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Sufficient Income and Sustainable Withdrawal Rates for Retirement

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Abstract

Since the passage of the Employee Retirement Income Security Act of 1974 (ERISA), numerous companies from throughout the United States have chosen to change from providing "Defined Benefit" pension plans to providing "Defined Contribution" pension plans. Issues with underfunding and increasing longevity caused the Defined Benefit to Defined Contribution switch. Now, successful retirement planning is an iterative process that requires the management of many variables. Some are random and unpredictable in scope and magnitude and others are choices we make as our retirement objectives change. It's essential that changes be incorporated expeditiously to minimize adverse outcomes. One can begin the process by estimating the annual income required to support one's "retirement lifestyle" if retirement occurred today. Then extrapolate that income to the planned retirement date based upon the expected rate of inflation. A "modified four percent rule" can then be used to estimate the portfolio value required to support 30 or more years in retirement. The financial planner and client should go through this process at least every two years or when major events suggest a change is required. To assist the planner, this paper extends the Four Percent Rule in the following ways: Time in retirement is 16 – 40 years in 2-year increments with an asset allocation range is 0% to 100% stocks in 15 equal steps. Three auxiliary tables (Panels A, B, & C) are provided to facilitate the iteration process.

I. Introduction

The passage of the Employee Retirement Income Security Act of 1974 (ERISA) set the stage for changes in the provision for retirement for individual workers. ERISA requires retirement plans to provide participants with plan information including important information about plan features and funding; sets minimum standards for participation, vesting, benefit accrual and funding; provides fiduciary responsibilities for those who manage and control plan assets; requires plans to establish a grievance and appeals process for participants; gives participants the right to sue for benefits and breaches of fiduciary duty; and, if a defined benefit plan is terminated, guarantees payment of certain benefits through a federally chartered corporation, known as the Pension Benefit Guaranty Corporation (PBGC).[1] Additionally, the Revenue Act of 1978 included a provision that became Internal Revenue Code (IRC) Sec. 401(k) (for which most Defined Contribution Plans are named), under which employees are not taxed on the portion of income they elect to receive as deferred compensation rather than as direct cash payments.[2]

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[1] Source: United States Department of Labor

[2] Public Law 95-600—November 6, 1978, 95th Congress of the United States of America

The above referenced Revenue Act added permanent provisions to the IRC, sanctioning the use of salary reductions as a source of plan contributions. The law went into effect on Jan. 1, 1980. Regulations were issued in November of 1981. Using the above provisions, numerous companies from throughout the United States chose to establish "Defined Contribution" pension plans and discontinue the provision of "Defined Benefit" pension plans. Under the defined benefit plan, participants were not concerned about withdrawing too much from their plan in the early years leaving them with a shortfall in funding at the end of their life. Providing their Defined Benefit Plan was sufficiently funded by the company for whom they had worked for many years, the participant was guaranteed a life annuity, possibly with survivor benefits for the participants' spouse. Issues with underfunding and increasing longevity, along with the aforementioned statutory changes, resulted in the Defined Benefit to Defined Contribution switch. Additionally, decreasing loyalty to one company and movement by members of the workforce to several different employers over the course of a career increased the attractiveness of Defined Contribution Plans. However, as participants in Defined Contribution plans approach retirement, the owner of the plan is faced with determining the appropriate rate of withdrawal. Each plan participant must continually answer the following two questions: (1) "Based upon my existing portfolio, my current income, and the risk that I am willing to take, what percentage (or amount) of my current income should I invest to reach the desired portfolio value so that I can have my desired income level in retirement?" and (2) "While in retirement, what proportion may I withdraw from my portfolio each year to ensure that I will have sufficient funds to provide for my needs for retirement that lasts for an uncertain period?"

A last and fundamental question: "Given my current age, what is my (and my spouse/partner) remaining life expectancy." Drawing on the Life Expectancy Calculator available on the Social Security Administration website, the life expectancy of a child born on January 1, 2020 is approximately 85 years.[3] An individual attaining age 40 in 2020 can expect to live an additional 43.5 years or to age 83.5 years. One reaching the earliest SSA retirement age of 62 may expect to live to be 84.8 years old. If retirement is chosen at full SSA retirement age (66 years in 2020), the individual may expect to reach the age of 86.3 years. Finally, one who retires at age 70 may expect to reach the age of approximately 85.5 years.[4] These age expectancies are averages indicating that fifty percent of those attaining the listed life expectancy will live longer. Of course, how much longer is unknown.

Retirement planning is an iterative process that attempts to keep one on a trajectory leading to a successful retirement. Several analytical approaches are available for estimating one's desired inflation adjusted annual income in retirement.[5] Retirement planners (Financial Planners and individuals) should then use this information to estimate the value of a portfolio that will produce the desired annual income for an appropriate number of years (life expectancy) and determine the **savings profile** that will achieve the income goal. Of course, there are challenges to this process. The planner faces an uncertain future that may change significantly over time. Changing employment opportunities, illnesses, lifestyle desires, unplanned obligations, unexpected monetary gains or losses, tax rates and regulations are examples. Most Financial Planners agree that financial plans should be reviewed at least every two years or when

[3] See https://www.ssa.gov/planners/lifeexpectancy.html
[4] The average age attained is computed as a simple average of the life expectancy of male and female at each age attained in 2016 added to the age attained that year.
[5] Please see Appendix 1 for a discussion of how this paper adjusts for inflation.

significant life changes occur. Doing so provides information and understanding to adjust early to minimize the magnitude of necessary changes. This paper proposes an adaptive process that uses historical data to estimate future portfolio and income values and sustainable withdrawal rates from *Defined Contribution* based retirement portfolios over reasonable retirement (life expectancy) horizons. The next section considers the approach taken in recently published literature. Section III presents the methods used to establish retirement portfolios and sustainable withdrawal rates from those portfolios. Section IV discusses simulations that provide information concerning sustainable withdrawal rates that a retiree may wish to utilize while Section V provides conclusions and potential areas for further study.

II. Previous Studies

Nearly thirty years have passed since the publication of Bengen's watershed article, "Determining Withdrawal Rates Using Historical Data" (1994) proposing the sustainably safe "four percent drawdown rule" as the Financial Planner's "rule of thumb" retirement withdrawal rate. Since then, others have refined, elaborated upon and, often, taken issue with Bengen's original work resulting in the evolution of two groups of retirement withdrawal studies. Research from one perspective has considered portfolio success or failure rates based on constant or inflation-adjusted withdrawals from continuously rebalanced portfolios. The portfolios utilize both stocks and bonds and are considered for specific payment periods or until the portfolios are exhausted. Modifications of withdrawal amounts and rates in the financial plan as well as changes in the portfolios' asset allocations during the payout period are likely. The second perspective examines whether variable or adaptive payouts over actuarial life expectancies and portfolio rebalancing improve the sustainability of withdrawal rates. Several studies suggest procedures for changing the withdrawal rate or amount in response to unanticipated changes in the value of the retirement portfolio.

Bierwirth (1994) used historic returns to compute terminal values of various portfolios of common stock, U.S. Treasury Bonds and U.S. Treasury Bills and concluded that sustainable withdrawal rates ranging from 2.66 percent to 5.19 percent were primarily related to the common stock allocations and showed that stock returns and inflation rates in the early years are extremely important to the success of the retirement portfolio.

Bengen's initial articles (1994, 1996, 1997) examined inflation-adjusted withdrawal rates with allocations to common stocks and longevities up to 50 years based on portfolios of common stocks, U.S. Treasury Bonds and U.S. Treasury Bills. Bengen (1994) recommended a "safe" withdrawal rate of 4% from portfolios of 50 to 75 percent common stock for clients just beginning retirement. Bengen (1996) factored in the client's age in determining the common stock allocation and recommended reducing the common stock allocation by 1 percentage point each year after retirement. Bengen (1997) examined the effects of adding small-company common stocks and U.S. Treasury Bills to retirement portfolios and showed that adding small-company stocks may increase the maximum "safe" withdrawal rate while T-Bills substituted for intermediate term Treasury Bonds will reduce the "safe" withdrawal rate. Bengen (2001) analyzed variable withdrawal plans that supported more spending in the early active peak retirement years, reduced spending in the less active years as health and travel activity decline, and more spending in the final years. He also suggested both a ceiling or floor for the rate of future withdrawals.

Cooley, Hubbard and Waiz (1998, 1999, 2001, 2003, 2005) authored a series of articles that tested the sustainability of fixed annual and inflation-adjusted withdrawals of 3 to 12% using Monte Carlo Simulation for portfolios of large company stocks and high-grade corporate bonds. Their results show that initial withdrawal rates of 5 to 7 percent from portfolios consisting of 50 to 75 percent common stock had success rates of 75 percent or more during the payment periods. Cooley, et. al. (2011) modified the approach based on the record variability of returns in the financial markets and showed that withdrawals must adapt to unexpected losses and require revision in response to unexpected gains and losses.

Pye (2000) calculated portfolio success rates using Monte Carlo simulation for inflation adjusted withdrawals up to 30 years. Key factors for sustainability are the initial withdrawal rate and how future withdrawals are adjusted in response to shortfalls in returns.

Guyton (2004) evaluated the maximum safe initial withdrawal rate during the 40-year period from 1973 to 2003. His analysis used a withdrawal rule that increased the withdrawal rate by the previous year's inflation rate unless the portfolio declined that year and concluded that the maximum "safe" initial withdrawal rate ranges as high as 5.8 percent to 6.2 percent depending on the percentage of the portfolio allocated to equities.

Spitzer, Streiter, and Singh (2007) used a bootstrap simulation to estimate the probability of running out of money in a retirement portfolio when annual withdrawals of a constant real amount are taken. Their results indicate that utilizing withdrawal rates as high as 5.5 to 6 percent indicates a 25 to 30 percent chance of running out of money exists with stock allocations of 75 to 100 percent.

Stout (2008) recommends using Stochastic Optimization to identify optimal portfolio asset allocations for desired withdrawal rates, and then managing withdrawals to reduce or manage the risk of exhausting the retirement portfolio. If the portfolio value rises more than expected, spending could be increased, but only after establishing a reserve fund to help maintain the value of the portfolio for the future. Consistent with other studies, Stout concludes that portfolio sustainability over long retirement horizons requires aggressive portfolios (50 to 75 percent equities). Greater withdrawal rates also require more aggressive portfolio allocations that subject the portfolio to greater market risk and probabilities of failure.

Spitzer, Streiter, and Singh (2008) utilized a bootstrap simulation to investigate the effect of alternative withdrawal strategies on the probability of running out of money over 30 years, assuming a portfolio of stocks and bonds. They examined six constant initial withdrawal rates of 3 percent to 5.5 percent and four adaptive withdrawal models. The adaptive models adjust the annual withdrawal rates in response to annual changes in the value of the retirement portfolio. They concluded if a person is lucky enough to retire when the marketing is rising, increasing withdrawal amounts are possible. But, several years of declining portfolio values at the beginning of the retirement period requires cutting back on withdrawals.

Blanchett and Frank (2009) recognize that sustainability decisions do not occur just once at retirement but change as situations warrant throughout the retirement years. They evaluate the ongoing sustainability of the withdrawal rate that is revisited every year and changed based on the probability of failure for the remaining target distribution period. Frank and Blanchett (2010) cite sequence risk as a timely issue. Sequence risk is defined as the effect of returns on the probability of success/failure for a distribution portfolio. The authors conclude that this probability is an important value to understand along with the withdrawal rate. Together they provide valuable tools for sequence risk evaluation.

Miller (2010) cited depressed real yields on bonds as the reason current retirees are likely to earn rates of return far below historical returns and shows that if return assumptions are adjusted using current bond yields, the failure rate of an initial 4 percent withdrawal rate increases dramatically. Miller also demonstrated that decreasing the allocation to bonds and increasing the allocation to stocks allows for increases in the safe withdrawal rate of a portfolio at a cost of dramatically changing the risk profile of the portfolio.

The idea of the appropriate rate of withdrawal continues to be of importance and discussion in more resent literature. For example, Gardner and Pittman (2013) discuss the risk of running out of money in retirement and that it is important to consider expected lifespan in determining withdrawal rates from retirement funds. Miller (2016) considers the impact of low bond yields on the success of retirement plans and shows that the probability of failure of retirement plans using a 50-50 balance of equities and bonds in a retirement portfolio dramatically increased. This implies that one must consider heavier weighting in equites (resulting in greater risk exposure) to cover the shortfall.

Bengen (2016) considers the impact of a lower return future investment horizon on a hypothetical year 2000 and year 2008 retiree using a 4.5 percent initial withdrawal rate that had previously been referred to as "safe," and whether they can have a successful retirement. He concludes that the answer is yes and that the retirees need not make any adjustments in their strategy and their retirement assets should last 30 years.

Blanchett (2017) discusses the concept of guaranteed income and dynamic withdrawals from retirement accounts and finds that guaranteed income levels suggest that a lower withdrawal rate in the initial stages of retirement may be important to the safe sustainability of retirement income. Blanchett and Cormier (2021) consider that some households underspend in retirement resulting in a "retirement consumption gap" due to the underspending in the early years. Reasons for underspending include not being sufficiently well prepared for retirement. Many adjust spending in ways that increase the probability that retirement income will continue throughout the retirement years. Others, who are better prepared may spend less because of the desire to leave bequests and/or provide for uncertain medical expenses.

Marwood and Minnen (2020) build on Bengen's 4% rule (Bengen 1994) to show that portfolios with greater equity exposure provide the possibility of increased withdrawal rates that are still safe for the retiree. Waggle, Moon, and Lee (2022) study the impact of extremely low bond yields on the viability of traditionally recommended withdrawal rates from retirement portfolios. They show that higher bond allocations lead to shortfalls in retirement funds. Should bond yields return to historical averages, this shortfall might be alleviated, however, retirees must know that holding bonds in an increasing yield environment will result in capital losses in the bond portion of their portfolio.

This study adds to our understanding of acceptable withdrawal rates by utilizing historic equity and debt returns to compute and simulate the impact of computed withdrawal rates on the longevity of defined contribution retirement portfolios. It considers varying combinations of equity and debt in determining possible rates of withdrawal.

III. METHODOLOGY

This section presents a straightforward method for estimating the income level that one might expect at retirement, the portfolio value and composition necessary to attain that income level, and a methodology that may be utilized to estimate the sustainable rate of withdrawal from that portfolio over a retirement period (life span) of up to 40 years.

