

Individual Stock Returns Volatility and Equity Anomalies

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Abstract

We examine the explanatory power of stock return volatility on well-documented equity anomalies. We estimate the time-varying volatility of stock returns using the EGARCH model. Using the estimated stock return volatility, we first form a volatility factor and incorporate it into a wide range of asset pricing models. Second, we adjust the loadings of risk factors in these asset pricing models. We then assess the impact of these two volatility-associated model modifications in mitigating equity anomalies. Our findings indicate that the volatility modifications enhance the explanatory power of asset pricing models. Notably, momentum effects are reduced when the volatility factor is applied alone. Size, value, and short-term reversal anomalies are mitigated when factor loadings are adjusted for volatility where the impacts are more pronounced when both volatility adjustments are applied. Regarding liquidity anomaly, we observe positive effects when both volatility adjustments are incorporated into more sophisticated models. Our results remain robust across two sub-sample periods—covering the latest 20 years and periods of economic recession.

Keywords: Stock Returns Volatility, Equity Anomalies, Momentum Effects, EGARCH Model

JEL Classifications: G12, G14

I. Introduction

The capital asset pricing model (CAPM) developed by Sharpe (1964), Lintner (1965), and Black (1972) has long been fundamental in asset pricing literature. However, subsequent research has challenged the notion that average stock returns can be fully explained by the market risk of CAPM (β) alone. As a result, numerous studies have explored additional risk factors and non-risk firm-specific factors to enhance the explanatory power of CAPM.¹ Fama and French, in their studies from 1992, 1993, and 1996, argue that firm-specific variables essentially scale stock prices and that well-specified risk factors should account for their effects. They introduced two market-wide risk factors, size (ME) and book-to-market equity (BE/ME), resulting in the SMB (small minus big) and HML (high minus low) factors, respectively. Fama and French claimed that their three-factor model, which includes market beta, captures most equity market anomalies overlooked by CAPM, except for the momentum effects identified by Jegadeesh et al. (1990, 1993).² However,

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¹ Such as size (Banz, 1981), leverage (Bhandari, 1988), book value of common equity (Rosenberg et al., 1985), book to market equity ratio (Chan et al., 1991), earnings-price ratios (Basu, 1977; Ball, 1978).

² Behavior explanations have been more devoted to explaining the momentum effects. See, for example, Barberis et al. (1998, conservatism bias), Daniel et al. (1998, self-attribution biased overconfidence), Grinblatt et al. (2005,

subsequent research has reported persistent market anomalies even after incorporating additional or alternative risk factors.

This paper is part of ongoing efforts to better explain stock return movements and mitigate existing equity market anomalies. Instead of searching for indirect variables linked to stock return movements, we focus directly on the individual stock return. Specifically, we examine the standard deviation as a direct risk measure of individual stock returns and investigate how its predictive power can mitigate well-documented equity market anomalies. Under the efficient market hypothesis, all publicly available information is reflected in stock prices, meaning the degree of stock return fluctuation should incorporate all firm-specific information and investor behavior biases, which are critical sources contributing to equity market anomalies.

Considerable research has explored why stock return volatility matters. Levy (1978) argued that stock return volatility could be a key factor within the CAPM framework, while Merton (1987) demonstrated that under imperfect information, volatility affects expected returns due to idiosyncratic risk. Campbell et al. (1993) and Campbell (1996) showed within the Intertemporal Capital Asset Pricing Model (ICAPM) framework that stock return volatility is significant because risk-averse investors hedge against changes in volatility, as it positively affects future expected stock returns. Ang et al. (2006) examined the pricing of aggregate volatility risk in the cross-section of stock returns and concluded that securities with high sensitivities to volatility risk hedge against substantial market declines. More recent studies by Cremers et al. (2015), Jordan et al. (2015), Daniel and Moskowitz (2016), Moreira et al. (2017), and Baltussen et al. (2018) further supported volatility risk as a priced factor and explored its implications on stock return movements. Additionally, several studies have examined the direct impact of individual stock return volatility on equity anomalies. For instance, Campbell et al. (2018) show that incorporating stochastic volatility into the ICAPM helps to explain some equity anomalies.³

We investigate the explanatory power of the standard deviation of stock returns on the cross-sectional differences in average stock returns. To capture the time-varying nature of stock return volatility, we use the exponential generalized autoregressive conditional heteroskedasticity (EGARCH) model, specifically EGARCH (1,1) with ARCH (1,0) as the mean equation. The estimated volatility is then applied to a range of asset pricing models in two ways. First, a volatility factor is developed using the Fama and French procedure of sorting individual stocks into 2x3 portfolios based on size and volatility, and it is added to tested asset pricing models: CAPM, the Fama and French three-factor model, the Fama and French three-factor plus liquidity and momentum factors, and the Fama and French five-factor model. Second, we condition and allow the factor loadings of asset pricing models to vary according to individual stock return volatility, thus individualizing market-wide risk factors to better incorporate stock-specific return variations, which helps alleviate equity anomalies. We then examine whether the volatility factor and the conditional beta loadings can enhance the explanatory power in addressing well-documented equity anomalies, specifically size, value, liquidity, and past return-initiated anomalies (momentum and short-term reversal).

disposition effect), and Hirshleifer et al. (2009, investors' limited ability to attention).

³ Campbell et al. (2018) estimated individual stock return volatility using a Vector Auto Regression (VAR) model, which is designed to capture low-frequency movements in volatility. They then integrated this stochastic volatility into the ICAPM framework. Their findings indicate that the three-beta ICAPM, incorporating stochastic volatility, outperforms the traditional two-beta ICAPM in explaining equity anomalies, such as growth versus value stocks and the low returns associated with certain beta exposures.

We focus on individual stocks rather than portfolios and options because this approach allows market-based risk factors to vary with individual stock-level features. Ang et al. (2020) showed that using portfolios to mitigate estimation errors in risk factor loadings does not necessarily result in smaller standard errors of cross-sectional coefficient estimates, as grouping stocks can destroy information contained in individual stock factor loadings.⁴ Studying individual stocks also addresses the information loss inherent in portfolio-based asset pricing tests (Litzenberger and Ramaswamy, 1979) and accounts for data-snooping biases (Lo and MacKinlay, 1990). Additionally, using individual stock returns rather than options for volatility estimation enables the study to investigate sensitivities to market-wide volatility factors and introduce various risk factors within the same model.

We observe offsetting relationships between individual stock return volatility and all tested anomaly constituents, suggesting that volatility may play a role in mitigating equity anomalies. Examining the impact of the two volatility-related modifications, we find that when the volatility factor is applied alone, it effectively reduces momentum effects and improves the size anomaly. However, it also exacerbates liquidity and short-term reversal anomalies, while the results for the value anomaly remain inconsistent. Beta conditioning enhances the models' explanatory power for size, value, and short-term reversal anomalies, but does little to address liquidity and momentum anomalies. When both volatility adjustments are applied together, the models show the most substantial improvements in mitigating size, value, and short-term reversal anomalies. However, in most cases, momentum effects worsen. For the liquidity anomaly, we observe positive effects when both volatility adjustments are incorporated into more sophisticated models—such as the Fama and French three-factor model with added liquidity and momentum factors, as well as the five-factor model.

