19.4****	-0.0048 *-13.7****	-0.0127 *-10.4****	-0.0117 *-10.5****

Table III (a)

The forecasting error of the original forecasting model by Jacquier, Kane, and Marcus and the simplified forecasting model by Bodie, Kane, and Marcus.

$$ERROR\ OF\ ORIGINAL\ MODEL = [\ G\ (F/E) + (\ G + 1/2\sigma^2\ )(\ 1 - (F/E)\ )\ ]\ - ACTUAL$$

$$ERROR\ OF\ SIMPLIFIED\ MODEL = [\ G\ (F/E) + \ \mu\ \ (\ 1 - (F/E)\ )\ ]\ - ACTUAL$$

Data Set One

For ten individual companies, the mean forecasting error based on monthly percent rate of return (t statistic in parenthesis)

Short-term forecast

Original model forecasting error

AIG	T	BA	BAC	DELL	DIS	GE	IBM	TXN	XOM
1.22 *5.04****	0.10 *0.32*	-0.24 *-0.80*	0.02 *0.15*	3.16 *7.18****	0.30 *1.01*	1.09 *3.69****	1.15 *4.29****	2.48 *4.80****	-0.07 *-0.36*

Simplified model forecasting error

AIG	T	BA	BAC	DELL	DIS	GE	IBM	TXN	XOM
1.22 *5.06****	0.09 *0.29*	-0.23 *-0.76*	0.01 *0.13*	3.10 *7.10****	0.3 *1.01*	1.09 *3.67****	1.13 *4.21****	2.45 *4.76****	-0.07 *-0.39*

Long-term forecast

Original model forecasting error

AIG	T	BA	BAC	DELL	DIS	GE	IBM	TXN	XOM
2.64 *24.6****	0.97 *15.3****	-0.12 *-0.85*	-0.25 *-2.45***	6.05 *13.8****	1.06 *9.10****	3.00 *39.4****	2.44 *19.4****	3.96 *13.0****	0.58 *12.6****

Simplified model forecasting error

AIG	T	BA	BAC	DELL	DIS	GE	IBM	TXN	XOM
2.64 *24.5****	0.97 *15.4****	-0.13 *-0.85*	-0.26 *-2.49***	6.09 *13.9****	1.07 *9.13****	3.00 *39.5****	2.45 *19.5****	4.00 *13.1****	0.58 *12.7****

Table III (b)

The forecasting error of the original forecasting model by Jacquier, Kane, and Marcus and the simplified forecasting model by Bodie, Kane, and Marcus.

$$ERROR\ OF\ ORIGINAL\ MODEL = [G\ (F/E) + (G + 1/2\sigma^2)(\ 1 - (F/E)\ )\ ] - ACTUAL$$

$$ERROR\ OF\ SIMPLIFIED\ MODEL = [G\ (F/E) + \mu\ \ (1 - (F/E))] - ACTUAL$$

Data Set Two

For five asset classes, the forecasting error of annual percent returns.

Short-term forecast

Original model forecasting error

Treasury Bills	Intermediate-Term Treasury Bonds	Long-Term Treasury Bonds	Large Stock	Small Stock
-2.02 *-8.04****	-1.99 *-4.75****	-1.57 *-3.01***	-1.83 *-1.45*	3.17 *1.99**

Simplified model forecasting error

Treasury Bills	Intermediate-Term Treasury Bonds	Long-Term Treasury Bonds	Large Stock	Small Stock
-2.02 *-8.05****	-2.00 *-4.76****	-1.58 *-3.02***	-1.91 *-1.51*	2.39 *1.49*

Long-term forecast

Original model forecasting error

Treasury Bills	Intermediate-Term Treasury Bonds	Long-Term Treasury Bonds	Large Stock	Small Stock
-3.17 *-14.65****	-2.40 *-5.74****	-1.62 *-2.66***	-1.41 *-1.02*	-1.61 *-0.93*

Simplified model forecasting error

Treasury Bills	Intermediate-Term Treasury Bonds	Long-Term Treasury Bonds	Large Stock	Small Stock
-3.17 *-14.65****	-2.39 *-5.74****	-1.62 *-2.65***	-1.32 *-0.95*	-0.74 *-0.43*

Table III (c)

The forecasting error of the original forecasting model by Jacquier, Kane, and Marcus and the simplified forecasting model by Bodie, Kane, and Marcus.

$$\textit{ERROR OF ORIGINAL MODEL} = [G\,(F/E) + (G + 1/2\sigma^2)(\,1 - (F/E)\,)\,] - \textit{ACTUAL}$$

$$\textit{ERROR OF SIMPLIFIED MODEL} = [G\,(F/E) + \mu\,(1 - (F/E))\,] - \textit{ACTUAL}$$

Data Set Three

For five optimized portfolios, the forecasting error of annual percent return.

Short-term forecast

Original model forecasting error

Minimum	Conservative	Moderate	Aggressive	Maximum
-2.55 *-8.46****	-0.82 *-0.96*	-0.17 *-0.16*	2.08 *2.18***	3.14 *3.93****

Simplified model forecasting error

Minimum	Conservative	Moderate	Aggressive	Maximum
-2.55 *-8.46****	-0.82 *-0.96*	-0.17 *-0.16*	2.07 *2.17***	3.13 *3.92****

Long-term forecast

Original model forecasting error

Minimum	Conservative	Moderate	Aggressive	Maximum
-3.15 *-9.60****	1.05 *1.67*	2.44 *2.86***	5.06 *7.29****	5.37 *7.20****

Simplified model forecasting error

Minimum	Conservative	Moderate	Aggressive	Maximum
-3.15 *-9.59****	1.05 *1.68*	2.44 *2.87***	*5.07* *7.30****	*5.38* *7.18****

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