A number of analytical approaches are available for predicting one's inflation adjusted annual income over an assumed retirement period (see Bengen (1994) and Cooley, et. al. (1998)). Retirement planners use this information to estimate the value of a portfolio that will "statistically" produce this annual income for a given number of years. The final step is to recommend a **saving profile** that will achieve this goal based on current information and our best guess of what the future holds. But there are many challenges! In general, the planner is faced with an uncertain future that may change significantly before retirement: job changes, unemployment, promotion, illnesses, "bucket list" changes, unplanned obligations, unexpected monetary gains and losses, taxes and regulation changes to name a few examples. Financial plans should be periodically reviewed and receive special attention when significant changes occur. A good rule is to review financial plans at least every two years. It is important to make adjustments early to minimize the magnitude of the adjustments. In this sense, retirement planning is an iterative process that seeks to identify a trajectory that leads to a successful retirement.

This paper proposes an adaptive process that uses historical data arrays to predict future values based upon identified retirement needs. The three data arrays utilized are 1.) Future Value Factors (FVFs) for Inflation, 2.) FVFs for Income, and Future Value Annuity Factors (FVAF) for estimating saving requirements.

Individuals should mathematically estimate funding needed if retirement occurs today using accepted financial models. One can then project that financial requirement to the actual retirement date using FVFs for inflation. Once this projection is completed an expanded "four-percent rule" is one method for staying on a successful retirement trajectory and it is the subject of this paper.

The **Solution Matrix Generator Program** uses **equation 1** (presented below) and 16 to 40 years of contiguous historical monthly data to compute **Lambda.** Lambda is the percent of the initial portfolio's value one can withdraw the first year (paid monthly) with a high probability of generating inflation adjusted monthly income for the assumed 16 to 40 years in retirement. The program uses historical financial data and stochastic modeling to build a histogram of **Lambdas**.

As shown in Table 1, this process uses three fixed arrays: The array at the top of the solution matrix is the assumed number of **years in retirement**, the array down the left side of the solution matrix is the **percent of stocks and bonds** in the portfolio, and the array at the top of Panel A is the number of **years to retirement that is used in the adaptive process**. For a given "Confidence Interval", Lambdas for all combinations of years in retirement and stock/bond allocations are represented in the solution matrix.

The derivation of the equation for computing Lambdas assumes the following: withdrawals are at the end of the period (Month), portfolio income is tax-deferred or tax-free, and the portfolio gain is not adjusted for transaction cost.

Table 1
Simulation Results: 90% Confidence Interval

	YEARS IN RETIREMENT												
% Stock	16	18	20	22	24	26	28	30	32	34	36	38	40
0.00	5.20	4.50	4.06	3.71	3.43	3.18	2.96	2.78	2.62	2.47	2.31	2.20	2.09
0.07	5.56	4.91	4.47	4.15	3.82	3.56	3.33	3.14	2.97	2.81	2.63	2.52	2.39
0.14	5.99	5.37	4.89	4.52	4.20	3.89	3.68	3.47	3.27	3.12	2.96	2.83	2.71
0.21	6.08	5.50	5.07	4.74	4.45	4.23	4.01	3.83	3.68	3.44	3.29	3.16	3.05
0.29	6.16	5.63	5.23	4.83	4.62	4.41	4.25	4.03	3.91	3.78	3.66	3.52	3.41
0.36	6.26	5.73	5.32	5.00	4.80	4.55	4.38	4.26	4.11	4.02	3.90	3.79	3.67
0.43	6.37	5.76	5.40	5.11	4.96	4.69	4.51	4.35	4.26	4.17	4.07	3.98	3.87
0.50	6.32	5.87	5.40	5.14	4.94	4.73	4.59	4.43	4.31	4.23	4.17	4.06	4.00
0.57	6.26	5.84	5.37	5.20	4.96	4.81	4.67	4.50	4.37	4.30	4.24	4.18	4.04
0.64	6.38	5.84	5.48	5.17	4.95	4.82	4.67	4.45	4.46	4.34	4.33	4.21	4.10
0.71	6.36	5.81	5.40	5.18	4.94	4.78	4.73	4.65	4.56	4.38	4.33	4.23	4.15
0.79	6.23	5.69	5.42	5.06	4.82	4.76	4.63	4.50	4.40	4.34	4.32	4.19	4.11
0.86	6.10	5.62	5.44	5.03	4.82	4.60	4.51	4.50	4.30	4.28	4.23	4.13	4.12
0.93	6.00	5.51	5.23	4.99	4.75	4.69	4.63	4.41	4.26	4.25	4.25	4.09	4.08
1.00	5.90	5.46	5.13	4.86	4.72	4.51	4.43	4.35	4.30	4.21	4.12	4.05	4.02

ALGORITHMS

1. COMPUTING LAMBDA

Let

$M =$ Assumed number of months in retirement

$j_m =$ Monthly inflation rate

$i_m =$ Monthly total return

The Present Value Annuity Factor for monthly data can be written as:

$$PVAF = \left(\frac{1}{1+i_0}\right)\sum_{m=0}^{M-1}\frac{N_m}{D_m}$$

Where

$$N_0 = D_0 = 1$$

When $m > 0$,

$$N_m = N_{m-1}(1+j_m)$$

$$D_m = D_{m-1}(1+i_m)$$

Lambda (λ) is the percent of an initial portfolio (P_0) one can withdraw the first year. And one can withdraw $(\lambda P_0/12)$ at the end of the first month and adjust this amount for inflation during subsequent monthly withdrawals with a high probability of ending with a positive portfolio.

Assuming income withdrawals are monthly, we have:

$$P_0 = \left(\frac{\lambda P_0}{12}\right)\left(\frac{1}{1+i_0}\right)\sum_{m=0}^{M-1}\frac{N_m}{D_m}$$

And the final equation for computing yearly Lambda is:

$$\lambda = \frac{12}{\left(\frac{1}{1+i_0}\right)\sum_{m=0}^{(M-1)}\frac{N_m}{D_m}}$$ **Eq. 1**

2. COMPUTING THRESHOLD

Let

λ_k = The Lambda associated with cell k

C_k = The sum of all observations placed in cell k

N_t = Total number of observations in the histogram **(4000)**

N_s = 8he number of cells added before the left side of Eq. **5** is equal the right side.

$p(x_k)$ = The probability associated with $Cell_k$ is equal to C_k / N_t

δ = Confidence level (**0.90, 0.95, or 0.99**) [Alpha, $\alpha = (1-\delta)$]

The probability that $X > k$ is given by:

$$P(X > k) = 1 - P(X \leq k)$$ **Eq. 2**

In Discrete terms

$$P(X > k) = \left\{1 - \underbrace{[p(x_1) + p(x_2) + \bullet\bullet\bullet + p(x_k)]}_{P(X \leq k)}\right\}$$ **Eq. 3**

Where

$$P(X \leq k) = [p(x_1) + p(x_2) + \bullet\bullet\bullet + p(x_k)] = (1-\delta)$$

Expressed in counts after the last cell is summed, the threshold is:

$$N_t[p(x_1) + p(x_2) + \bullet\bullet\bullet + p(x_k)] = (1-\delta)N_t$$ **Eq. 4**

Equation 3 can be written as

$$\sum_1^{N_s}\frac{C_k}{N_t} = \left(\frac{1}{N_t}\sum_1^{N_s}C_k\right) = (1-\delta)$$

or

$$\sum_1^{N_s} C_k = (1-\delta)N_t \quad \text{Where } k = 1, 2, \bullet\bullet\bullet, N_s$$ **Eq. 5**

Continue summing until left side of Eq. 5 is equal to (or greater than) the right side. Assume this occurs when k = N_s. The left side may be slightly greater due to histogram quantization (< 0.01 percent).

$$\sum_{1}^{N_s} C_k = (1-\delta)N_t$$

Note: The right side of the equation is the **expected value of the sum of observations from 1 to** N_s and is referred to as **the Threshold.** The **Solution Matrix Value is:**

$$\lambda_k = 0.01(N_s)$$

The expression $(1-\delta)$ has the following values:

=0.01 when Confidence interval is 99% (0.99)
=0.05 when Confidence interval is 95% (0.95)
=0.10 when Confidence interval is 90% (0.90)

3. COMPUTING MARGIN OF ERROR

1. The margin of error (ME) is expressed as:

$$ME = z\sqrt{\frac{\hat{p}(1-\hat{p})}{N_t}} \qquad \text{Eq. 6}$$

Where,

$\hat{p}$ = An estimate of the sample proportions: It is the percent of the sample population required to compute a solution matrix entry.

N_t = The sample size.

z = Z-values (critical values) for selected confidence intervals.

δ = Confidence Levels: 90%, 95%, and 99%

2. The following constraints must be observed.

$(N_t)\hat{p} \geq 10$, $N_t(1-\hat{p}) \geq 10$, **and** $N_t(1-\delta) \geq 10$

Estimates of $\hat{p}$ for the various confidence intervals are:

Sample	Confidence	$z^* - val$	$\hat{p}$	$(1-\hat{p})$	ME
4000	90%	1.645	0.1	0.90	0.0078
4000	95%	1.96	0.05	0.95	0.0067
4000	99%	2.58	0.01	0.99	0.0040

Note: All Margin of Errors are less than one percent.

IV. SIMULATION RESULTS

Table 1 shows the results of the above simulation for a number of retirement periods and estimates of the different parameters utilized in the simulation for the 90% confidence interval. The "sample size" indicates that the results are presented for 4000 simulations utilizing data randomly selected from 1926 through 2018 for both stock and bonds to simulate portfolio returns. "Years in Retirement" show that the rate of withdrawal that one can take from their retirement account and be 90% confident that retirement savings accumulated during ones working years are sufficient to provide the level of income in retirement that one desires. The rate of savings necessary will depend upon the desired income level in retirement and should be established utilizing the financial information and resources available to the prospective retiree.

Concentrating on Table 1, one can see the various withdrawal rates available for retirees based upon varying combinations of stocks and bonds and for different numbers of years of expected retirement. For example, if one expects to live in retirement for 30 years and invests in a portfolio that consists of 57% stocks and 43% bonds, an annual withdrawal rate of 4.5% from the portfolio with leave the retiree with a positive portfolio balanced with 90% confidence.[6] It is evident from the simulation that for one to achieve their desired income levels in retirement for significant time periods, the portfolio must include both stocks and bonds as the maximum withdrawal levels flow from portfolios with approximately 57% to 65% stocks for all expected "years in retirement".

Table 2 provides the same information as Table 1 except that retirees can be 95% confident that funds will last the specified number of years, while Table 3 provides the information for 99% confidence. From the 30 years in retirement column we see that the withdrawal rates must be reduced slightly to have the increased level of confidence with the rate approaching 4%, the commonly used rule of thumb for withdrawal from retirement accounts.

Table 2
Simulation Results: 95% Confidence Interval

	YEARS TO RETIREMENT												
% Stock	16	18	20	22	24	26	28	30	32	34	36	38	40
0.00	4.93	4.36	3.90	3.48	3.29	3.02	2.85	2.67	2.49	2.29	2.18	2.02	1.92
0.07	5.27	4.76	4.32	3.91	3.66	3.36	3.24	2.90	2.77	2.66	2.55	2.39	2.23
0.14	5.70	5.15	4.70	4.30	3.98	3.63	3.50	3.26	3.12	2.93	2.79	2.69	2.55
0.21	5.96	5.40	4.93	4.59	4.29	4.02	3.83	3.64	3.46	3.36	3.17	3.05	2.91
0.29	6.06	5.49	5.06	4.75	4.51	4.30	4.11	3.98	3.78	3.66	3.50	3.38	3.26
0.36	6.10	5.54	5.17	4.90	4.65	4.47	4.28	4.14	4.01	3.90	3.81	3.71	3.58
0.43	6.11	5.58	5.17	4.88	4.69	4.48	4.34	4.23	4.12	4.04	3.92	3.87	3.80
0.50	6.07	5.61	5.18	4.91	4.67	4.51	4.35	4.25	4.17	4.10	4.03	3.95	3.88
0.57	6.09	5.48	5.16	4.93	4.66	4.51	4.38	4.27	4.20	4.07	4.05	3.97	3.92
0.64	6.00	5.56	5.15	4.88	4.64	4.52	4.34	4.29	4.16	4.11	4.05	4.01	3.95
0.71	5.99	5.44	5.12	4.86	4.61	4.47	4.38	4.23	4.17	4.12	4.08	4.00	3.96
0.79	5.89	5.42	5.08	4.82	4.63	4.45	4.32	4.24	4.24	4.10	4.05	3.99	3.95
0.86	5.82	5.35	5.05	4.71	4.56	4.40	4.28	4.20	4.16	4.09	4.04	4.00	3.91
0.93	5.74	5.27	4.89	4.63	4.46	4.30	4.21	4.10	4.10	4.00	3.97	3.91	3.87
1.00	5.59	5.07	4.79	4.53	4.44	4.23	4.18	4.04	4.01	3.97	3.92	3.86	3.84

[6] Data utilized for this study was taken randomly from the Ibbotson *SBBI Yearbook*.