We contribute to the asset pricing literature by exploring methods to mitigate equity anomalies, building on prior research that has expanded the foundational CAPM model. Since the introduction of the CAPM, numerous studies have shown that incorporating additional risk factors enhances its explanatory power (e.g., Fama and French, 1992, 1993, 1996; Jegadeesh et al., 1990, 1993; Pastor and Stambaugh, 2003). Several studies have also examined how stock return volatility can help address equity anomalies (Levy, 1978; Merton, 1987; Campbell et al., 1993, 1996; Ang et al., 2006; Cremers et al., 2015; Jordan et al., 2015; Daniel and Moskowitz, 2016; Baltussen et al., 2018; Campbell et al., 2018). In line with this body of research, we apply two volatility-related modifications to various asset pricing models and find that these adjustments effectively mitigate equity anomalies, depending on the application. Specifically, adding a volatility factor is most effective in addressing the momentum anomaly, while beta conditioning proves useful in mitigating size, value, and short-term reversal anomalies. Liquidity anomaly is effectively mitigated when both volatility adjustments are applied to more sophisticated models. Moreover, we show that these adjustments in volatility are effective across different market segments and under varying economic conditions, which highlights the importance of including volatility adjustments when analyzing stock returns.

The rest of this paper is organized as follows. Section II describes the methodology, Section III details the sampling process and provides summary statistics, Section IV analyzes the data and reports empirical findings, Section V provides robustness tests, and Section VI concludes.

⁴ See Jegadeesh et al. (2019) for further arguments on the shortcomings of using portfolios to address the errors-in-variables (EIV) problem.

II. Methodology

The asset pricing tests in this study are based on the monthly absolute alphas from various equity anomaly-initiated investment strategies. Zero-investment portfolios are constructed by longing the top quintile of stocks and shorting the bottom quintile within a given month. If the asset pricing models adjusted with stock return volatility measures are effective, we should observe a decrease in these absolute alphas. To estimate the key variable, individual stock return volatility, we use the EGARCH model, which accounts for the time-varying and asymmetric properties of stock return volatility.⁵ This volatility characteristic of individual stock returns is applied to the asset pricing models in two ways: first, by developing a volatility factor using the Fama and French procedure, and second, by allowing the beta loadings of risk factors in each asset pricing model to vary with individual stock return volatility.

We then examine whether the volatility factor and the conditional beta loadings can enhance the explanatory power in addressing well-documented equity anomalies. In addition, previous literature suggests that returns of NASDAQ stocks differ from returns of NYSE-AMEX stocks, we conduct our analyses separately for a sample that only contains NASDAQ stocks (Avramov and Chordia, 2006).

Time series monthly absolute alphas and beta loadings

To compute time series monthly absolute alphas, we run time-series regressions of individual stocks' excess returns on various asset pricing models, recording alphas (realized excess returns minus model-predicted excess returns) for each stock. Each month, we sort individual stocks by equity anomaly constituents—size, value, liquidity, momentum, and short-term reversals—into five portfolios with equal numbers of stocks. The top portfolio contains the 20% of stocks with the highest anomaly attributes, and the bottom portfolio contains the 20% with the lowest. We then calculate time series average absolute alphas by constructing zero-investment portfolios that long the top portfolio and short the bottom portfolio.

Beta loadings of each risk factor are estimated on a rolling basis over 60 months. Specifically, from July 1963 to June 2018, we use the previous 60 months of stock return data to estimate beta loadings for each risk factor, following the methodology of Fama and French (1992). For stocks with less than 60 months of prior data, we require at least 24 return observations, starting from the 25th month. For instance, for a stock with return data starting in July 1963, the rolling estimation process begins in July 1965 with 24 return observations. In August 1965, factor loadings are estimated using 25 observations, and this continues until the rolling window reaches 60 months, after which the duration remains constant at 60 months.

Volatility estimation: EGARCH model

Due to the time-varying properties of stock return variance—such as clustering, mean-reversion, and leptokurtic distribution—we estimate stock return variance using the EGARCH model rather than relying solely on realized past return variance.

After Engle (1982) introduced the autoregressive conditional heteroskedasticity (ARCH) model to capture time-varying volatility, Bollerslev (1986) extended it to the generalized ARCH

⁵ We calculate individual stock return volatility using the previous 60 months of monthly returns.

(GARCH) model, which incorporates a more dynamic structure of conditional variances, providing a more flexible framework. Nelson (1991) further advanced this by developing the Exponential GARCH (EGARCH) model, designed to capture the asymmetric effects of shocks (e.g., policy changes, newly announced information) in the stock market by differentiating the impacts of good and bad news. EGARCH model posits that good news tends to stabilize stock returns and reduce volatility, while bad news tends to destabilize returns and increase volatility. Pagan and Schwert (1990) found Nelson’s EGARCH model to be the best fit for estimating monthly U.S. stock return volatility, which is further supported by the evidence of Engle and Ng (1991). We employ a straightforward EGARCH framework: EGARCH (1,1) with ARCH (1,0) as the mean equation. The functional forms are as follows:

$$R_{i,t} = \alpha_1 R_{i,t-1} + \mu_{i,t}; \mu_{i,t} \sim N(0, \sigma_{i,t}^2) \quad (1)$$

$$\ln \sigma_{i,t}^2 = \alpha_i + \sum_{i=1}^q \omega_{i,t} \left| \frac{\mu_{i,t-1}}{\sqrt{\sigma_{i,t-1}^2}} \right| + \sum_{i=1}^q \gamma_{i,t} \left| \frac{\mu_{i,t-1}}{\sqrt{\sigma_{i,t-1}^2}} \right| + \sum_{k=1}^p \theta_k \ln \sigma_{i,t-k}^2 \quad (2)$$

where $\omega_{i,t}$ represents the ARCH effects, $\gamma_{i,t}$ captures the asymmetric effects, θ_k denotes the GARCH effects, and the logarithm of variance, $\ln \sigma_{i,t}^2$, makes the leverage effect exponential rather than the quadratic as in the standard GARCH model.

Volatility factor

To construct a volatility factor, we follow the Fama and French procedure. First, we sort all individual stocks for a given month into two size groups and three volatility groups, with breakpoints calculated using NYSE stocks. The volatility breakpoints are set at the 30th and 70th percentiles of the individual stocks’ one-month lagged standard deviation. This 2x3 sorting on size and volatility results in six portfolios. The volatility factor is then computed as the equal-weighted average of the returns of the two high-volatility portfolios minus the equal-weighted average of the returns of the two low-volatility portfolios.

$$\begin{aligned} \text{Volatility factor} & \quad (3) \\ &= \frac{1}{2} (\text{small size high volatility} + \text{big size high volatility}) \\ & - \frac{1}{2} (\text{small size low volatility} + \text{big size low volatility}) \end{aligned}$$

Conditional beta framework: varying individual stock level beta loadings

Conditioning the beta loadings of risk factors in asset pricing models involves tailoring market-

wide risk factors to better reflect individual stock-level characteristics. This approach allows asset pricing models to more effectively incorporate fluctuations in stock returns, potentially reducing deviations between realized and model-predicted excess returns. Equity anomalies—systematic deviations documented in stock returns that correlate with certain firm characteristics or past returns—are among these discrepancies. By adjusting risk factor loadings to account for key stock-level features, modified asset pricing models can improve their ability to explain these anomalies.