Table 3
Simulation Results: 99% Confidence Interval

	Years in Retirement												
% Stock	16	18	20	22	24	26	28	30	32	34	36	38	40
0.00	4.63	4.07	3.58	3.28	3.01	2.79	2.59	2.43	2.29	2.16	2.05	1.94	1.83
0.07	4.95	4.39	3.95	3.62	3.34	3.13	2.94	2.77	2.61	2.47	2.36	2.25	2.13
0.14	5.26	4.71	4.29	3.98	3.70	3.48	3.28	3.11	2.95	2.82	2.67	2.57	2.44
0.21	5.59	5.08	4.65	4.30	4.06	3.85	3.64	3.47	3.32	3.18	3.04	2.91	2.80
0.29	5.89	5.37	4.98	4.66	4.40	4.18	3.99	3.83	3.67	3.53	3.38	3.27	3.14
0.36	5.96	5.43	5.01	4.74	4.51	4.30	4.15	4.02	3.92	3.79	3.68	3.57	3.43
0.43	5.93	5.41	5.02	4.72	4.51	4.32	4.21	4.06	3.97	3.88	3.84	3.76	3.68
0.50	5.83	5.37	5.00	4.73	4.51	4.34	4.20	4.11	4.03	3.91	3.86	3.83	3.76
0.57	5.80	5.36	4.96	4.73	4.53	4.34	4.21	4.10	3.99	3.94	3.89	3.83	3.78
0.64	5.74	5.28	4.95	4.70	4.49	4.35	4.19	4.11	4.02	3.96	3.90	3.84	3.78
0.71	5.64	5.21	4.89	4.68	4.45	4.32	4.19	4.09	4.01	3.95	3.91	3.86	3.84
0.79	5.58	5.15	4.85	4.58	4.40	4.26	4.19	4.09	4.04	3.98	3.89	3.84	3.83
0.86	5.44	5.08	4.79	4.55	4.40	4.23	4.11	4.02	3.96	3.92	3.86	3.85	3.79
0.93	5.27	4.87	4.54	4.35	4.19	4.12	4.00	3.94	3.86	3.87	3.79	3.79	3.77
1.00	4.85	4.51	4.23	4.00	3.86	3.80	3.70	3.65	3.68	3.58	3.55	3.53	3.51

Table 4 shows three auxiliary **Panels A, B, and C** that can be used to aid the financial planner in adjusting savings and/or spending requirements for the 90% confidence interval. The array at the top of Panel A is the number of **years to retirement** and it is used to select the appropriate values from Panels A, B, and C when adjustments are required. (The percent of stock to the left of Panel A is used to identify the source of the data but **is not used** in the computation.[7]) Tables 5 and 6 provide the same information for the 95% and 99% confidence interval, respectively.

Table 4
Adjustment Factors 90% Confidence Interval

			YEARS IN RETIREMENT PANEL A: FVF for INFLATION										
% Stock	16	18	20	22	24	26	28	30	32	34	36	38	40
0.43	1.30	1.41	1.51	1.68	1.74	1.95	2.10	1.97	2.12	2.38	2.12	2.24	2.70
0.50	1.31	1.41	1.52	1.66	1.77	1.98	1.80	2.04	1.97	1.94	2.14	2.24	2.55
0.57	1.30	1.42	1.52	1.66	1.75	1.97	1.94	2.24	2.12	1.94	2.23	2.14	2.39
0.64	1.31	1.42	1.53	1.63	1.77	1.99	2.13	2.26	2.04	2.11	1.98	2.24	2.37
0.71	1.30	1.40	1.48	1.65	1.75	1.96	1.97	1.99	1.97	2.18	2.20	2.12	2.28
W			PANEL B: FVF for INCOME										
0.43	2.35	2.74	3.21	3.70	4.19	4.94	5.90	6.64	7.49	8.64	9.50	10.84	12.03
0.50	2.45	2.80	3.35	3.88	4.63	5.63	6.65	7.97	8.83	10.14	11.39	13.09	14.61
0.57	2.50	2.86	3.45	4.07	4.81	6.14	7.49	9.11	10.62	11.81	13.51	15.66	16.85
0.64	2.47	2.92	3.52	4.13	5.26	6.70	8.45	10.41	12.21	13.97	15.67	18.40	20.24
0.71	2.44	2.94	3.54	4.34	5.31	7.11	9.02	11.49	13.91	16.51	18.29	20.99	23.52
			PANEL C: FVAF for INCOME										
0.43	24.83	30.58	36.67	44.17	51.17	60.83	71.50	83.25	98.25	115.42	131.50	152.25	177.50
0.50	25.08	30.67	37.92	45.58	54.75	65.50	74.92	92.92	106.75	125.25	148.42	173.50	201.92
0.57	25.17	31.00	38.33	47.08	56.42	67.58	80.83	101.58	123.83	139.33	169.50	194.50	227.67
0.64	25.08	31.08	38.83	47.33	58.33	71.25	87.42	110.25	132.08	158.00	183.42	221.50	258.25
0.71	24.92	31.42	38.83	48.50	59.00	72.50	88.42	109.25	137.00	177.58	211.50	242.25	288.33

[7] An example of utilizing the adaptive process is available from the authors upon request.

Table 5
Adjustment Factors 95% Confidence Interval

	YEARS TO RETIREMENT					PANEL A: FVF for INFLATION							
% Stock	16	18	20	22	24	26	28	30	32	34	36	38	40
0.43	1.06	1.29	1.40	1.52	1.55	1.57	1.64	1.69	1.73	1.77	1.81	1.90	1.96
0.50	1.06	1.29	1.40	1.51	1.56	1.57	1.61	1.69	1.72	1.76	1.82	1.92	2.08
0.57	1.05	1.29	1.39	1.50	1.56	1.56	1.60	1.69	1.74	1.78	1.81	1.90	2.04
0.64	1.04	1.35	1.39	1.44	1.56	1.57	1.60	1.69	1.73	1.77	1.81	1.90	2.01
0.71	1.05	1.29	1.39	1.51	1.56	1.57	1.61	1.69	1.73	1.77	1.81	1.88	1.97
	PANEL B: FVF for INCOME												
0.43	2.19	2.59	2.95	3.24	3.86	4.62	5.55	5.99	6.96	7.81	9.12	9.43	11.47
0.50	2.22	2.68	2.94	3.32	4.10	5.06	6.32	7.15	8.27	8.74	10.46	10.94	13.52
0.57	2.18	2.71	3.06	3.42	4.46	5.38	7.11	7.86	9.34	10.55	12.42	13.03	15.91
0.64	2.13	2.75	2.81	3.48	4.69	5.76	7.88	9.03	10.94	12.67	14.24	15.94	18.38
0.71	2.16	2.72	3.01	3.65	4.80	6.12	8.60	9.61	12.16	14.41	16.69	18.25	21.31
	PANEL C: FVAF for INCOME												
0.43	23.00	27.83	33.42	40.08	47.17	54.67	63.42	75.42	86.83	105.17	119.58	139.58	163.08
0.50	23.42	28.83	33.33	39.00	46.08	56.83	66.58	86.33	95.50	108.83	132.92	150.50	184.17
0.57	23.08	27.92	33.58	38.75	50.17	57.58	72.83	84.75	99.67	121.25	141.00	166.33	203.00
0.64	22.58	28.92	32.33	38.33	50.33	63.17	70.42	89.92	110.50	129.83	153.58	190.08	216.17
0.71	22.83	26.75	33.00	38.83	47.50	57.58	71.92	90.33	113.17	140.50	169.00	208.08	243.33

Table 6
Adjustment Factors 99% Confidence Interval

	Years in Retirement												
% Stock	16.00	18.00	20.00	22.00	24.00	26.00	28.00	30.00	32.00	34.00	36.00	38.00	40.00
			PANEL A: FVF FOR INFLATION										
0.43	0.95	1.01	1.13	1.36	1.40	1.52	1.52	1.52	1.65	1.69	1.73	1.75	1.86
0.50	0.94	1.00	1.17	1.36	1.39	1.51	1.52	1.56	1.64	1.69	1.72	1.74	1.86
0.57	0.91	1.01	1.16	1.36	1.40	1.51	1.52	1.56	1.64	1.69	1.72	1.74	1.86
0.64	0.95	0.99	1.18	1.36	1.35	1.52	1.52	1.57	1.64	1.66	1.72	1.77	1.81
0.71	0.94	0.99	1.16	1.36	1.39	1.51	1.52	1.56	1.64	1.69	1.72	1.75	1.83
			PANEL B: FVF FOR INCOME										
0.43	2.01	2.10	2.30	2.89	3.37	4.19	4.78	5.51	6.37	7.17	8.45	8.92	9.47
0.50	1.97	2.13	2.49	3.05	3.54	4.57	5.21	6.31	7.27	8.19	9.69	9.87	10.89
0.57	1.93	2.05	2.26	3.06	3.67	4.90	5.82	6.90	8.14	9.09	11.22	11.85	13.07
0.64	1.87	2.04	2.28	3.09	3.91	5.21	6.30	7.77	8.86	10.15	12.66	13.41	14.75
0.71	1.82	1.98	2.15	3.20	3.94	5.51	6.75	8.52	9.60	11.27	14.50	15.08	16.68
			PANEL C: FVAF for INCOME										
0.43	20.42	24.33	28.75	33.92	40.42	47.58	57.00	65.67	79.58	92.83	108.75	126.25	145.50
0.50	19.42	23.67	28.42	33.17	39.92	47.75	57.75	69.67	83.17	98.92	116.25	136.58	159.75
0.57	18.50	22.42	26.92	32.08	38.83	47.58	58.75	72.83	87.00	103.08	124.75	148.25	174.00
0.64	17.58	21.75	25.92	30.83	38.50	47.00	59.33	73.00	89.25	107.92	131.25	161.00	188.33
0.71	16.83	20.67	24.58	30.42	37.50	46.33	58.92	74.08	91.25	113.17	138.58	168.25	202.75

V. CONCLUSION

One cannot develop a static retirement plan at age 25 and feel secure about accumulating the resources required to retire successfully. Appropriate use of time is critical for a successful outcome. Consider the following:

1. **Starting too late:** Charles Schwab estimates you will need to save 10 – 15% of your pre-tax income if you start saving for retirement in your 20s. And you will need 15 – 25% if you begin in your 30s.

2. **Taking your eyes off the ball:** Planning for and during retirement requires periodically reviewing retirement needs and current position to achieve your goals. It is imperative that timely adjustments be made to maintain your retirement trajectory. This paper provides a roadmap to assist investors and their retirement planners to realize their financial objectives throughout their retirement years.

Through all of this, the rate of withdrawal available to retirees with confidence that they can maintain a positive portfolio balance will vary with the expected years in retirement. While none of us know just how long our retirement time will be, these simulations provide some direction for withdrawal rates that retirees can draw upon based on their individual family history.

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APPENDIX 1

This paper uses the following process to adjust for inflation:

1. During saving for retirement each payment is increased by the historical level of inflation before discounting the inflation adjusted amount back to time 0 for each period.
2. The historically derived return to discount each amount.
3. The sum then provides the present value of all future inflation adjusted income and the present value of the portfolio to provide the future inflation adjusted income for the period of interest.

On the Models to Evaluate a Merger/Acquisition Project

Chengho Hsieh and Yannan Shen

Abstract

We compare and contrast the existing merger/acquisition evaluation models and based on the results, propose a model that is more complete. The model emphasizes on the need to value not only the target firm, but also the acquiring firm such that 1) the increase in debt capacity for each firm due to the coinsurance benefit that stems from the diversification effect by pooling two firms together can be considered and 2) the synergy gains accrued to each firm can be valued by the firm's own discount rate. The model provides methods to account for the effect of the increased debt capacity on the valuations.

I. Introduction

Corporate mergers/acquisitions are common in the business world; thus, the models to evaluate a merger/acquisition project are widely discussed. The main task in the evaluation is to value the acquiring and the target firms. There are two broad categories of valuation methods: the discounted cash flow (DCF) and the market multiple methods. Between them, the former appears to be the mainstream. In this study, we focus on the DCF method.

To find the merger/acquisition evaluation models, we do a comprehensive search on Amazon.com for textbooks related to 1) financial management, 2) corporate finance, and 3) merger and acquisition valuations. The result is a list that includes the books used in this study plus others listed in Appendix A. The appendix also explains why these books are not included in the study.

We specify the main features in each model to facilitate analysis. From the analysis, we find that within the DCF method, the authors use divers approaches, leaving some clarifications desirable, and in our opinion, the models are incomplete. We then propose our model. The model emphasizes on the need to value not only the target firm, but also the acquiring firm such that 1) the increase in debt capacity for each firm due to the coinsurance benefit that stems from the diversification effect by pooling two firms together can be considered and 2) the synergy gains accrued to each firm can be valued by the firm's own discount rate. The model also provides methods to account for the effect of the increased debt capacity on the discount rate for the valuation.

Section II reviews the models and list their main features. Based on the results, we offer some discussions in Section III. In Section IV, we lay out what a more complete model should be based on the discussions. Section V concludes this paper.

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II. Review on Existing Evaluation Models

Before we discuss the evaluation models for mergers/acquisitions, a brief review on the DCF valuation method is in order. The method involves 4 common steps for all its variants.

Step 1:
Estimate the annual free cash flows (FCF's) for the next N years.

Step 2:
Based on the FCF for year N, estimate the horizon value (HV), which is the present value at time N of all the FCF's beyond year N.

Step 3:
Add the HV to year N's FCF.

Step 4:
Find the present value of all the N cash flows.

This four-step procedure is applied in three variants.

1. The corporate valuation method
   In this method, the free cash flows are those generated from the firm's operations, and the resulting present value from Step 4 would be the total value of the firm.[1] The discount rate used to find the present value is the weighted average cost of capital, WACC.

   $$WACC = W_D R_D (1 - T) + W_S R_S \qquad (1)$$

   In this equation, the W's represent the chosen weights of debt (D) and equity (S), R's are the costs to the firm of each type of financing, and T represents the tax rate. Let $V=D+S$; thus, $W_D = \frac{D}{V}$. W_D is called the "debt ratio." WACC assumes that the firm will continuously rebalance its debt to maintain a constant debt ratio. Subtracting the market value of the debt from the total value of the firm, one obtains the total value of the equity.
2. The equity free cash flow method
   In this method, the free cash flows are those that flow to the equity-holders and the resulting present value from Step 4 would be the total value of the equity. The discount rate used to find the present value is the cost of equity (either levered or unlevered, depending on whether the firm is levered or not). The cost of equity, R_S, is determined by the capital asset pricing model (CAPM),

   $$R_S = R_F + (R_M - R_F)\beta_S. \qquad (2)$$

   In this equation, R_F is the risk-free rate, R_M is the expected rate of return of the market portfolio, $(R_M - R_F)$ is the equity market risk premium, and β_S is the market risk of the equity.

[1] Assume the firm has no non-operating assets for simplicity. Otherwise the total value of the non-operating assets should be added to the present value from Step 4 to find the total value of the firm.

3. The adjusted present value method
   In this method, there are two streams of cash flows, and the aforementioned four-step procedure is applied to each stream. The first is the same as in the corporate valuation method above, the free cash flows from the firm's operations. The second is the tax shields. The two present values from Step 4 are then added together to find the adjusted present value. The discount rate for both streams is the firm's unlevered cost of equity if the firm will maintain a constant debt ratio, otherwise the discount rate for the second stream would be the cost of debt. Subtracting the market value of the debt from the adjusted present value, one obtains the total value of the equity. The cost of unlevered equity, R_A, is also determined by the CAPM,

$$R_A = R_F + (R_M - R_F)\beta_A. \qquad (3)$$

   In this equation, β_A is the market risk of the unlevered equity. The unlevered cost of equity is also equal to the weighted average of the cost of debt and cost of equity.

$$R_A = W_D R_D + W_S R_S \qquad (3a)$$

We review the evaluation models for mergers/acquisitions found in finance textbooks and specify their respective main features in the following.

Berk and DeMarzo (2020)
The model uses the adjusted present value method. It values the target firm only, including all synergy gains. It is not concerned with debt ratio and instead, uses a pre-determined debt schedule to calculate the tax shields. The total cost of the acquisition (including the cost paid to acquire the target firm) is subtracted from the adjusted present value to arrive at the net present value (NPV) of the merger project.

Brealey, Myers, and Allen (2020)
Two models are presented. One uses the corporate valuation method and the other the adjusted present value method. In the former, the target firm will maintain the pre-merger debt ratio into the future. In the latter, the target firm will have a pre-determined debt schedule. Synergy gains are not included in the valuations. Let PV_A be the stand-alone value of the acquiring firm and PV_B of the target firm. Let PV_{AB} be the value of the merged firm, including all synergy gains. The total synergy gain is equal to PV_{AB}-(PV_A+PV_B). Let C be the cost paid to acquire the target firm. The net cost of the acquisition is equal to C-PV_B. The NPV of the merger project is equal to the total synergy gain minus the net cost of the acquisition.

$$NPV = PV_{AB} - (PV_A + PV_B) - (C - PV_B) = PV_{AB} - PV_A - C$$

Oddly, the authors do not explain how to estimate PV_{AB}. They also do not explain if synergy gains are not included, why it is necessary to estimate PV_A and PV_B.

Brigham and Ehrhardt (2020)
The model uses the adjusted present value method. It values the target firm only, taking into account all synergy gains. After the merger, the debt will be adjusted annually to maintain a

constant debt ratio. Subtracting the pre-merger debt from the adjusted present value results in the value of the equity. The NPV of the merger project is equal to this value minus the pre-merger equity value. Although the model assumes a constant capital structure, the authors point out that after the merger, the firm usually has a non-constant capital structure during the N-year post-merger valuation period (in Step 1); therefore, the adjusted present value method is more appropriate than others.

Brigham and Houston (2019)
The model uses equity free cash flow method. It values the target firm only, including all synergy gains. The authors point out that in a merger, the acquiring firm often assumes the target firm's existing debt, and the debt the acquiring firm structures to finance the merger would thus have two components: the debt assumed and any new debt obtained. Because the debt assumed and the new debt usually have differing interest rates, it is more appropriate to calculate the interest expenses for each year during the N-year post-merger valuation period to find the free cash flows. Thus, the authors prefer to use the equity free cash flow method to value the target firm instead of the corporate valuation method.[2] Nevertheless, the model still assumes that the target firm will be able to maintain a constant debt ratio that is the same as its pre-merger debt ratio. Therefore, the discount rate for the free cash flows is the pre-merger cost of equity. The NPV of the merger project is equal to the equity value (from Step 4) minus the pre-merger equity value.

Emery, Finnerty, and Stowe (2018)
The model uses corporate valuation method. It values the target firm only, including all synergy gains. Instead of the WACC described above, the model uses a weighted average cost of capital developed by Miles and Ezzell (1980), which assumes annual debt rebalancing.[3] The NPV of the merger project is equal to the total value of the target firm minus the total acquisition cost (including the cost paid to acquire the target firm).

Arzac (2008)
The model uses the corporate valuation method. It values the acquiring and the target firms separately using their respective WACC. Each firm includes the synergy gains accrued to it. Each firm has its own target debt ratio. The author doesn't mention whether the target debt ratio is the same as the firm's pre-merger debt ratio. Nor does he mention whether the R's in the WACC are pre-merger ones or estimated based on the target debt ratio if the target debt ratio is different from the pre-merger one. The total value of the firm minus the value of the firm's existing debt is equal to the value of the equity. The estimated stock price per share is equal to this value divided by the number of shares outstanding. The author does not calculate the NPV of the merger project, but mentions that the project would be beneficial to the acquiring firm as long as the price per share offered to the target firm's shareholders is less than the estimated stock price. For the acquiring

[2] In the corporate valuation method, the interest expenses is not included in the calculation of the free cash flow. Instead, the cost of debt is accounted for by the discount rate, WACC. If the firm has multiple debts with differing interest rates, it is difficult to determine the value for the cost of debt, R_D, in the WACC.

[3] The Miles and Ezzell (1980) weighted average cost of capital is shown in the following,

$$WACC^* = R_A - LR_D T^* \left(\frac{1+R_A}{1+R_D}\right).$$

In this equation, R_A is the unlevered cost of equity, L is the debt ratio, R_D is the cost of debt, and T^* is the net-benefit-to-leverage factor. Emery, Finnerty, and Stowe (2018) however, do not explain what T^* means and how to obtain its value. In Miles and Ezzell (1980), it is the firm's income tax rate.

firm, the estimated stock price is lower than the pre-merger price; the possible explanation offered by the author is that the market has impounded a higher growth rate for the firm than assumed by the model, but the author doesn't explain how this difference plays a role in making the decision for the merger project.

Grinblatt and Titman (1998)
The model uses the equity free cash flow method. Only the target firm is valued, including the synergy gains accrued to it. The synergy gains accrued to the acquiring firm are value separately using the acquiring firm's cost of equity.[4] The results of these two valuations are then added together to obtain the total value. The authors do not calculate the NPV of the merger project, but mention that if the total value is higher than the target's pre-merger equity value, the project should be pursued.

III. Discussions

If the post-merger debt ratio can't be kept constant, one should use the equity free cash flow or the adjusted present value method to specify the interest expenses for each year during the N-year post-merger valuation period. However, for the equity free cash flow method, it would be difficult to determine the cost of equity, R_S, because strictly speaking, R_S should be changing from year to year during this period, depending on the debt ratio in each year. In the adjusted present value method, the discount rate for the free cash flows is the unlevered cost of equity, which stays the same over time regardless of the debt ratio. Therefore, if the debt ratio cannot be kept constant post-merger, the adjusted present value method is preferred.

Except Arzac (2008), the models discussed in this study only value the target firm. In Arzac (2008), the estimated stock price for the acquiring firm is lower than the pre-merger price, implying the possibility of negative synergy gains for the firm. It may be more robust if both firms are valued such that each firm can include the synergy gains accrued to it and the gains can be valued by the firm's own discount rate. If only the target firm is valued, the synergy gains for both firms would be lumped together and valued at the target firm's cost of capital. This is problematic if the firms have different discount rates, which is especially likely to happen if the merger is a conglomerate one. In Grinblatt and Titman (1998), although only the target firm is valued, the synergy gains accrued to the acquiring firm are valued separately.

Most of the models assume a constant debt ratio post-merger and the debt ratio is the same as the pre-merger one. The literature posits that at least in theory, WACC should be based on the firm's target debt ratio, instead of the current debt ratio; that is, W_D in Equation (1) should be the target debt ratio, implying that the R's in the equation should also be based on the target debt ratio.

[4] The authors offer a hypothetical example for this treatment. Hilton Hotels is considering acquiring a German hotel chain, Welch Hotels. After the merger, Welch will have a higher occupation rate due to Hilton's reputation and its worldwide reservation network. This stream of gains would have the same discount rate as Welch's original cash flows because they all are subject to the state of the German economy. Hilton will also benefit from the merger; the presence in Germany results in more German people coming into contact with the Hilton name, which increases the likelihood that these people will choose Hilton hotels when they travel outside Germany. The increased cash flows are determined by the demand for Hilton's hotels outside Germany; therefore, they should have a discount rate that is the same as the firm's original cash flows.

Therefore, to value the acquiring and the target firms, the WACC for each firm should be based on the firm's post-merger target debt ratio. The determination of this ratio should take into account the increased debt capacity due to the co-insurance effect from the merger. Unless the operating cash flows of both firms are perfectly correlated, the merger creates diversification effect that results in coinsurance benefit for both firms' debts and in turn, raises their debt capacities.[5] It means that both firms can afford to pursue a higher target debt ratio. Empirical results have indicated that post-merger firms have more debt capacity than the combined total for the pre-merger firms (Kim and McConnell (1977)). A higher target debt ratio decreases the firm's WACC and raises the value of the firm. None of the models reviewed in this study take into account these effects. In Arzac (2008), the acquiring and the target firm each has its own post-merger target debt ratio; however, it doesn't say whether this ratio is the same as the pre-merger debt ratio, nor does it mention the coinsurance effect.

Based on the analysis above, we believe that a complete evaluation model for merger/acquisition should contain the following elements:

1. It values both the acquiring and the target firm;
2. It estimates the synergy gain accrued to each firm separately;
3. The synergy gain accrued to each firm is calculated using the firm's own discount rate;
4. It uses the post-merger target debt ratio of the firm 1) to estimate the costs of debt and equity and thus the WACC for the corporate valuation method; 2) to estimate the cost of equity for the equity free cash flow method, and 3) to construct the debt schedule to estimate tax shields for the adjusted present value method. The post-merger target debt ratio should take into account the increased debt capacity due to the coinsurance effect from the merger;
5. For the corporate valuation method, it provides an academically accepted methodology to estimate the costs of debt and equity (and thus the WACC) if the post-merger target debt ratio differs from the existing one. The methodology obeys the constraint that the unlevered cost of equity (R_A in Equation (3a)) stays the same after the change in debt ratio unless there is a change in the firm's asset pool.
6. For the equity free cash flow method, it provides an academically accepted methodology to estimate the cost of equity if the post-merger target debt ratio differs from the existing one. It points out that although the cost of debt is not needed in the model, it should be estimated based on the post-merger target debt ratio. Then the resulting costs of equity and debt should be used to calculate the post-merger R_A based on Equation (3a) to make sure it is equal to the pre-merger R_A to obey the constraint mentioned above. If not, adjustments should be made.

Based on these elements, we point out the deficiencies of each evaluation model reviewed in this study in Appendix B.

[5] In a perfect market, corporate diversification does not add value because investors can diversify on their own. However, if imperfections exist, such as corporate taxations, corporate diversification increases debt capacity and potentially raises frim value due to increased interest tax shields stemming from the increased debt capacity.

IV. Our Proposal

In our model, the acquiring and the target firms are valued separately. Each firm's free cash flows include the synergy gains accrued to it. For the corporate valuation method, based on Equation (1), we specify the pre- and the post-merger WACC's in the following.

$$WACC^{Pre} = W_D^{Pre} R_D^{Pre} (1 - T) + W_S^{Pre} R_S^{Pre} \quad (4)$$

$$WACC^{Post} = W_D^{Post} R_D^{Post} (1 - T) + W_S^{Post} R_S^{Post} \quad (5)$$

In Equation (4), all values for the variables are pre-merger and in Equation (5), post-merger. Assume that T stays the same. If the post-merger target debt ratio is the same as the pre-merger debt ratio, $WACC^{Post} = WACC^{Pre}$. That is, $WACC^{Pre}$ can be used to discount the free cash flows and there is no need to estimate $WACC^{Post}$. If the post-merger target debt ratio is not the same as the pre-merger debt ratio due to increased debt capacity or any other reason, $WACC^{Post}$ can be obtained by the following procedure.[6]

1) Calculate the firm's unlevered cost of equity, R_A, using the firm's R_D^{Pre}, R_S^{Pre}, and W_D^{Pre}. Notice that $W_S^{Pre} = 1 - W_D^{Pre}$.

$$R_A = W_D^{Pre} R_D^{Pre} + W_S^{Pre} R_S^{Pre} \quad (6)$$

2) Based on the post-merger target debt ratio, W_D^{Post}, estimate R_D^{Post}.[7]
3) Based on W_D^{Post}, R_D^{Post}, and R_A, calculate R_S^{Post}.
4) Based on W_D^{Post}, R_D^{Post}, R_S^{Post}, and T, calculate $WACC^{Post}$.

This procedure is applied to the acquiring and the target firms to obtain the discount rates to calculate the total values of the firms respectively. WACC assumes that the firm will have a constant debt ratio that is also the target debt ratio, or at least, "will move over time toward the target debt ratio." If this assumption is not valid, one should simply use the adjusted present value method.

Subtracting the market value of the debt from the total value of the firm, one obtains the total value of the equity. Let S_A^{Post} be the acquiring firm's post-merger total value of equity and S_T^{Post} the target firm's. Let S_A^{Pre} be the acquiring firm's pre-merger total value of equity and S_T^{Pre} the target firm's. Let NC be the net acquisition cost. The NPV of the merger project is equal to the combined post-merger equity value minus the combined pre-merger equity value and minus the net acquisition cost.

$$NPV = (S_A^{Post} + S_T^{Post}) - (S_A^{Pre} + S_T^{Pre}) - NC \quad (7)$$

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[6] This procedure comes from Chapter 19 in Brealey, Myers, and Allen (2020).

[7] If $W_D^{Post} > W_D^{Pre}$, R_D^{Post} would be higher than R_D^{Pre}, and vice versa. The managers would need to rely on their knowledge and experience in the business to determine W_D^{Post} and thus, R_D^{Post}. Alternatively, they can consult with their bankers. In the example in Chapter 19 in Brealey, Myers, and Allen (2020), R_D^{Post} is assumed to be equal to R_D^{Pre} for simplicity.

For the equity free cash flow method, based on Equation (2), we specify the pre- and the post-merger R_S in the following.

$$R_S^{Pre} = R_F + (R_M - R_F)\beta_S^{Pre} \quad (8)$$

$$R_S^{Post} = R_F + (R_M - R_F)\beta_S^{Post} \quad (9)$$

If the post-merger target debt ratio is the same as the pre-merger debt ratio, $R_S^{Post} = R_S^{Pre}$. That is, R_S^{Pre} can be used to discount the free cash flows and there is no need to estimate R_S^{Post}. If the post-merger target debt ratio is not the same as the pre-merger debt ratio due to increased debt capacity or any other reason, R_S^{Post} can be obtained by the following procedure.