We follow the beta-varying methodology outlined by Avramov and Chordia (2006), allowing the factor loadings in asset pricing models to vary based on individual stocks' one-month lagged standard deviation.⁶ The beta scaling framework is illustrated as follows:

$$r_{j,t} = \alpha_j + \sum_{i=1}^N \beta_{j,(2i-1)} F_{k,t} + \sum_{i=1}^N \beta_{j,(2i-1)} F_{k,t} Sd_{j,(t-1)} + \mu_{j,t} \quad (4)$$

Where $r_{j,t} = R_{i,t} - R_{F,t}$ the excess return of the individual stocks, $F_{k,t}$ denotes various risk factors, and $Sd_{i,(t-1)}$ is the one-month lagged standard deviation of individual stocks. For example, applying this conditional beta modeling to the Fama-French three-factor model, the time-varying factor loadings can be specified as follows:

$$r_{j,t} = \alpha_j + (\beta_{j1} + \beta_{j2} Sd_{j,(t-1)}) \times r_{m,t} + (\beta_{j3} + \beta_{j4} Sd_{j,(t-1)}) \times SMB_t + (\beta_{j5} + \beta_{j6} Sd_{k,(t-1)}) \times HML_t \quad (5)$$

Alternatively, this can be expressed as:

$$r_{j,t} = \alpha_j + \beta_{j1} r_{m,t} + \beta_{j2} r_{m,t} \times Sd_{j,(t-1)} + \beta_{j3} SMB_t + \beta_{j4} SMB_t \times Sd_{j,(t-1)} + \beta_{j5} HML_t + \beta_{j6} HML_t \times Sd_{j,(t-1)} \quad (6)$$

where $r_{m,t}$ is the excess return on the value-weighted market index, SMB and HML are size and value factors in the Fama-French three factor model, respectively.

III. Data

We collect monthly stock returns for NYSE, AMEX, and NASDAQ stocks from the Center for Research in Security Prices (CRSP) and firm-specific variables—such as size (market capitalization), book-to-market ratio (book equity divided by market capitalization), and turnover (trading volume divided by total shares outstanding)—from COMPUSTAT. Our sample period spans from July 1963 to June 2019, totaling 672 months.⁷ To be included in the sample for a given

⁶ Avramov and Chordia (2006) allow beta loadings to change as a function of firm-specific variables, such as size and book-to-market ratio, thus extending the framework of Brennan et al. (1998), where rolling regressions enable beta loadings to evolve over time. Avramov and Chordia's (2006) approach also differ from that of Shanken (1990), Ferson and Harvey (1999), and Lettau and Ludvigson (2001) in that the economic quantity obtained by multiplying firm-specific variables by factors does not constitute an additional risk factor.

⁷ Trading volume data of NASDAQ stocks, a variable in computing firm turnover rate, is available after November 1982. Thus, our sample for NASDAQ stocks is from July 1983. We exclude COVID 19 period to mitigate the

month, stocks must meet the following criteria. Stock returns data must be available for at least 100 consecutive months to ensure stable convergence in the EGARCH estimation of individual stock returns volatility. Firm-specific data necessary to calculate key variables, such as size and turnover ratio from CRSP and the book-to-market ratio as of December of the previous year from COMPUSTAT, must be available. After applying these criteria, our sample consists of 7,939 different firms (including 4,556 NASDAQ firms) over the 672-month period, with an average of 2,666 firms (1,883 NASDAQ firms) included per month.

Given the substantial negative skew in our key variables, we use logarithmic transformations for firm-specific variables such as size, book-to-market ratio, and turnover ratio, while stock returns data—including lagged returns for momentum and short-term reversal effects—remain untransformed. Following the methodology of Fama and French (1992), we calculate book equity for July of year $t-1$ to June of year t using accounting data from the end of year $t-2$. Following Brennan et al. (1998) and Avramov and Chordia (2006), firm characteristics and stock return volatility are lagged one month with respect to excess returns. Risk factors—including market portfolio return, SMB, HML, liquidity, momentum, profitability, and investment factors—are obtained from Kenneth R. French's data library.⁸ To mitigate the impact of extreme observations, stock return standard deviation estimates, and stock return measures (such as excess returns and past returns for momentum and short-term reversals) are winsorized at the 0.995 and 0.005 percentiles.

Table 1: Summary Statistics

	Mean	Median	Standard Deviation	Skewness
Panel A: NYSE, AMEX, and NADAQ stocks				
Excess return (%)	0.14	-0.27	10.07	0.03
Ln(Firm size) (\$ million)	2.25	2.18	0.96	0.32
Ln(Book-to-market ratio)	-0.23	-0.20	0.39	-0.73
Ln(Turnover) (%)	1.66	1.67	0.57	-0.25
Ret2_12 (%)	7.44	4.82	36.99	0.18
Ret1 (%)	0.47	0.00	10.05	0.02
Standard Deviation (of returns)	12.41	11.38	5.27	0.49
Panel B: NASDAQ stocks				
Excess return (%)	0.08	-0.31	10.64	0.02

impact of unusual observations.

⁸ <http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/index.html>

Ln(Firm size) (\$ million)	2.05	2.00	0.83	0.35
Ln(Book-to-market ratio)	-0.27	-0.23	0.41	-0.73
Ln(Turnover) (%)	1.73	1.77	0.60	-0.45
Ret2_12 (%)	6.64	3.75	39.51	0.16
Ret1 (%)	0.35	0.00	10.63	0.02
Standard Deviation (of returns)	13.67	13.05	5.68	0.16

This table presents the descriptive statistics of stocks listed on the NYSE, AMEX, and NASDAQ. Panel A includes all stocks in the sample, averaging 2,666 firms per month from July 1963 to June 2019, while Panel B focuses on NASDAQ stocks, averaging 1,883 firms per month from July 1982 to June 2019. Excess return is defined as the monthly raw stock return minus the one-month T-bill rate. Firm size is measured by the market value of equity in millions of dollars. The book-to-market ratio is calculated as the fiscal year-end book value of common equity divided by market equity. Turnover is defined as the monthly share trading volume divided by shares outstanding. Due to the substantial negative skewness of these firm-specific variables, all firm characteristics are presented as natural logarithms. Ret2_12 represents cumulative returns over the second through twelfth months, while Ret1 denotes the previous month's stock return, capturing short-term reversals. Standard deviations of individual stock returns are estimated using the EGARCH model. To mitigate the influence of extreme observations, standard deviation estimates, excess return, Ret2_12, and Ret1 are winsorized at the 0.005 and 0.995 percentiles.

Table 1 provides key descriptive statistics for the entire sample. It highlights differences in firm-specific information and stock return characteristics between NASDAQ stocks and those listed on NYSE and AMEX, supporting the need for separate analyses. Additionally, focusing on individual stocks rather than portfolios reveals that return measures, including standard deviation estimates, exhibit a broader range around the average.

IV. Empirical analysis

This section evaluates the effectiveness of existing asset pricing models by integrating individual stock return volatility, measured as standard deviation, into the analysis. The goal is to assess whether incorporating this direct risk measure helps mitigate well-known equity anomalies. The asset pricing models we examine include: (i) the Capital Asset Pricing Model (CAPM), (ii) the Fama and French three-factor model (1992), (iii) the Fama and French three-factor model augmented with the liquidity factor of Pastor and Stambaugh (2003), (iv) the Fama and French three-factor model augmented with the momentum factor of Jegadeesh and Titman (1993), (v) the Fama and French three-factor model incorporating both liquidity and momentum factors, and (vi) the Fama and French five-factor model (2015). The equity anomalies considered are size (market capitalization), value (book-to-market ratio), liquidity (turnover ratio), momentum (past two-to-twelve-month returns), and short-term reversals (past one-month returns).

Our approach involves estimating individual stock return volatility using the EGARCH model, with these volatility measures incorporated into the asset pricing models either as an additional factor or by conditioning factor loadings on volatility. We then compare the absolute alphas from zero-investment strategies—going long on the top quintile and short on the bottom quintile of stocks sorted by equity anomaly characteristics—against those from the baseline

models. A reduction in absolute alphas would indicate that the volatility-related modifications improve the models' ability to explain the returns associated with equity anomalies.

Table 2: Portfolios Sorted by Volatility

Rank	Size	Value	Liquidity	Momentum	Short-term Reversal
Panel A: NYSE, AMEX, and NADAQ stocks					
1 Low	2.73	-0.20	1.34	11.01	0.57
2	2.65	-0.22	1.48	12.99	0.78
3	2.44	-0.20	1.56	13.05	0.92
4	2.17	-0.19	1.63	13.58	1.05
5 High	1.71	-0.21	1.70	13.87	1.66
Panel B: NASDAQ stocks					
1 Low	2.28	-0.17	1.40	13.57	0.76
2	2.30	-0.24	1.67	15.71	0.89
3	2.14	-0.28	1.81	15.60	0.98
4	1.96	-0.32	1.87	14.31	1.13
5 High	1.67	-0.36	1.89	14.63	2.43

This table presents equal-weighted quintile portfolios for each equity anomaly constituent, sorted by individual stock return volatility (standard deviation). Panel A includes all stocks in the sample from July 1963 to June 2019, while Panel B focuses on NASDAQ stocks from July 1982 to June 2019. Individual stock return volatility is estimated using the EGARCH model. Portfolios are rebalanced monthly, and the statistics in each column represent the time-series averages over the entire sample period. Rank 1 corresponds to portfolios of stocks with the lowest volatilities, while Rank 5 corresponds to those with the highest volatilities. Size is measured by the log of market capitalization; Value represents the log of the book-to-market ratio; Liquidity is defined as the monthly turnover ratio; Momentum denotes cumulative returns over the last twelve months, excluding the previous month; and Short-term Reversal refers to the stock return from the past month.

Table 2 presents time-series averages of equal-weighted quintile portfolios sorted by individual stock return volatility. The results show that stock return volatility significantly reduces the magnitude of equity anomalies. In Panel A, Column 2 reveals a negative relationship between firm size and volatility, where higher volatility is associated with a smaller average firm size, implying that strategies targeting the size anomaly may generate positive alphas but come with higher risk. Similarly, Column 4 shows that higher momentum (past two-to-twelve-month returns) is linked to increased stock return volatility, indicating that momentum-based strategies, while potentially profitable, carry higher risk. This trade-off between alpha generation and risk is observed across all anomaly factors. Panel B, which focuses on NASDAQ stocks, supports these

findings with only minor variations in the magnitude of the relationships. The results suggest incorporating individual stock return volatility into asset pricing models substantially mitigates equity anomalies. In the following subsections, we will explore how individual stock return volatility interacts with each asset pricing model in more detail.

The capital asset pricing model (CAPM)

Table 3 presents the average monthly absolute alphas from zero-investment strategies based on the CAPM for various equity anomalies. The first column reports results using the baseline CAPM, which assumes fixed betas without any volatility adjustments. The second column incorporates a volatility factor into the CAPM. In the third column, the CAPM allows betas to vary with the standard deviation of stock returns. The fourth column shows the results from a CAPM model that includes both a volatility factor and scaled factor loadings based on stock return volatility.

Column 1 of Table 3 reveals that the average monthly absolute alphas from the baseline CAPM are significantly different from zero, suggesting that all the tested equity anomalies can generate considerable excess returns when stock returns are priced solely on market beta. However, introducing a volatility factor in Column 2 leads to noticeable improvements in the model's ability to capture some of the anomalies. For instance, the absolute alpha for the size anomaly decreases from 0.7023 to 0.4348, for the value anomaly from 0.9809 to 0.8796, and for the momentum anomaly from 0.4928 to 0.4012. On the other hand, the volatility factor exacerbates certain anomalies, particularly liquidity and short-term reversal, with the absolute alphas increasing from 0.0178 to 0.1467 for liquidity and from 1.4959 to 1.6928 for short-term reversal.

Conditioning beta loadings, as shown in Column 3 of Table 3, helps reduce the absolute alphas for the size and value anomalies to 0.6742 and 0.8579, respectively, with a noticeable decrease in the short-term reversal anomaly to 1.4385. However, this approach worsens the momentum anomaly, increasing the absolute alpha to 0.6163, and the liquidity anomaly to 0.0527. The improvements in size and value anomalies are more pronounced when both volatility modifications are applied. Column 4 of Table 3 shows a further reduction in absolute alphas, with the size anomaly dropping to 0.3664 and the value anomaly decreasing to 0.6807. Nevertheless, the model still struggles with liquidity, momentum, and short-term reversal anomalies, as these absolute alphas remain elevated.

Panel B, which focuses on NASDAQ stocks, reveals similar patterns. While the baseline approach shows higher average monthly alphas across all anomaly strategies (except for momentum), the effects of the two volatility modifications remain consistent. Adding a volatility factor improves the model's explanatory power for size, value, and momentum anomalies but worsens its performance for liquidity and short-term reversal anomalies. Beta conditioning provides slight improvements for size, value, and short-term reversal anomalies, though it further exacerbates the liquidity and momentum anomalies. When both volatility modifications are applied, the model demonstrates more pronounced positive impacts on size and value anomalies, but continues to exhibit negative effects on liquidity, momentum, and short-term reversal anomalies.

Table 3: Time Series Average Absolute Alphas: the CAPM

Baseline	Volatility Factor	Conditional Betas	Volatility Factor & Conditional Betas
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Panel A: NYSE-AMEX and NASDAQ Stocks				
Size	0.7023 [-21.14]	0.4348 [-20.47]	0.6742 [-22.18]	0.3664 [-20.00]
Value	0.9809 [31.69]	0.8796 [44.86]	0.8579 [30.27]	0.6807 [39.39]
Liquidity	0.0178 [0.57]	0.1467 [-12.4]	0.0527 [1.66]	0.1216 [-10.81]
Momentum	0.4928 [16.51]	0.4012 [28.05]	0.6163 [27.08]	0.5816 [54.78]
Short-term Reversal	1.4959 [-53.59]	1.6928 [-66.02]	1.4385 [-52.51]	1.5661 [-73.72]
Panel B: NASDAQ stocks				
Size	1.3834 [-21.91]	1.3567 [-25.15]	1.2804 [-20.64]	1.0490 [-21.76]
Value	1.5706 [19.5]	1.4362 [27.81]	1.3543 [17.25]	1.0759 [19.46]
Liquidity	0.0451 [-0.51]	0.1767 [-4.48]	0.0746 [-0.85]	0.1796 [-5.09]
Momentum	0.0386 [0.37]	0.0111 [0.15]	0.3067 [3.25]	0.3449 [5.89]
Short-term Reversal	1.697 [-22.74]	1.8932 [-32.13]	1.6309 [-20.34]	1.8067 [-35.75]

This table reports the average monthly absolute alphas from well-documented equity anomaly strategies derived from the Capital Asset Pricing Model (CAPM). Panel A includes all stocks from July 1963 to June 2019, while Panel B focuses on NASDAQ stocks from July 1982 to June 2019. The second column ("Baseline") reports time-series absolute alphas using the CAPM; the third column adds a volatility factor; the fourth column allows the CAPM beta to vary with individual stock return standard deviation; and the final column augments the CAPM with a volatility factor while also allowing the beta to be conditional on volatility. The Newey-West (1987) corrected t-statistics are reported in the bracket.