1) Obtain β_S^{Pre} from public sources. It can also be estimated using historical stock price data.
2) Convert β_S^{Pre} into unlevered equity beta, β_A, based on the Hamada model (1972).

$$\beta_A = \beta_S^{Pre}\left[\frac{1}{1+(1-T)\left(\frac{D^{Pre}}{S^{Pre}}\right)}\right] \quad (10)$$

In this equation, D^{Pre} and $S^{Pre}$are the pre-merger market values of the firm's debt and equity respectively.

3) Let $\left(\frac{D^{Post}}{S^{Post}}\right)$ be the post-merger debt-to-equity ratio based on the post-merger target debt ratio. Based on this debt-to-equity ratio and β_A, calculate the post-merger equity beta, β_S^{Post}, using again, the Hamada model (1972). Assume the tax rate, *T*, stays the same.
4) Based on β_S^{Post} and Equation (9), calculate R_S^{Post}.

This procedure is applied to the acquiring and the target firms to obtain the discount rates to calculate the equity values of the firms respectively. As pointed out in the previous section, although it is not needed in the model, the cost of debt should also be estimated based on the post-merger target debt ratio. Then the new costs of equity and debt should be used to calculate the post-merger R_A based on Equation (3a) to make sure it is equal to the pre-merger R_A. If not, adjustments should be made.

The use of R_S^{Post} assumes that the firm will have a constant debt ratio that is also the target debt ratio, or at least, "will move over time toward the target debt ratio." If this assumption is not valid, one should simply use the adjusted present value method. The NPV of the merger project can be determined by Equation (7).

For the adjusted present value method, the unlevered cost of equity stays the same pre- and post-merger (unless there is a change in the firm's asset pool due to the merger); thus, the pre-merger unlevered cost of equity can be used to discount the free cash flows from the firm's operations. However, the post-merger tax shields would need to be estimated based on post-merger debt schedule. This schedule should take into account the aforementioned change in debt capacity. If, after the adjustments for the debt, the firm is unlikely to have a constant debt ratio after the merger, the discount rate for the tax shields should be the post-merger cost of debt. Subtracting the value of the debt from the NPV, one obtains the value of the equity. The NPV of the merger project can be determined by Equation (7).

V. Conclusion

Corporate mergers/acquisitions are common in the business world; thus, the models to evaluate a merger/acquisition project are widely discussed. The main task in the evaluation is to value the acquiring and the target firms. There are two broad categories of valuation methods: the discounted cash flow (DCF) and the market multiple methods. In this study, we focus on the DCF method. We collect the evaluation models for mergers/acquisitions from finance textbooks and specify the main features in each model to facilitate analyses. We then propose our model. The model emphasizes on the need to value not only the target firm, but also the acquiring firm such that 1) the increase in debt capacity for each firm due to the coinsurance benefit that stems from the diversification effect by pooling two firms together can be accounted for and 2) the synergy gains accrued to each firm can be valued by the firm's own discount rate. The model provides methods to account for the effect of the increased debt capacity on the discount rate for the valuation.

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Appendix A

To find the merger/acquisition evaluation models, we did a comprehensive search on Amazon.com for textbooks related to 1) financial management, 2) corporate finance, and 3) merger and acquisition valuations. The result is a list that includes those books used in this study plus the following ones.

1. Block, Stanley B., Geoffrey A. Hirt, and Bartley R. Danielson, "Foundation of Financial Management," McGraw Hill Education, 17th edition, 2019.
2. Ross, Stephen A., Randolph W. Westerfield, and Bradford D. Jordan, "Fundamentals of Corporate Finance," McGraw Hill Education, 12th edition, 2019.
3. Koller, Tim, Mark Goedhart, and David Wessels, "Valuation – Measuring and Managing the Value of Companies," Wiley, 7th edition, 2020.
4. Bacon, Frank, Eun Kang, Min Xu, and Suk kim, "Corporate Financial Management," XanEdu, 4th edition, 2020.
5. Titman, Sheridan, Arthur J. Keown, and John D. Martin, "Financial Management – Principles and Applcations," Pearson, 13th edition, 2019.
6. Petitt, Barbara S. and Kenneth R. Ferris, "Valuation for mergers and Acquisitions," FT Press 2nd edition, 2013.
7. Gaughan, Patrick A., "Mergers, Acquisitions, and Corporate Restructurings," Wiley, 7th edition, 2018.
8. Depampi, Donald M., "Mergers Acquisitions, and other Restructuring Activities," Academic Press, 10th edition, 2019.

Books No. 1 through 4 above contain a merger/acquisition chapter. In the chapter, some topics are discussed but no models are used to value either the acquiring or the target firm. Book No. 5 does not contain a merger/acquisition chapter. These books, therefore, are not mentioned in the study.

Book No. 6 values the target firm only by assuming some numbers as the free cash flows over the valuation period. It does not use either the corporate valuation method, or the equity free cash flow method, or the adjusted present value method to estimate the free cash flows. To discount the free cash flows, it uses the current market data and capital structure to estimate the weighted average cost of capital (WACC) although it mentioned that the estimation should be based on the target capital structure.

Book No. 7 uses the corporate valuation method to value only the target firm. To discount the free cash flows, it uses the current market data and capital structure to estimate the WACC.

Book No. 8 uses the corporate valuation method to value only the target firm. Although it points out that the WACC should be estimated based on the target debt ratio, it uses the current data (thus based on the current capital structure) to estimate the cost of equity. For the cost of debt, it is not clear whether the book uses the current data or estimates based on the target debt ratio.

Compared to the books included in this study, the books above do not add value for the analysis; therefore, they are not included.

Appendix B

Based on the six elements outlined in the Discussions section, we list the deficiencies of each evaluation model reviewed in this study in the following. The numbers of the elements absent from the model are specified under each model.

Berk and DeMarzo (2020)
Elements 1, 2, 3, and 4

Brealey, Myers, and Allen (2020)
Elements 2, 3, 4, and 5

Brigham and Ehrhardt (2020)
Elements 1, 2, and 3

Brigham and Houston (2019)
Elements 1, 2, 3, 4, and 6

Emery, Finnerty, and Stowe (2018)
Elements 1, 2, and 3

Arzac (2008)
Elements 4 and 5

Grinblatt and Titman (1998)
Elements 1, 4, and 6

Robust Testing for Bollinger Band, Moving Average and Relative Strength Index

Matthew Lutey

Abstract

We test whether the moving average indicator is profitable in trend following and compare the result with common momentum indicators such as Relative Strength Index (RSI) and Bollinger Band (BB). Our sample runs from January 1, 1963, through December 31. 2019. We test whether the signals generated are profitable and compare their success in timing the portfolios based on the previous year's volatility using the Center for Research in Security Prices (CRSP) data. We note the portfolios sorted by volatility to be trend following and use common entry points for timing them. We find all indicators to be more profitable than the Buy and Hold (BH). The moving average (MA) has the strongest return, while BB and RSI show positive return they do worse than the MA. The results hold across all portfolio deciles sorted by both size and volatility. The indicators for RSI and BB are robust to other specifications for entry and parameters. We also show price level analysis for both RSI and BB as well as a trailing three year analysis for the volatility deciles and moving average.

I. Introduction

We obtained daily price levels and returns of these ten portfolios from the Center for Research in Security Prices (CRSP). These portfolios were constructed from NYSE, AMEX, and NASDAQ stocks and stocks in the Center for Research in Security Prices (CRSP) and are assigned to a portfolio based on their annual standard deviation. The annual standard deviation is calculated as the standard deviation of daily returns within the prior year. An equally weighted price index will be computed daily for each portfolio. The portfolios are rebalanced each year based on the standard deviation of the previous year.

Covel's (2009) that trending markets are characteristic of higher highs and higher lows provide intuition for this rule. Higher highs and higher lows characterize an uptrend. In this type of environment, a trend-following strategy should be used. (Covel, 2009). Thus, buying around the oversold levels and riding the highs will produce profitable returns.

Typical strategies for trend following include a moving average crossover with price crossing over a short-term moving average. Han et al. (2013) test this study on the volatility sorted decile portfolios. They are confirming the ability of a moving average strategy to work well at times the portfolios. According to the definitions by Hayden (2003) and Bollinger (2001), a market of higher highs and higher lows would be traded differently than a market with a primarily sideways activity where prices range between typical highs and lows. Hayden (2003) and Bollinger (2001) develop a set of strategies for trading the Relative Strength Index (RSI) and Bollinger Bands (BB) on this type of market. Their definitions are consistent with the trend following ideas in Covel (2009). and thus, a moving average crossover would not fare well in a sideways market. We can test whether the volatility decile portfolios are trend following.

We employ an additional set of technical trading rules to the portfolios, namely the RSI and Bollinger Band outlined in Henry (1999) and Bollinger (2001). These indicators are typically thought of as oscillators which should be used for sideways markets. In technical trading circles, it is common to use these indicators for trend following by buying when the indicators reach their oversold levels (prices fall below the bottom of the band). As the market has upward momentum, prices will shoot higher as they deviate from the trend (unless a new, downward trend is starting).

A moving average first constructs the Bollinger Bands (see Bollinger 1991). A length of 20 is typical in Bollinger 1991. Then standard deviation bands are added above and below the band. They constructed by taking the standard deviation of the MA(20) and adding subtracting two times the value to the moving average.

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Covel (2009) discusses the trend following to be characterized by a series of higher highs and higher lows for an uptrend which is the case of the volatility decile portfolios of Han et al. (2013). Bollinger (1991) shows that the bands can ride the band's top when prices cross through the band from below, which is the signal we use.

Similar oscillators common in technical trading circles include the Relative Strength Index (RSI) and Stochastic (PK). The construction limits them to values between 0 and 100 with below 20 oversold and above 80 overbought.

John Henry (1993) notes that the RSI can be traded using a trend following strategy by buying when it is above oversold. He also notes that the RSI may push higher and higher when it reaches overbought (lower and lower for oversold). We employ this strategy to the volatility decile portfolios by buying when RSI crosses 20 from below and earning the return on the volatility decile portfolio. We earn the T-bill rate when it crosses back below 20 from above. The indicators all work at various levels below the midpoint of overbought and oversold, which Pring (2002) discussed as the cutoff for buy/sell signals. RSI 40 is standard in Hayden (2003).

Pring, 2002), Edwards and Magee (1948) observations on moving averages suggest that one should buy when prices revert to the moving average from above and touch the average. A mean-reverting signal is referred to for trend following markets.

Indicators that typically oscillate between highs and lows can predict tops and bottoms in sideways markets. (Hayden, 2003, Bollinger, 2001). Since our market comprises higher highs and higher lows, we test buying the dips when oscillators such as RSI and Bollinger Bands revert from their means. We suggest that stock prices are mean-reverting when sorted by volatility rather than following a random walk.

Fama and Blume (1966) define the weak form of market efficiency because past prices are already incorporated in the current price. Therefore any information in the past is already incorporated in the current price. The semi-strong form and super-string form are even more restrictive. They suggest that public and private information (semi-strong) and all information, including insider information (super strong), are already incorporated in the stock price. This study would have implications for the weak form of market efficiency.

This paper tests the busy signals of oversold regions for popular oscillators in literature and technical trading circles. The implications of these indicators suggest that markets are mean-reverting when in an uptrend and that the volatility decile portfolios may be better timed by buying the oversold regions of oscillators than using a moving average crossover.

Constructing a factor of moving averages scaled by price and finding different levels of the indicator is done in Zhou et al (2016) and Arvamov et al (2020) and done in Chung et al (2021). MA trend might be a better indicator than the standard MA definition for comparing to oscillators.

Future work would include combining several indicators in a genetic algorithm to time the portfolios and use neural networks to predict the tops and bottoms. Lo, Mamaysky and Wang (2000) suggested that future work include using technical indicators and optimizing a specific objective function. Optimizing indicators over Volatility Decile may be an appropriate study for this line of work. Or comparing the trend factor of MA to oscillators.

Our focus here is on uptrend stock prices that are mean-reverting. There are several indicators (including moving averages) that pick up on this behavior. We show that buying extreme oversold levels of oscillators (while in an uptrend) acts as a buy signal comparable to a moving average crossover. This type of buy signal has been outlined by Edwards and Magee (1948), Pring (2002), Covel (2009), Hayden (2003), and Bollinger (2001). It has not yet been adopted by academic literature. Thus, we introduce new ways to interpret several technical indicators common in technical trading circles for decades.

We use CRSP daily prices from the start of the year 1963 through the year ended 2019. We find the results in this paper hold outside of the sample for data before 1963, starting as early as 1925.

Daily prices are obtained, and then stocks are sorted into decile portfolios based on their previous year's annual volatility. We construct our technical indicators from these portfolios.

We also test the indicators on simulated geometric Brownian motion (random walk) stock prices. This is shown in the section 3.

To carry this out we use the Center for Research in Security Prices (CRSP) 1929-2019 and simulated a random walk by taking the mean and standard deviation of every stock file. We then follow the geometric Brownian motion procedure by estimating the next stock price based on the current stock price, the mean, and the standard deviation.

We take buy rules common in trend following. Following Henry (1994), Bollinger (2002) and Pring (2002). We see that buying oscillators which smooth out momentum indicators when they are oversold is similar to buying when prices touch the moving average from above which is a common buy rule in stock trading.

Typically investors stack several indicators on a chart at a time. They use the various buy and sell signals to create informed decisions.

We show a sample chart below of common indicators using the SPY from the last 6 months ending 08/31/2021 daily data from Barchart.com. We add the main indicators studied in this paper and mark the buy signals which we test rigorously.

The highlighted yellow regions denote buy signals. They are when prices fall below the bottom of the band, or touch the lower band (which would be falling below the lower band on a tighter standard deviation). We use a 20 period moving average and 2 standard deviations to construct the Bollinger band. The equations for this and our other indicators are in the following section.

We show the Relative Strength Index (RSI) with a standard time frame of 14 periods with an oversold region of 30 and overbought of 70. We see the buy signals when RSI falls below 40, and 50 (which is tested following the equations and are also common oversold areas used for RSI (See Henry 1994, Pring 2002 among many others).