The Fama and French three factor model

Panel A of Table 4 presents the average monthly absolute alphas from zero-investment strategies based on the Fama and French three-factor model, assessing how well the model captures equity anomalies. The baseline model's performance improves with the addition of two volatility modifications, showing patterns and magnitudes similar to those seen with the CAPM, though with similar challenges.

As with the CAPM, adding a volatility factor helps mitigate size, value, and momentum anomalies, but worsens the model's ability to explain liquidity and short-term reversal anomalies. When the factor loadings for beta, SMB (small-minus-big), and HML (high-minus-low) are conditioned on individual stock return volatility, the modified model further reduces size, value, and short-term reversal anomalies, though it continues to perform worse for liquidity and momentum anomalies. A key difference is that beta conditioning has a larger impact on mitigating size and value anomalies in the Fama and French model compared to the CAPM, where the volatility factor appeared to be more effective. When both volatility modifications are applied, the

improvements are amplified, with roughly half of the abnormal returns from size and value anomalies disappearing. Additionally, the model provides a better explanation for short-term reversal, though it continues to struggle with liquidity and momentum effects.

Table 4: Time series average absolute alphas: the Fama and French 3 Factor Model

	Baseline	Volatility Factor	Conditional Betas	Volatility Factor & Conditional Betas
Panel A: NYSE-AMEX and NASDAQ Stocks				
Size	0.2926 [35.64]	0.2502 [33.44]	0.1618 [25.70]	0.1469 [21.80]
Value	0.6380 [37.27]	0.6075 [60.31]	0.3446 [29.14]	0.2861 [32.34]
Liquidity	0.0144 [0.66]	0.0981 [8.67]	0.0206 [-1.16]	0.0332 [3.77]
Momentum	0.3931 [18.5]	0.3423 [32.66]	0.5796 [44.36]	0.5015 [70.02]
Short-term Reversal	1.6117 [-62.49]	1.7126 [-69.9]	1.4800 [-61.63]	1.3775 [-77.3]
Panel B: NASDAQ stocks				
Size	1.1201 [20.85]	1.0537 [23.65]	0.7865 [17.57]	0.5696 [14.44]
Value	1.2663 [25.54]	1.1833 [31.21]	0.8435 [22.77]	0.6485 [16.58]
Liquidity	0.0288 [-0.55]	0.1375 [4.30]	0.0148 [-0.34]	0.0275 [1.06]
Momentum	0.0437 [0.49]	0.0191 [-0.29]	0.4622 [7.08]	0.3610 [7.01]
Short-term Reversal	1.7829 [-23.2]	1.8934 [-28.18]	1.6075 [-21.84]	1.4235 [-31.95]

This table reports the average monthly absolute alphas from well-documented equity anomaly strategies derived from the Fama and French 3 factor model. Panel A includes all stocks from July 1963 to June 2019, while Panel B focuses on NASDAQ stocks from July 1982 to June 2019. The second column ("Baseline") reports time-series absolute alphas using the Fama and French 3 factor model; the third column adds a volatility factor; the fourth column allows the Fama and French 3 factor model betas to vary with individual stock return standard deviation; and the final column augments the Fama and French 3 factor model with a volatility factor while also allowing the beta to be conditional on volatility. The Newey-West (1987) corrected t-statistics are reported in the bracket.

Panel B shows similar results for NASDAQ stocks, with two notable distinctions. First, the impact of adding the volatility factor on momentum effects is particularly significant, reducing the alpha from 0.0437 to 0.0191. Second, the liquidity anomaly diminishes meaningfully, from 0.0288 to 0.0148 when factor loadings are conditioned on volatility, and to 0.0275 when both volatility modifications are included in the baseline model.

An important observation is the baseline performance of the Fama and French three-factor model, which captures equity anomalies more effectively than the CAPM. For example, comparing the second columns in Panel A of Tables 3 and 4, the absolute alphas for size and value strategies

decline from 0.7023 to 0.2926 and from 0.9808 to 0.6380, respectively, when the SMB and HML factors are added to the CAPM. Similar improvements are observed for liquidity and momentum anomalies, with the exception of short-term reversal. Moreover, when the Fama and French model is enhanced with both volatility modifications, all absolute alphas, including for short-term reversal, are smaller than those in the CAPM with the same modifications. These findings contrast sharply with several previous studies, notably Avramov and Chordia (2006), which argued that the SMB and HML factors in the Fama and French model fail to account for the effects of firm characteristics like size and book-to-market ratio on individual stock returns. Our results suggest that the Fama and French model, particularly when combined with volatility adjustments, is more capable of explaining equity anomalies than previously thought.

The Fama and French three factor model augmented with the liquidity factor

Table 5 presents the average monthly absolute alphas from zero-investment strategies based on the Fama and French three-factor model augmented with a liquidity factor. The first consideration is how the addition of the liquidity risk factor affects each equity anomaly before incorporating stock return volatility adjustments. The baseline results in both Panel A and Panel B demonstrate that the inclusion of the liquidity factor markedly improves the model's explanatory power, providing a stronger ability to account for equity anomalies compared to the Fama and French three-factor model on its own.

Consistent with previous findings, incorporating a volatility factor into the baseline model results in notable improvements for the size, value, and especially momentum anomalies. However, this adjustment leads to a worsening of liquidity and short-term reversal anomalies. When beta loadings are conditioned on volatility, the model shows even greater improvements in explaining size and value anomalies, as well as noticeable reductions in the short-term reversal anomaly. On the other hand, beta conditioning exacerbates liquidity and momentum anomalies. When both volatility modifications are applied together, the model further improves its ability to explain size and value anomalies, but the liquidity and momentum anomalies are still exacerbated. The results for NASDAQ stocks, as shown in Panel B, follow similar patterns, indicating that the volatility-related modifications have consistent effects across different market segments.

Table 5: Time series average absolute alphas: the FF 3 Factor + Liquidity Factor Model

	Baseline	Volatility Factor	Conditional Betas	Volatility Factor & Conditional Betas
Panel A: NYSE-AMEX and NASDAQ Stocks				
Size	0.2491 [31.2]	0.2133 [29.64]	0.1581 [25.98]	0.1090 [16.19]
Value	0.5647 [33.63]	0.5375 [52.46]	0.2559 [22.83]	0.2065 [24.55]
Liquidity	0.0026 [0.12]	0.0720 [6.66]	0.0382 [-2.21]	0.0119 [1.50]
Momentum	0.4190 [19.58]	0.3709 [34.19]	0.5786 [45.98]	0.5470 [76.44]
Short-term Reversal	1.4744 [-61.24]	1.5758 [-69.15]	1.1417 [-52.12]	1.2172 [-73.79]
Panel B: NASDAQ stocks				
Size	1.0435 [19.72]	0.9907 [23.38]	0.6478 [14.92]	0.4593 [11.75]
Value	1.1931 [24.09]	1.1140 [30.25]	0.6382 [16.65]	0.4830 [10.46]
Liquidity	0.0579 [-1.05]	0.1037 [3.35]	0.0693 [-1.62]	0.0229 [-0.79]
Momentum	0.0738 [0.84]	0.0214 [0.32]	0.4052 [5.88]	0.2716 [5.97]
Short-term Reversal	1.597 [-20.97]	1.6912 [-27.53]	1.1853 [-18.32]	1.1979 [-32.61]

This table reports the average monthly absolute alphas from well-documented equity anomaly strategies derived from the Fama and French 3 factor model plus the liquidity factor. Panel A includes all stocks from July 1963 to June 2019, while Panel B focuses on NASDAQ stocks from July 1982 to June 2019. The second column ("Baseline") reports time-series absolute alphas using the Fama and French 3 factor model plus the liquidity factor; the third column adds a volatility factor; the fourth column allows betas in the Fama and French 3 factor model plus the liquidity factor to vary with individual stock return standard deviation; and the final column augments the Fama and French 3 factor model plus the liquidity factor with a volatility factor while also allowing the beta to be conditional on volatility. The Newey-West (1987) corrected t-statistics are reported in the bracket.

The Fama and French three-factor model augmented with the momentum factor

Table 6 presents the average monthly absolute alphas from zero-investment strategies of anomaly constituents, based on the Fama and French three-factor model plus the momentum factor. Notably, the addition of the momentum factor does not enhance the model's explanatory power; in fact, two of the anomalies worsen. When compared to adding a liquidity factor to the Fama and French three-factor model, the momentum factor generally performs worse at capturing most equity anomalies, with the exception of the momentum effect itself. The baseline column consistently reports higher alphas across both samples (all stocks and NASDAQ stocks), except for the momentum effects. This trend persists throughout all columns, even when models are further adjusted for volatility, with the momentum effects remaining an outlier.

Regarding the two volatility adjustments, we find consistent results. Adding the volatility factor alone to the baseline model reduces size and momentum anomalies but continues to struggle with liquidity and short-term reversal. Unlike the previous sections, adding the volatility factor increases value anomaly. Conditioning on beta alone mitigates size, value, and short-term reversal anomalies, though it exacerbates momentum and liquidity anomalies. Applying both volatility adjustments has the most positive impact on size and value anomalies, with noticeable improvement in the short-term reversal anomaly. However, momentum and liquidity anomalies still increase. These patterns are also evident when examining NASDAQ stocks, as shown in Panel B.

Table 6: Time series average absolute alphas: the FF 3 Factor + Momentum Factor Model

	Baseline	Volatility Factor	Conditional Betas	Volatility Factor & Conditional Betas
Panel A: NYSE-AMEX and NASDAQ Stocks				
Size	0.2897 [33.96]	0.2442 [33.36]	0.1972 [32.21]	0.1111 [16.82]
Value	0.6559 [40.93]	0.6722 [79.4]	0.3817 [38.88]	0.3337 [47.27]
Liquidity	0.0102 [0.52]	0.1093 [11.03]	0.0188 [-1.16]	0.0521 [6.70]
Momentum	0.3959 [17.7]	0.3204 [36.41]	0.5061 [65.29]	0.4505 [84.76]
Short-term Reversal	1.6172 [-66.73]	1.6880 [-55.85]	1.3394 [-54.06]	1.3758 [-63.52]
Panel B: NASDAQ stocks				
Size	1.1281 [20.98]	0.9650 [23.73]	0.6344 [14.89]	0.3669 [9.65]
Value	1.2861 [26.31]	1.2114 [35.77]	0.6680 [18.96]	0.5439 [11.54]
Liquidity	0.0217 [-0.47]	0.1288 [4.82]	0.1313 [-3.64]	0.0252 [-1.12]
Momentum	0.0535 [0.60]	0.0136 [-0.21]	0.4675 [7.82]	0.3159 [7.00]
Short-term Reversal	1.7942 [-25.01]	1.8408 [-24.99]	1.3180 [-23.59]	1.3421 [-32.2]

This table reports the average monthly absolute alphas from well-documented equity anomaly strategies derived from the Fama and French 3 factor plus the momentum factor. Panel A includes all stocks from July 1963 to June 2019, while Panel B focuses on NASDAQ stocks from July 1982 to June 2019. The second column ("Baseline") reports time-series absolute alphas using the Fama and French 3 factor plus the momentum factor; the third column adds a volatility factor; the fourth column allows betas in the Fama and French 3 factor plus the momentum factor to vary with individual stock return standard deviation; and the final column augments the Fama and French 3 factor plus the momentum factor with a volatility factor while also allowing the beta to be conditional on volatility. The Newey-West (1987) corrected t-statistics are reported in the bracket.

The Fama and French three-factor model plus the liquidity and momentum factors

Table 7 presents the average monthly absolute alphas from zero-investment strategies of anomaly constituents, based on the Fama and French three-factor model augmented with both liquidity and momentum factors. The results largely align with previous findings. Baseline results indicate that adding these two risk factors enhances the explanatory power of the Fama and French three-factor model. Momentum effects are most effectively mitigated when the volatility factor alone is introduced. Size and value anomalies are significantly reduced, and short-term reversals shrink noticeably when beta loadings are conditioned. The combined volatility adjustments yield the most pronounced positive impacts. Notably, the abnormal returns associated with size are the smallest across all asset pricing models tested.

Table 7: Time series average absolute alphas: the FF 3 factor + Liquidity factor + Momentum factor

	Baseline	Volatility Factor	Conditional Betas	Volatility Factor & Conditional Betas
Panel A: NYSE-AMEX and NASDAQ Stocks				
Size	0.2619 [36.39]	0.2091 [29.37]	0.135 [22.65]	0.0634 [10.06]
Value	0.5869 [43.38]	0.5995 [70.97]	0.2725 [30.08]	0.2239 [31.88]
Liquidity	0.0222 [-1.07]	0.0829 [8.72]	0.0384 [-2.51]	0.0112 [1.54]
Momentum	0.4359 [34.97]	0.3447 [37.55]	0.5506 [75.29]	0.4973 [89.40]
Short-term Reversal	1.4516 [-56.82]	1.5502 [-59.85]	1.1701 [-54.72]	1.2056 [-66.04]
Panel B: NASDAQ stocks				
Size	0.9991 [20.20]	0.8995 [22.63]	0.5312 [12.23]	0.2805 [7.65]
Value	1.1360 [24.70]	1.1411 [33.58]	0.4573 [10.87]	0.3780 [8.89]
Liquidity	0.1530 [-2.89]	0.1184 [4.51]	0.1803 [-5.05]	0.0207 [-0.88]
Momentum	0.1183 [1.62]	0.0231 [0.36]	0.3980 [8.07]	0.3398 [8.24]
Short-term Reversal	1.5150 [-23.71]	1.6382 [-26.50]	1.0712 [-23.12]	1.0971 [-28.92]

This table reports the average monthly absolute alphas from well-documented equity anomaly strategies derived from the Fama and French 3 factor model plus liquidity and momentum factor. Panel A includes all stocks from July 1963 to June 2019, while Panel B focuses on NASDAQ stocks from July 1982 to June 2019. The second column ("Baseline") reports time-series absolute alphas using the Fama and French 3 factor model plus liquidity and momentum factor; the third column adds a volatility factor; the fourth column allows betas in the Fama and French 3 factor model plus

liquidity and momentum factor to vary with individual stock return standard deviation; and the final column augments the Fama and French 3 factor model plus liquidity and momentum factor with a volatility factor while also allowing the beta to be conditional on volatility. The Newey-West (1987) corrected t-statistics are reported in the bracket.