Figure 1. S&P 500 last 6 months with BB and RSI

II. Data and Methodology

We obtained 10 volatility decile portfolios returns and prices from Center for Research in Security Prices (CRSP) from the start of the year 1963 through the year ended 2019. We chose the 1963-2019 period to match up with Fama-French factors. We find the results in this paper hold outside of the sample for data prior to 1963 starting as early as 1925. These portfolios were constructed from NYSE, AMEX and NASDAQ stocks and are assigned to a decile portfolio based on their annual standard deviation. The standard deviations are calculated as the standard deviation of daily returns within the prior year. Once the stocks are assigned a portfolio, we then compute daily an equally weighted price index for each portfolio. These portfolios are rebalanced annually based on the standard deviation of the previous year.

Next, we describe a series of momentum indicators we tested. For each indicator we specify a timing strategy that requires us to invest when the timing signal suggests to be in the portfolio, otherwise to invest in a 30-day treasury bill. In other words, the timing portfolio earns the portfolio rate of return each day it is in the market and the risk-free rate on all other days. We then compare the performance of the timing portfolio with the performance of a buy and hold volatility portfolio for the period. We present the excess returns where the excess returns are the returns of the timing portfolio minus the returns of the buy and hold strategy. We also report the accuracy of the timing portfolio. The rule is accurate on a given day if it chose the asset (the portfolio or the T-Bill) earning the higher return.

For completeness, we first examine the moving average strategy examined by Han et al (2013). This is the simple mean of the current close and the previous 9 trading days close, MA(10). The buy signal is triggered when the price crosses the MA(10) from below. When this happens we are in the market. We stay in the market until price crosses the MA(10) from above. We earn the t-bill rate of return until price crosses back above the MA(10). The moving average crossover is a momentum signal that indicates when prices cross above the moving average from below they will continue to rise.

The formal statement of the investment strategy is:

$$\mathrm{MA}_n = \frac{P_{\mathrm{close}} + P_{t-1} \ldots + P_{t-n-1}}{n} \tag{1}$$

The rule becomes: (2)

$$\begin{cases} 1 & P_{t-1} > \mathrm{MA10}_{t-1} \\ 0 & P_{t-1} < \mathrm{MA10}_{t-1} \end{cases}$$

Where MA_n *is an n day (in our case 10 day) moving average of prices*; 1 *is* owning the decile portfolio and 0 is owning a risk-free asset.

The Relative Strength indicator, RSI, is another momentum indicator that examines the speed and the strength of recent price changes to determine whether the portfolio is overbought or undersold. We buy the volatility decile portfolios when RSI crosses 20 from below and continue to hold until it crosses 80 from below at which point, we sell the portfolio and earn the t-bill rate.

The second momentum strategy we tested is the Bollinger Band Indicator (BBAND). The indicator uses an upper band (UB), above a simple moving average, and below the simple moving average, a lower band (LB). The distance between the upper and lower band is determined by the moving standard deviation. To test this strategy we used a 10-day moving average and the upper band is determined by adding 2-standard deviations to the simple average and the lower band is set by subtracting 2-standard deviations from the moving average. We buy the volatility decile portfolios when the moving average is above the lower band and continue to hold until it crosses below the lower band at which point, we sell the portfolio and earn the t-bill rate. Formally, the investment strategy is presented below:

Bollinger Band Indicator

$$\text{BBAND} = \text{UB}, \text{LB}, \text{MA10} \tag{3}$$

$$\text{UB} = \text{MA}_{10} + 2\sigma \tag{4}$$

$$\text{LB} = \text{MA}_{10} - 2\sigma \tag{5}$$

σ *is the volatility of the* $MA10$

The rule becomes: (6)

$$\begin{cases} 1 & P_{t-1} > LB_{t-1} \\ 0 & P_{t-1} < LB_{t-1} \end{cases}$$

The third momentum strategy we tested is Relative Strength Index (RSI). The indicator uses an oversold region typically below 20 (30, 40 and 50 are also commonly used in technical trading), a middle line (50), and an upper line 80 (50, 60 and 70 are also commonly used in technical trading). We buy the volatility decile portfolios when the RSI is above the oversold region and continue to hold until it crosses below the oversold region again at which point, we sell the portfolio and earn the t-bill rate. Formally, the investment strategy is presented below:

Relative Strength Index (RSI)

$$\text{RSI} = 100 - [\frac{100}{1 + \text{U}\bar{\Delta}\text{P}_{14}/\text{D}\bar{\Delta}\text{P}_{14}}] \tag{7}$$

$\text{U}\bar{\Delta}\text{P}_{14}$ = 14 Average of Upward Price Change over last 14 Periods
$\text{D}\bar{\Delta}\text{P}_{14}$ = Average of Downward Price Change over last 14 periods (8)

Upward Price Change = (9)

$$\begin{cases} P_{\text{close}} & P_{\text{close}} > P_{t-1} \\ 0 & P_{\text{close}} < P_{t-1} \end{cases}$$

Downward Price Change = (10)

$$\begin{cases} P_{\text{close}} & P_{\text{close}} < P_{t-1} \\ 0 & P_{\text{close}} > P_{t-1} \end{cases}$$

∴ **Relative Strength Index** (11)

The rule becomes: $\begin{cases} 1 & RSI_{t-1} > 40 \\ 0 & RSI_{t-1} < 40 \end{cases}$

All of these indicators provide a robust test of additional technical indicators for timing the volatility decile portfolios. Indicators that have a positive MAP (the returns in excess of the buy and hold are greater than 0) are tested using the CAPM and Fama and French 3 Factor models. We also test these indicators on portfolios sorted by size. We report the CAPM and Fama and French alphas for size portfolios that have a positive MAP.

III. Results

We use CRSP daily prices from the start of the year 1963 through the year ended 2019. We use the momentum indicator from Han, Yang and Zhou (2013) deemed MA(10). This is the simple mean of the current close and the previous 9 trading days close. The market in signal is triggered when the price crosses the MA(10) from below. We stay in the market until the price crosses the MA(10) from above. We earn the t-bill rate of return until the price crosses back above the MA(10).

We annualize the returns by using 252 trading days. We also annualize the standard deviation of the returns by multiplying by the square root of 252.

Table 1 presents the results for the MA(10) rule in panels A through C. This updates the results of Han et al. (2013). Panel A provides the annualized average returns and annualized standard deviations for the buy and hold strategy for the decile volatility portfolios and for the high (decile 10) minus the low (decile 1). The average returns are, with the exceptions of deciles 7 and 8, an increasing function across the deciles ranging from 10.1% to 28.11%. Similarly, the MA timing portfolios are an increasing function across the deciles ranging from 14.2% to 32.82%. The timing portfolio returns are considerably higher than the buy and hold strategy and the standard deviations are only about two-thirds of the volatility decile portfolios. This is clearly reflected by Sharpe ratios in Panel B. Panel C shows the return difference between MA timing portfolios and the buy and hold portfolios (MAP). All of the returns in this panel are statistically significant. Showing an average excess return of 8.80 percent annualized. With a standard deviation of 15.33 percent. The success rate (fraction of trading days when the MA timing strategy timed well) of this strategy is about 55%, meaning 55% of the of time the strategy placed the investor in the higher earning asset (the portfolio or risk-free asset). Contrary to the suggestion of Mclean and Pontiff (2016) that such profitable studies on financial time series fall apart post-publication, these results held up well even after the results were published by Han et al. (2013).

Table 2 presents the results for the Relative Strength Indicator. When RSI is above 80% we sell the portfolio and invest in 30-day t-bills. We invest in the portfolio when RSI hits 20%. Panel A of Table 2 provides the average returns and standard deviations for the Relative Strength indicator timing portfolios and for the high (decile 10) minus the low (decile 1). The average returns across the portfolios range from 4.84% to 14.31%. The return difference between RSI timing portfolios and the buy and hold portfolios (MAP) (Panel B) is consistently negative across all deciles implying that RSI timing portfolio returns are not any better than the Buy and Hold strategy. Clearly, buy and hold strategy performs better than the RSI timing strategy across all portfolios. Less than 50% of the time this strategy placed the investor in the higher earning asset. This shows poor excess returns using the RSI as a timing tool.

Next, we present the results of Bollinger Band's timing strategy. We calculate the 20-day moving average (MA) prices each day using the last 20 days' closing prices including the current closing price, and 2 standard deviations above and below the MA denoted upper and lower bands respectively. We compare the lower band's price with the current price as the timing signal. If the current price is above the lower band, it is an in-the-market signal, and we will invest in the decile portfolios for the next trading day and we will be fully invested in the portfolio until the price exceeds the upper band at which point we sell the portfolio and invest in the 30-day risk-free T-bill for the next trading day. Table 3 has the results of this timing strategy. The average returns of this timing strategy are all positive but not better than the buy and hold strategy. The return difference between this timing strategy and the buy and hold portfolios (MAP) is negative across all deciles indicating that Bollinger Bands is not an effective timing tool.

Next, we present the results of MACD's timing strategy. We calculate the 12,26-day fast and slow moving averages that make up the MACD indicator. We then use a 9 day average of the difference of this indicator to compute the MACD signal line. We take a signal of 1 if the signal line prices each day using the last 14 days' average of up and down closing prices including the current closing price. The up closing price is recorded if the close is greater than the previous close. The down closing price is recorded if the close is lower than the previous close. RS is the ratio of the simple mean of the up closing price to the down closing price. RSI is 100-100/(1+RS). The value ranges from 0 to 100. If the RSI value is above 20 it is an in-the-market signal, and we will invest in the decile portfolios for the next trading day; otherwise it is an out-of-the-market signal, and we will invest in the 30-day risk-free T-bill for the next trading day. We use the 10 NYSE/AMEX volatility decile portfolios as the investment assets. We report the average return (Avg Ret). And the standard deviation (Std Dev) for the buy-and-hold benchmark decile portfolios (Panel A), the RSI timing decile portfolios (Panel B), and the RSI portfolios (MAP RSI) that are the differences between the RSI timing portfolios and the buy-and-hold portfolios (Panel C). The results are annualized and in percentages. We also report the annualized Sharpe ratio (Sratio) for the buy-and-hold portfolios and the MA timing portfolios, and we report the success rate for the MAPs. The sample period if from January 1, 1963, to Dec. 31, 2018; t-statistics are in parentheses. ** and * indicate significance at the 1% and 5% levels, respectively. Other periods of 30, 40, and 50 may be used for an in the market signal but are not recorded.

Last, we present the results of OBV's timing strategy. We calculate the 12-day On Balance Volume (OBV) using the 5-day highest close and lowest close including the current close. We multiply 100 by the ratio of the difference of the current close and the lowest close to the difference of the highest close and the lowest close. This value scales between 0 and 100. If it is above 20 (oversold) it is an in-the-market signal, and we will invest in the decile portfolios for the next trading day; otherwise it is an out-of-the-market signal, and we will invest in the 30-day risk-free T-bill for the next trading day. We use the 10 NYSE/AMEX volatility decile portfolios as the investment assets. We report the average return (Avg Ret). And the standard deviation (Std Dev) for the buy-and-hold benchmark decile portfolios (Panel A), the %K timing decile portfolios (Panel B), and the %K portfolios (MAP %K) that are the differences between the %K timing portfolios and the buy-and-hold portfolios (Panel C). The results are annualized and in percentages. We also report the annualized Sharpe ratio (Sratio) for the buy-and-hold portfolios and the %K timing portfolios, and we report the success rate for the MAP %K. The sample period if from January 1, 1963, to Dec. 31, 2018; t-statistics are in parentheses. ** and * indicate significance at the 1% and 5% levels, respectively.

The time frames we use are common for the indicators and are not optimized. We show robustness checks with different thresholds for the RSI, Stochastic, and Bollinger Band and determine the results are as good as, or better with these thresholds. We show results for the default parameters.

We then analyze the average returns, standard deviations, excess returns, sharpe ratio and success rate for the Relative Strength Index (RSI). We use the typical parameters in technical trading to analyze these. RSI 14, 20,80.

We test our indicators in an excess return format following Han et al (2013) where excess return is defined as the MAP (Moving Average Portfolio) where the indicator return is enacted by taking the dummy variable (1 in the market, 0 out of the market) multiplied by the market return for that day. When the indicator is out of the market it earns the risk free rate of return. Thus excess returns are earned by being out of the market on down (negative return) days. We summarize the excess returns for all portfolios.

We then try to explain the returns away using the CAPM and Fama and French 3 factor models. The excess returns we use are the returns in excess of the buy and hold strategy which is a little more aggressive than the typical Risk-Free rate of return. Noted that the market return is much higher than the risk-free rate. The models tell us whether the portfolios are generating returns by taking on excess risk.

Table 1

We calculate the 5-day Stochastic (PK) using the 5-day highest close and lowest close, including the current close. The indicator construction and rules are in the methodology. If it is above 20 (oversold), it is an in-the-market signal, and we will invest in the decile portfolios for the next trading day; otherwise, it is an out-of-the-market signal, and we will invest in the 30-day risk-free T-bill for the next trading day. We use the 10 NYSE/AMEX volatility decile portfolios as the investment assets. We report the average return (Avg Ret). And the standard deviation (Std Dev) for the buy-and-hold benchmark decile portfolios (Panel A), the PK timing decile portfolios (Panel B), and the PK portfolios (MAP PK) are the differences between the PK timing portfolios and the buy-and-hold portfolios (Panel C). The results are annualized and in percentages. We also report the annualized Sharpe ratio (Sratio) for the buy-and-hold portfolios and the PK timing portfolios, and we report the success rate for the MAP PK. The sample period is from January 1, 1963, to December 31, 2018. Y-statistics are in parentheses. ** and * indicate significance at the 1% and 5% levels, respectively. Other 30, 40, and 50 may be used for an in-the-market signal, and the MAPs are shown in the

		Panel A. Volatility Decile Portfolios				Panel B. BB (20) Timing Portfolios			Panel C. MAP BB	
Rank	Avg Ret	Avg Ret	Std Dev	Sratio	Avg Ret	Std Dev	Success	Std Dev	Sratio
Low	10.10 **	11.37**	6.23	106.37	1.27**	2.52	57.33	6.73	79.69
	(11.13)	(13.52)			(3.72)				
2	12.02 **	13.43**	8.79	98.90	1.41**	2.96	56.55	9.28	78.46
	(9.60)	(11.32)			(3.53)				
3	13.45 **	14.75**	10.79	92.82	1.30**	3.48	56.07	11.34	76.85
	(8.79)	(10.13)			(2.77)				
4	14.10 **	15.68**	12.50	87.55	1.58**	3.76	56.37	13.06	71.70
	(8.00)	(9.30)			(3.12)				
5	14.45 **	15.96**	14.15	79.33	1.52**	3.81	55.83	14.66	66.22
	(7.30)	(8.36)			(2.95)				
6	15.35 **	16.73**	15.18	79.01	1.38**	4.15	55.86	15.74	67.44
	(7.23)	(8.17)			(2.46)				
7	14.87 **	16.78**	16.48	73.10	1.92**	4.70	55.69	17.14	59.09
	(6.43)	(7.55)			(3.03)				
8	14.71 **	17.35**	17.88	70.58	2.64**	4.95	55.71	18.55	53.76
	(5.88)	(7.20)			(3.96)				
9	16.49 **	18.43**	18.82	72.75	1.94**	5.27	55.47	19.54	60.11
	(6.25)	(7.26)			(2.73)				
High	38.21 **	39.76**	19.91	175.85	1.55**	5.07	57.32	20.56	162.80
	(13.78)	(14.80)			(2.26)				
High-Low	28.11 **	28.39**	17.39	136.00	0.28	4.60	57.30	17.61	132.76
	(11.83)	(12.10)			(0.45)				

previous table.