In addition, the model with the combined volatility adjustments begins to address the liquidity anomaly. Abnormal return expectations from liquidity decrease from 0.0222 to 0.0112 for all stocks, and from 0.1530 to 0.0207 for NASDAQ stocks. These findings are significant because neither the volatility factor nor beta conditioning alone effectively mitigate the liquidity anomaly—in previous cases, they mostly exacerbate it. However, despite these improvements, the model with added volatility and beta conditioning still struggles to address the momentum anomaly.

The Fama and French five-factor model

Table 8 presents the average monthly absolute alphas from zero-investment strategies of anomaly constituents, based on the Fama and French five-factor model. The results are largely consistent with previous findings. Adding the volatility factor alone helps mitigate size and value anomalies. However, unlike in earlier sections where the treatment had positive impacts, it exacerbates the momentum effects, but only marginally. Including beta conditioning reduces size, value, and short-term reversal anomalies but fails to address momentum and liquidity anomalies. When both volatility adjustments are applied together, we observe the most significant improvements in size, value, and short-term reversal anomalies, along with continued progress in mitigating the liquidity anomaly. Similar results are found when analyzing NASDAQ stocks.

Consistent patterns observed across the previous sections can be summarized as follows. First, adding additional risk factors generally enhances the ability of asset pricing models to capture most equity anomalies, both with and without volatility adjustments. Second, when the volatility factor is added alone, it effectively reduces momentum effects and further improves the size anomaly. However, it exacerbates liquidity and short-term reversal anomalies. The findings on the value anomaly are inconsistent. Third, beta conditioning strengthens the models' explanatory power for size, value, and short-term reversal anomalies, but fails to address liquidity and momentum anomalies. Fourth, when both volatility adjustments are applied together, the models show the most significant improvements in mitigating size, value, and short-term reversal anomalies. However, momentum effects worsen in most cases. The impact on the liquidity anomaly is mixed; however, more sophisticated models—such as the Fama and French three-factor model with added liquidity and momentum factors, as well as the five-factor model—are effective at mitigating the liquidity anomaly. Fifth, NASDAQ-listed stocks generally exhibit higher levels of anomalies, except for the momentum effect. Nonetheless, the effects of adding the volatility factor and conditioning on beta for NASDAQ stocks are broadly similar to those observed for all stocks.

Table 8: Time series average absolute alphas: the Fama and French 5 Factor Model

	Baseline	Volatility Factor	Conditional Betas	Volatility Factor & Conditional Betas
Panel A: NYSE-AMEX and NASDAQ Stocks				
Size	0.3367 [34.25]	0.2343 [28.40]	0.2243 [28.86]	0.1017 [15.34]
Value	0.5037 [34.92]	0.4794 [49.91]	0.1655 [17.47]	0.0923 [13.05]
Liquidity	0.0563 [-3.21]	0.0971 [10.01]	0.0979 [-7.65]	0.0052 [0.81]
Momentum	0.3268 [15.68]	0.3290 [27.46]	0.5047 [44.57]	0.5084 [70.80]
Short-term Reversal	1.7288 [-61.87]	1.7701 [-60.54]	1.4887 [-57.18]	1.4198 [-62.39]
Panel B: NASDAQ stocks				
Size	1.0005 [17.30]	0.8628 [18.17]	0.5937 [11.46]	0.2959 [6.87]
Value	1.0545 [24.31]	0.9910 [27.32]	0.5797 [13.38]	0.4417 [10.80]
Liquidity	0.1260 [-2.50]	0.1099 [3.16]	0.1775 [-5.12]	0.0619 [-2.28]
Momentum	0.0533 [0.55]	0.0548 [0.72]	0.2909 [6.63]	0.3318 [9.11]
Short-term Reversal	1.9273 [-28.79]	1.9076 [-32.03]	1.6690 [-26.39]	1.4518 [-34.34]

This table reports the average monthly absolute alphas from well-documented equity anomaly strategies derived from the Fama and French 5 factor model. Panel A includes all stocks from July 1963 to June 2019, while Panel B focuses on NASDAQ stocks from July 1982 to June 2019. The second column ("Baseline") reports time-series absolute alphas using the Fama and French 5 factor model; the third column adds a volatility factor; the fourth column allows betas in the Fama and French 5 factor model to vary with individual stock return standard deviation; and the final column augments the Fama and French 5 factor model with a volatility factor while also allowing the beta to be conditional on volatility. The Newey-West (1987) corrected t-statistics are reported in the bracket.

V. Robustness Check

To ensure the robustness of the two volatility-related model modifications, we perform tests on two sub-samples from distinct time periods. First, we evaluate the application of these methods over the most recent 20 years to assess their effectiveness in a market where the significance of anomaly effects has arguably weakened. Second, we analyze how these volatility measures perform during recession periods, when the market experiences heightened stress and volatility.

The case of recent years (latest 20 years)

McLean and Pontiff (2016) investigated the post-publication return predictability of variables known to forecast cross-sectional stock returns, finding that returns from publication-informed trading are, on average, 32% lower. To assess whether individual stock return volatility continues

to mitigate equity anomalies, even as the magnitude of anomaly effects has declined, we analyze data from July 1999 to June 2019.

Table 9: Robustness Tests: The Case of Recent Years (Latest 20 Years)

	Fama and French 5 Factor model			
	Baseline	Volatility Factor	Conditional Betas	Volatility Factor & Conditional Betas
Panel A: NYSE-AMEX and NASDAQ Stocks				
Size	0.5068 [15.04]	0.3717 [15.63]	0.3026 [15.33]	0.1708 [10.84]
Value	0.7261 [12.38]	0.7521 [21.72]	0.2681 [8.60]	0.2643 [11.12]
Liquidity	0.2454 [4.03]	0.3654 [13.23]	0.1796 [4.59]	0.2105 [12.15]
Momentum	0.1068 [-1.28]	0.0811 [3.19]	0.2578 [6.79]	0.1907 [13.51]
Short-term Reversal	0.8568 [-17.07]	0.9347 [-16.85]	0.6258 [-14.68]	0.5703 [-19.17]
Panel B: NASDAQ stocks				
Size	0.9127 [8.66]	0.8369 [10.85]	0.4949 [7.40]	0.3845 [7.32]
Value	0.9643 [11.41]	1.0302 [17.18]	0.4639 [8.66]	0.4525 [9.89]
Liquidity	0.0235 [0.25]	0.2570 [4.61]	0.1063 [1.96]	0.0196 [0.60]
Momentum	0.1769 [-1.93]	0.1510 [-3.05]	0.2749 [5.20]	0.1921 [8.91]
Short-term Reversal	1.1952 [-13.63]	1.2414 [-15.09]	0.8884 [-12.21]	0.7744 [-15.58]

This table reports the recent 20 years of average monthly absolute alphas from well-documented equity anomaly strategies derived from the CAPM and the Fama and French 5 factor model. Panel A includes all stocks from July 1999 to June 2019, while Panel B focuses on NASDAQ stocks from July 1999 to June 2019. From the second to the fifth columns, time-series monthly absolute alphas are reported, starting with no modifications (baseline) and then with the addition of a volatility factor, where factor loadings are conditioned for both the CAPM and the Fama and French 5-factor model, respectively. The Newey-West (1987) corrected t-statistics are reported in the bracket.

Table 9 compares the average monthly absolute alphas from zero-investment strategies of anomaly constituents based on the Fama and French five-factor model over the past 20 years.⁹ The results align closely with the general patterns observed in earlier sections. Momentum and size anomalies are effectively reduced when the volatility factor is added alone. When beta loadings are conditioned, size, value, and short-term reversal anomalies decrease, while momentum effects are exacerbated. Regarding the liquidity anomaly, we observe mixed results:

⁹ We find consistent results using the other asset pricing models tested.

positive impacts on all stocks but negative impacts for NASDAQ stocks. When both volatility modifications are incorporated into the baseline model, we see more pronounced improvements in size, value, and short-term reversal anomalies, with noticeable positive effects on the liquidity anomaly. However, the modified model fails to enhance its explanatory power for the momentum anomaly.

Another point of interest is the comparison between the entire sample (Table 8) and the most recent 20 years. In recent years, anomaly return expectations derived from firm characteristics such as size, value, and liquidity have moderately increased, while those based on past return variables have significantly decreased. For NASDAQ stocks, return expectations have declined across all anomaly strategies except for momentum effects. This finding is particularly interesting as it suggests that market participants remain hesitant to fully exploit certain equity anomaly constituents, despite numerous studies highlighting strong relationships between stock portfolio returns and these anomalies. This result contrasts with the general conclusions of McLean and Pontiff (2016). Moreover, the difference in anomaly returns between all stocks and NASDAQ-listed stocks have narrowed considerably in the recent 20-year subsample.

The case of recessions

Several researchers have proposed that fluctuations in stock return volatility during recession periods can significantly influence portfolio returns. For instance, Daniel and Moskowitz (2016) suggest that momentum strategies can experience infrequent but persistent strings of negative returns during periods of market stress or panic. They recommend an optimal dynamic momentum strategy that hedges against these momentum crashes by adjusting portfolio weights based on both return premia and risk premia (variance). Similarly, Moreira and Muir (2017) advocate for volatility-managed portfolios during market crises, which reduce risk by scaling down exposure when volatility is high. These portfolios, they report, yield high alphas and Sharpe ratios. Against this backdrop, we further examine how stock return volatility impacts equity anomalies during recession periods.

Table 10 presents the average monthly absolute alphas for all stocks from zero-investment strategies of anomaly constituents based on the Fama and French five-factor model during recession periods.¹⁰ Notably, abnormal return expectations from liquidity and short-term reversal constituents are significantly higher than those estimated for the entire sample, both in the baseline figures and with the two volatility modifications. This disparity likely reflects the distinctive behavior of stock returns during recessions.

The impacts of volatility adjustments largely align with previous findings, though with somewhat enhanced performance during recessions. When the volatility factor is added alone, we observe substantial improvements in the size, value, liquidity, and momentum anomalies, though the short-term reversal anomaly slightly increases. When beta loadings are conditioned, size, value, liquidity, and short-term reversal anomalies decrease, but momentum effects worsen. When both volatility modifications are applied to the baseline model, we find consistent improvements in size, value, and short-term reversal anomalies, with significant positive effects on the liquidity anomaly. However, the modified model continues to fall short in improving its explanatory power for the momentum anomaly.

¹⁰ We identify 6 recession periods (83 months) during our sample period, using NBER database.

Table 10: Robustness Tests: The Case of Recessions

	Fama and French 5 Factor model			Volatility Factor & Conditional Betas
	Baseline	Volatility Factor	Conditional Betas	
NYSE-AMEX and NASDAQ Stocks				
Size	0.4314 [4.75]	0.1792 [3.70]	0.0522 [1.26]	0.1832 [-7.17]
Value	0.4057 [2.27]	0.2526 [3.22]	0.2534 [4.12]	0.3500 [-9.99]
Liquidity	1.0488 [-16.75]	0.6777 [-10.66]	0.9632 [-20.93]	0.6606 [-14.97]
Momentum	0.2123 [-0.71]	0.1198 [1.26]	0.2402 [2.52]	0.4295 [8.15]
Short-term Reversal	2.9810 [-22.30]	3.1451 [-27.49]	2.4723 [-18.84]	2.3277 [-25.89]

This table reports the average monthly absolute alphas from well-documented equity anomaly strategies derived from the CAPM and the Fama and French 5 factor model during the 6 recession periods identified by NBER (83 months). From the second to the fifth columns, time-series monthly absolute alphas are reported, starting with no modifications (baseline) and then with the addition of a volatility factor, where factor loadings are conditioned for both the CAPM and the Fama and French 5-factor model, respectively. The Newey-West (1987) corrected t-statistics are reported in the bracket.

VI. Conclusions

We address well-documented equity anomalies by enhancing existing asset pricing models with a direct risk measure: the volatility of individual stock returns. In an efficient market, fluctuations in stock returns should reflect all available information, including firm characteristics and various investor behavior biases, which are known contributors to equity anomalies. Our findings indicate that individual stock return volatility exhibits offsetting relationships across all tested anomaly constituents, suggesting that volatility has significant potential to mitigate equity anomalies and enhance the parsimony of asset pricing models.

Examining the effects of two volatility modifications on various asset pricing models, we find that adding the volatility factor alone effectively reduces momentum effects and further improves the size anomaly. Beta conditioning strengthens the models' explanatory power for size, value, and short-term reversal anomalies. When both volatility adjustments are applied together, we observe the most significant improvements in mitigating size, value, and short-term reversal anomalies; however, the inclusion of beta conditioning worsens momentum effects in the baseline models. Regarding the liquidity anomaly, both volatility adjustments prove effective in mitigating this anomaly within more sophisticated models. NASDAQ-listed stocks generally exhibit higher levels of anomalies, except for the momentum effect. Nonetheless, the effects of adding the volatility factor and conditioning on beta for NASDAQ stocks closely resemble those observed for all stocks.

While the estimated volatility of individual stock returns is an important consideration, it is crucial to recognize that the effectiveness of capturing equity anomalies varies based on how the volatility measure is integrated into asset pricing models. Despite consistent offsetting relationships between volatility and all anomaly constituents, the volatility factor mitigates momentum effects across most tested models when applied alone; its impact, however, diminishes when combined with the beta conditioning process. This does not imply that the volatility factor is unimportant; rather, it suggests that certain model frameworks could better harness the potential of individual stock return volatility to alleviate equity anomalies. These findings open avenues for future research, such as exploring how the incorporation of other factors alongside the volatility factor can more effectively explain variations in stock returns. Additionally, investigating alternative frameworks or methods for utilizing stock return volatility—beyond merely constructing a volatility factor or employing beta conditioning—could provide deeper insights into addressing equity anomalies.

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