This table shows that the risk-adjusted returns are superior to the buy and hold with lower risk measured by standard deviation. MAPs show excess returns at each decile, and all the returns are statistically significant. PK is the most substantial result. It shows highly significant results and has returns that are much higher than the buy and hold.

This table shows that the risk-adjusted returns are superior to the buy and hold with lower risk measured by standard deviation. The MAPs show the excess returns at each decile. All the returns are statistically significant. The most accurate decile is the top decile with 57.98 percent accuracy. They were showing an average excess return of 8.80 percent annualized with a

This table shows that the risk-adjusted returns are superior to the buy and hold with lower risk measured by standard deviation. The MAPs show the excess returns at each decile. All the returns are statistically significant.

The results are lower than the MA10, but this suggests that the Bollinger Band can be used as a timing tool on the volatility decile portfolios. It is common in technical trading to combine several technical indicators. We explore the ability of the RSI and Stochastic to time the volatility decile portfolios in the following tables.

For the default parameter, the Bollinger Band has several success rates above 57, while the moving average of 10 only had one. Our average excess return is a bit lower; however, it is met by much lower volatility. We see similar Sharpe ratios in the triple digits.

Table 1 (b)

We calculate the 14-day relative strength index (RSI) prices each day using the last 14 days' average of up and down closing prices, including the current closing price. The up-closing price is recorded if the close is greater than the previous close. The down closing price is recorded if the close is lower than the previous close. RS is the ratio of the simple mean of the up-closing price to the down closing price. RSI is 100-100/(1+RS). The value ranges from 0 to 100. If the RSI value is above 20, it is an in-the-market signal, and we will invest in the decile portfolios for the next trading day; otherwise, it is an out-of-the-market signal, and we will invest in the 30-day risk-free T-bill for the next trading day. We use the 10 NYSE/AMEX volatility decile portfolios as the investment assets. We report the average return (Avg Ret). And the standard deviation (Std Dev) for the buy-and-hold benchmark decile portfolios (Panel A), the RSI timing decile portfolios (Panel B), and the RSI portfolios (MAP RSI) are the differences between the RSI timing portfolios and the buy-and-hold portfolios (Panel C). The results are annualized and in percentages. We also report the annualized Sharpe ratio (Sratio) for the buy-and-hold portfolios and the MA timing portfolios, and we report the success rate for the MAPs. The sample period is from January 1, 1963, to December 31, 2018; t-statistics are in parentheses. ** and * indicate significance at the 1% and 5% levels, respectively. Other periods of 30, 40, and 50 may be used for an in-the-market signal but are not recorded.

	Panel B. RSI (14) Timing Portfolios			Panel C. MAP RSI		
Rank	Avg Ret	Std Dev	Sratio	Avg Ret	Std Dev	Success
Low	11.52**	5.65	120.02	1.42**	3.65	56.84
	(15.11)			(2.88)		
2	13.66**	8.51	104.94	1.64**	3.70	56.73
	(11.91)			(3.29)		
3	15.14**	10.28	101.13	1.68**	4.77	56.64
	(10.91)			(2.61)		
4	15.73**	11.87	92.62	1.63*	5.44	55.86
	(9.82)			(2.22)		
5	15.79	13.55	81.58	1.35	5.59	55.34
	(8.64)			(1.78)		
6	16.81**	14.59	82.73	1.46	5.89	55.64
	(8.54)			(1.84)		
7	16.88**	15.94	76.19	2.01**	6.30	55.63
	(7.85)			(2.37)		
8	17.91**	17.18	76.67	3.20**	6.98	55.45
	(7.73)			(3.40)		
9	18.94**	17.89	79.40	2.45**	7.86	55.28
	(7.85)			(2.31)		
High	38.25**	19.05	175.95	0.04	7.74	56.64
	(14.89)			(0.04)		
High-Low	26.74	17.28	127.30	(1.38)	7.39	55.93

This table shows that the risk-adjusted returns are superior to the buy and hold with lower risk measured by standard deviation. The RSI is significant at all levels except for the 5th and 6th deciles, high and high minus low. Results are mixed for excess returns using the RSI as a timing tool.

This indicator has similar results as the above two indicators; however, the excess returns are lower and insignificant in the top decile. The success rate is above 55 percent for all the deciles, which is like the above results. Thus, we have a positive expectation for these portfolios. The 20 thresholds may be too constricting, which is why we don't see better results. The robustness check excess returns may show better results because 20 could be too low of a threshold for buying. Prices may revert far above 20 and never hit the threshold. They could also, at times, be using 20 to correct to a new trend. Henry (1999) suggests this.

Table 1 (c)

We calculate the 10-day moving average (MA) prices each day using the last ten days' closing prices, including the current closing price, and compare the MA price with the current price as the timing signal. If the current price is above the MA price, it is an in-the-market signal, and we will invest in the decile portfolios for the next trading day; otherwise, it is an out-of-the-market signal, and we will invest in the 30-day risk-free T-bill for the next trading day. We use the 10 NYSE/AMEX volatility decile portfolios as the investment assets. We report the average return (Avg Ret). And the standard deviation (Std Dev) for the buy-and-hold benchmark decile portfolios (Panel A), the MA timing decile portfolios (Panel B), and the MA portfolios (MAPs) are the differences between the MA timing portfolios and the buy-and-hold portfolios (Panel C). The results are annualized and in percentages. We also report the annualized Sharpe ratio (Sratio) for the buy-and-hold portfolios and the MA timing portfolios, and we report the success rate for the MAPs. The sample period is from January 1, 1963, to December 31, 2019; t-statistics are in parentheses. ** and * indicate significance at the 1% and 5% levels, respectively.

	Panel B. MA (10) Timing Portfolios			Panel C. MAP MA		
Rank	Avg Ret	Std Dev	Sratio	Avg Ret	Std Dev	Success
Low	14.20 **	3.78	250.65	4.10 **	5.55	56.82
	(27.88)			(5.48)		
2	15.41 **	5.75	185.62	3.39 **	7.28	55.21
	(19.78)			(3.45)		
3	17.52 **	6.86	186.43	4.07 **	9.02	54.77
	(18.94)			(3.34)		
4	18.90 **	7.94	178.42	4.80**	10.35	55.01
	(17.65)			(3.44)		
5	19.90 **	8.98	168.80	5.46 **	11.56	54.64
	(16.42)			(3.50)		
6	21.59 **	9.76	172.63	6.25 **	12.31	55.44
	(16.39)			(3.76)		
7	21.71 **	10.70	158.67	6.84 **	13.37	55.43
	15.05			(3.80)		
8	25.90 **	11.63	181.98	11.19 **	14.40	55.90
	(16.51)			(5.76)		
9	28.10 **	12.28	190.26	11.62 **	15.14	56.10
	(16.96)			(5.69)		
High	47.01 **	13.60	310.88	8.80 **	15.33	57.98
	(25.63)			(4.26)		
High-Low	32.82 **	12.89	217.78	4.70 **	13.31	54.09
	(18.87)			(2.62)		

Table 2 (a)

Table 2 reports the alphas, betas, and adjusted R^2 of the regressions of the MAPs formed from the 14-day RSI timing strategy on the market factor (Panel A) and the Fama-French (1993) 3 factors (Panel B), respectively. The alphas are annualized and in percentages. The sample period is from January 1, 1963, to December 31, 2018.

	Panel a. CAPM			Panel B. Fama-French				
Rank	α	βmkt	R^2 %	α	βmkt	βsmb	βhml	R^2 %
Low	1.85**	(0.07)**	9.27	2.00**	(0.07)**	(0.02)**	(0.02)**	9.71
	(3.95)	(-37.63)		(4.27)	(-38.48)	(-5.36)	(-6.09)	
2	2.11**	(0.08)**	10.36	2.05**	(0.07)**	(0.02)**	0.02	10.75
	(4.46)	(-40.01)		(4.34)	(-38.31)	(-5.44)	(6.05)	
3	2.33**	(0.11)**	12.05	2.23**	(0.10)**	(0.01)**	0.03**	12.28
	(3.86)	(-43.57)		(3.69)	(-41.45)	(-2.49)	(5.81)	
4	2.38**	(0.12)**	12.53	2.38**	(0.12)**	(0.05)**	0.02**	13.10
	(3.47)	(-44.54)		(3.47)	(-43.48)	(-8.75)	(4.45)	
5	2.10**	(0.12)**	11.87	2.11**	(0.12)**	(0.06)**	0.03**	12.82
	(2.96)	(-43.19)		(3.00)	(-42.40)	(-11.48)	(5.23)	
6	2.22**	(0.12)**	10.87	2.18**	(0.12)**	(0.08)**	0.05**	12.34
	(2.96)	(-41.11)		(2.92)	(-40.07)	(-13.28)	(8.15)	
7	2.81**	(0.13)**	10.50	2.66**	(0.13)**	(0.08)**	0.07**	12.33
	(3.50)	(-40.31)		(3.34)	(-38.63)	(-13.14)	(11.51)	
8	4.04**	(0.14)**	9.58	4.06**	(0.14)**	(0.13)**	0.07**	12.46
	(4.52)	(-38.31)		(4.60)	(-38.08)	(-19.64)	(9.38)	
9	3.48**	(0.17)**	11.16	3.59**	(0.17)**	(0.13)**	0.05**	13.28
	(3.48)	(-41.71)		(3.63)	(-41.75)	(-17.70)	(6.07)	
High	0.92	(0.14)	8.42	0.98	(0.15)	(0.13)	0.06	10.66
High-Low	(0.93)	(0.07)**		(1.02)**	(0.07)**	(0.11)**	0.08**	
	(-0.94)	(-18.31)		(-1.04)	(-17.60)	(-14.89)	(9.81)	

This table shows the remaining alpha is still positive and significant at all deciles except for the high decile after using the market factor and the Fama and French 3-Factors. The Market, SMB, and HML factors are all adverse in explaining the excess returns, except for the HML in the top decile. We explore RSI in the following table.

Table 2 (b)

Table 2 reports the alphas, betas, and adjusted R^2 of the regressions of the MAPs formed from the 20-day BB timing strategy on the market factor (Panel A) and on the Fama-French (1993) 3 factors (Panel B), respectively. The alphas are annualized and in percentages. The sample period is from January 1, 1963, to December 31, 2018.

	Panel a. CAPM			Panel B. Fama-French				
Rank	α	βmkt	R^2 %	α	βmkt	βsmb	βhml	R^2 %
Low	1.44**	(0.03)**	4.10	1.44**	(0.03)**	0.00	0.00	4.12
	(4.40)	(-24.36)		(4.33)	(-23.28)	(-1.46)	(-1.11)	
2	1.70**	(0.05)**	5.95	1.81**	(0.05)**	0.00	(0.02)**	6.33
	(4.37)	(-29.59)		(4.66)	(-30.42)	(0.38)	(-7.74)	
3	1.66**	(0.06)**	7.06	1.76**	(0.06)**	(0.00)**	(0.02)**	7.25
	(3.67)	(-32.44)		(3.88)	(-32.73)	(-0.02)	(-5.53)	
4	1.96**	(0.06)**	6.57	2.11**	(0.07)**	(0.01)**	(0.03)**	6.92
	(4.00)	(-31.21)		(4.31)	(-32.14)	(-3.44)	(-6.40)	
5	1.88**	(0.06)**	5.96	2.02**	(0.06)**	(0.02)**	(0.02)**	6.33
	(3.77)	(-29.61)		(4.05)	(-30.40)	(-6.19)	(-4.05)	
6	1.77**	(0.06)**	5.84	1.95**	(0.07)**	(0.04)**	(0.02)**	6.49
	(3.26)	(-29.31)		(3.59)	(-30.44)	(-8.87)	(-4.00)	
7	2.38**	(0.08)**	6.31	2.58**	(0.08)**	(0.05)**	(0.02)**	7.04
	(-3.88)	(-30.54)		(4.22)	(-31.68)	(-9.68)	(-3.69)	
8	3.10**	(0.07)**	5.51	3.25**	(0.08)**	(0.06)**	(0.00)	6.36
	(4.77)	(-28.44)		(5.02)	(-29.12)	(-11.21)	(-0.38)	
9	2.40**	(0.07)**	4.90	2.53**	(0.08)**	(0.07)**	(0.01)	5.97
	(3.46)	(-26.74)		(3.67)	(-27.29)	(-12.57)	(1.27)	
High	1.93**	(0.06)**	3.78	2.02**	(0.07)**	(0.05)**	0.01	4.38
	(2.88)	(-23.35)		(3.02)	(-23.58)	(-9.35)	(1.13)	
High-Low	0.47	(0.03)**		0.58	(0.03)	(0.05)**	0.00**	
	(0.75)	(-12.24)		(0.94)	(0.63)	(-10.97)	(-13.06)	

The BB Timing shows significant positive returns at every decile except for the high-low in both sorts.

Table 2 (c)

Table 2 reports the alphas, betas, and adjusted R^2 of the regressions of the MAPs formed from the 10-day MA timing strategy on the market factor (Panel A) and the Fama-French (1993) 3 factors (Panel B), respectively. The alphas are annualized and in percentages. The sample period is from January 1, 1963, to December 31, 2018.

	Panel a. CAPM			Panel B. Fama-French				
Rank	α	βmkt	R^2 %	α	βmkt	βsmb	βhml	R^2 %
Low	5.23**	(0.18)**	26.90	5.41**	(0.19)**	(0.04)**	(0.02)**	27.29
	(8.16)	(-71.38)		(8.47)	(-71.24)	(-7.35)	(-4.04)	
2	5.24**	(0.30)**	42.07	5.71**	(0.31)**	(0.08)**	(0.06)**	43.30
	(7.01)	(-100.26)		(7.72)	(-102.16)	(-13.71)	(-9.96)	
3	6.54**	(0.40)**	49.29	7.21**	(0.42)**	(0.11)**	(0.08)**	50.86
	(7.55)	(-116.00)		(8.44)	(-118.98)	(-16.55)	(-12.20)	
4	7.72**	(0.48)**	52.00	8.68**	(0.50)**	(0.17)**	(0.11)**	54.65
	(7.98)	(-122.46)		(9.21)	(-128.23)	(-23.65)	(-14.66)	
5	8.75**	(0.54)**	52.83	10.02**	(0.57)**	(0.25)**	(0.14)**	56.96
	(8.16)	(-124.51)		(9.78)	(-133.82)	(-31.45)	(-16.90)	
6	9.71**	(0.56)**	51.49	11.03**	(0.60)**	(0.31)**	(0.12)**	56.36
	(8.39)	(-121.22)		(10.04)	(-131.16)	(-36.21)	(-13.48)	
7	10.54**	(0.60)	49.84	12.03**	(0.64)**	(0.37)**	(0.12)**	55.73
	(8.25)	(-117.28)		(10.02)	(-128.55)	(-40.41)	(-12.48)	
8	15.09**	(0.64)**	47.93	16.61**	(0.67)**	(0.45)**	(0.08)**	54.85
	(10.76)	(-112.88)		(12.71)	(-124.60)	(-44.93)	(-7.98)	
9	15.59 **	(0.65) **	44.98	17.12**	(0.69)**	(0.52)**	(0.05)**	53.09
	(10.29)	(-106.37)		(12.23)	(-118.39)	(-48.48)	(-4.36)	
High	12.26**	(0.56)**	33.28	13.89**	(0.60)**	(0.55)**	(0.05)**	42.07
	(7.26)	(-83.09)		(8.81)	(-92.72)	(-45.38)	(-4.26)	
High-Low	7.04**	(0.38)**		8.47**	(0.42)**	(0.51)**	(0.03)**	
	(4.38)	(-59.00)		(5.64)	(-66.98)	(-2.75)	(-44.48)	

This table shows the remaining alpha is still positive and significant at all deciles after using the market factor and the Fama and French 3-Factors. The Market, SMB, and HML factors are all adverse in explaining the excess returns. We explore the remaining indicators in the following tables.

We change the analysis of these indicators to time the portfolios sorted on size, which is a value-weighted portfolio instead of equally weighted. We know that equally weighting may overstate the emphasis of small stocks in the analysis. What we expect to see is that the technical indicators can time the size sorted portfolios. That size acts as a volatility decile portfolio with the highest volatility in the small stocks.

Table 3. Size Decile Portfolios

We report the average returns (Avg Ret) and the Fama-French (1993) alphas (FF) of the MAPs when they are constructed with 10 NYSE/AMEX/NASDAQ value-weighted market cap decile portfolios by using MA10, RSI, and Stochastic (%k) respectively. The results are annualized and in percentages. The sample period is from January 1, 1963, to December 31, 2018.

	MAP(RSI)		MAP(BB)		MAP(MA)	
Rank	Avg Ret	FF α	Avg Ret	FF α	Avg Ret	FF α
Large	0.77	0.41**	0.64**	0.88**	0.88	1.90**
	(0.54)	(4.48)	(2.69)	(3.82)	(0.47)	(4.39)
2	3.79**	3.32**	0.71**	0.95**	3.88**	3.34**
	(2.32)	(8.10)	(3.14)	(4.36)	(2.37)	(8.06)
3	6.20**	4.14**	0.72**	0.93**	6.27**	4.43**
	(3.67)	(10.45)	(3.32)	(4.46)	(3.71)	(10.40)
4	6.68**	4.36**	0.56**	0.79**	6.78**	4.63***
	(4.00)	(10.90)	(2.48)	(3.62)	(4.06)	(11.03)
5	7.71**	4.58**	0.95**	1.16**	7.77**	4.95**
	(4.78)	(11.88)	(4.47)	(5.67)	(4.82)	(11.89)
6	9.67**	4.91**	0.93**	1.15**	9.68**	5.60**
	(6.29)	(13.63)	(4.37)	(5.58)	(6.30)	(13.72)
7	10.82**	5.19**	0.90**	1.10**	10.80**	5.61**
	(7.94)	(14.49)	(4.48)	(5.68)	(7.94)	(14.51)
8	10.58**	5.17**	0.86**	1.00**	10.49**	5.24**
	(8.55)	(14.23)	(4.72)	(5.67)	(8.50)	(14.16)
9	10.05**	4.86**	0.79**	0.93**	9.96**	5.02**
	(8.26)	(13.49)	(4.60)	(5.59)	(8.22)	(13.43)
Small	9.18**	4.15**	0.56**	0.70**	9.10**	4.93**
	(6.10)	(10.56)	(3.00)	(3.82)	(6.09)	(10.66)
Small-Large	8.37**	3.00**	(0.09)	(0.21)	8.27**	2.95**
	(5.65)	(5.46)	(-0.33)	(-0.77)	(5.58)	(5.38)

From the table we see that RSI does comparable to the MA at size deciles. This is interesting because RSI does not beat the MA on the volatility sort. PK does a little better in all deciles. More notably is the large stock decile. The MA does the worst (a little under one percent) in the large stock decile while PK does nearly 5 percent. Percent R does better than the MA indicator at the Large-9 decile.

Robustness to alternate specifications.

We discuss various entry thresholds for our indicators below. We show the default settings in the main tables of the paper. They are not necessarily the best results but the more common ones.

Additional levels for entry and exit (as discussed above) are commong for technical trading. We show the excess returns in the tables below with the standard deviations and accuracy.

Table 4. RSI Threshold Levels

	RSI 30			RSI 40			RSI 50		
Panel A.	AVRet	Sdret	ACC	AVRet	Sdret	ACC	AVRet	Sdret	ACC
1	2.29	4.55	57.01	1.43	5.62	54.73	1.43	5.62	54.73
2	1.75	5.26	56.53	0.29	7.29	53.59	0.29	7.29	53.59
3	2.61	6.21	56.52	1.26	9.17	53.70	1.26	9.17	53.70
4	2.43	7.23	56.00	1.73	10.50	53.98	1.73	10.50	53.98
5	3.93	7.78	55.63	3.26	11.70	54.40	3.26	11.70	54.40
6	3.55	8.17	55.78	2.44	12.59	54.29	2.44	12.59	54.29
7	3.89	8.91	56.22	3.87	13.60	54.44	3.87	13.60	54.44
8	6.17	10.13	55.98	7.68	14.77	54.91	7.68	14.77	54.91
9	5.70	10.45	55.92	7.35	15.45	54.98	7.35	15.45	54.98
10	1.96	11.03	57.08	3.93	15.71	56.53	3.93	15.71	56.53
hml	(0.33)	9.89	50.78	2.50	13.64	54.60	2.50	13.64	54.60

Table 5. BB Threshold Levels

	BB 2 10			BB 1 20			BB 1 10		
Panel C.	AVRet	Sdret	ACC	AVRet	Sdret	ACC	AVRet	Sdret	ACC
1	1.35	3.25	57.62	3.73	4.92	57.59	5.08	4.69	59.11
2	1.44	3.51	56.57	3.38	6.14	56.76	4.60	5.89	57.14
3	1.49	4.84	56.21	5.11	7.64	56.64	5.82	7.28	57.26
4	1.41	5.51	56.15	4.41	8.74	56.26	5.49	8.13	56.57
5	2.56	6.07	56.00	6.17	9.58	56.30	7.67	9.25	56.78
6	2.91	6.21	55.95	6.52	10.33	56.71	7.35	9.66	57.20
7	2.84	6.73	55.89	7.47	11.17	56.71	9.88	10.72	57.50
8	4.02	7.04	55.64	10.30	12.25	56.66	11.34	11.49	57.22
9	3.40	7.21	55.56	10.38	12.79	57.01	12.10	12.04	57.38
10	3.00	7.58	57.59	9.51	12.85	58.72	9.89	12.03	59.19
hml	1.65	7.01	48.66	5.78	11.19	53.38	4.81	10.73	54.21

BB 1,10 shows some promise for excess returns so does BB 1,20.

You could take any of these thresholds in the table above to recreate the tables below or generate timing signals. The best signal is BB(1,10), where 1 denotes the standard deviation and 10 denotes the moving average period. We show more common or default indicator settings in the main tables.

We show the MAPs for these technical rules below. We can see that they are much better than the MA10 and are shared among indicators used in technical trading. The added advantage of these new indicators for volatility decile portfolios is that they have areas for overbought and oversold, which moving averages don't have. We obtain similar but different signals than the MA10. Despite tweaking the Average Moving Length, we cannot get results as good as the results shown in the table below. These are by changing the thresholds on the standard parameters. Changing the parameter length may yield different results.

The above analysis suggests that the indicators used may be compared to the MA. The return of the Stochastic (PK) indicator is seemingly more significant than the MA. The robustness check of the RSI 40,50 and Bollinger Band (1,10) and (1,20) seems to do better than having broader volatility for the band (i.e., 2,10 and 2,20) than the standard time frame results we show in the tables. BB (1,10) and BB (1,20) are slightly better than the MA, along with PK(5) 40 and PK(5) 50. The advantage is that the MA can only change length while the BB and PK can both change in length and entry signal. Looking at how far away price is to the MA might better relate the MA to these other indicators. We suggest that different parameters for these indicators may have a better timing ability than the MA and are commonly used in practice alongside it. Additional useful indicators in asset pricing studies.

Price Analysis

Table 6. BB and RSI levels and returns

BB	ret	sigma
sigma	5.91	0.30
+	1.85	0.16
MA10	0.35	0.12
-	-1.00	0.10
sigma	**-5.91**	**0.21**

RSI	ret	sigma
OVERSOLD	**-2.74**	**0.18**
40.00	-1.14	0.16
50.00	0.32	0.14
60.00	1.46	0.14
OVERBOUGHT	2.99	0.14

The table shows the BB has positive return when above the MA10. The return is greatest when it comes towards the upper band. The returns are negative when price is at low levels near the lower band.

The RSI has high returns near overbought and low returns near oversold.

Table 7. Summary Statistics RSI

	Mean	Std Dev	T (p)
RSI20	*0.43*	0.19	693.15
ret	*0.21*	0.20	0.00

Table 7a. Summary Statistics Bollinger Band

	Mean	Std Dev	T (p)
BB(LB)	*1.58*	0.58	2000.00
ret	*0.21*	0.70	0.00

We break our sample into price levels. $1-$25, $26-$50, $50+. We look at the summary statistics inside this.

Table 8. Price Level Analysis

	Bollinger Band		*RSI*		Buy and Hold	
Prc	**Mean**	**Std Dev**	**Mean**	**Std Dev**	**Mean**	**Std Dev**
$1-$25	179.9%	65.81%	45.0%	76.72%	17.5%	79.28%
$26-$50	104.0%	30.96%	38.3%	36.38%	28.6%	37.38%

>$50	97.8%	32.83%	41.6%	36.48%	34.5%	37.14%

We control for maximum draw down using 20 day rolling windows over the first peak to next trough.

Table 9. Maximum Drawdown Full Sample

	Max DD	Sigma
RSI	*51.88%*	23.81%
Bollinger B	*19.39%*	*14.67%*
BH	*79.53%*	26.24%

Table 10 t-test

	t-test	
	In Sample (1926-1950)	*Out of Sample (1950-2021)*
Bollinger B	262.1428	1.2+e03
RSI	113.58	463.28

Table 11 in sample and out of sample analysis.

Trailing 3 year Sample Analysis

We break the sample in to the three years prior to the publication by Han et al (2014) and three years after the publication. Leaving one year for the market to adjust. We find that the results hold both in and out of sample.

	In Sample (1926-1950)			*Out of Sample (1950-2021)*		
	Return	**Std Dev**	**Sharpe**	**Return**	**Std Dev**	**Sharpe**
Bollinger B	134.80%	43.66%	3.04	142.65%	43.82%	3.21
RSI	44.22%	72.67%	0.58	42.64%	67.03%	0.61
Buy Hold	26.32%	74.76%	0.33	20.55%	69.97%	0.27

All Decile	2012-2015	2015-2018
MAT	5.73%	7.97%
Wret	5.90%	5.74%

We see a slight improvement after the one year period leading through 12/31/2018. This provides a guide for indicating the indicators will still work after the article was published, in contrast to McLean and Pontiff (2016) which states that excess return studies fall apart post publication.

Decile 10	2012-2015	2015-2018
MAT	22.92%	19.15%
Wret	14.74%	8.89%

For Decile 10 we see that the results are similar between both of the periods prior to publication and after publication. We see from the above results that the MA10 is one of the strongest indicators and we expect that the other indicators would hold similarly.

IV. Conclusion

We can see that technical indicators common in technical trading circles can be used to time the volatility decile portfolios. The primary buy and sell signals are buying when indicators rebound from oversold and selling when they reach new oversold levels. The intuition behind the success of this rule is following Covel (2009) that trending markets are characteristic of higher highs and higher lows. Thus, buying around the oversold levels and riding the highs will produce profitable returns.

We can conclude that additional indicators such as the Relative Strength Index, Bollinger Bands, and Stochastic %k can be used to time the volatility decile portfolios. From a practical perspective, fund managers who hold many securities for clients could sort their portfolios by risk and use these types of indicators to enter and exit the market daily.

There isn't a one-size-fits-all technical indicator, and it is common to stack many on a chart.

We take to buy and sell signals when the indicators show that the price deviates from the trend. In practice, this would be done when many indicators make signals at a similar time. Chartists use their eyes to distinguish signals from noise which makes the scientific study of it difficult.

Future work would include combining several indicators in a genetic algorithm to time the portfolios and use neural networks to predict the tops and bottoms. We see from the excess returns that the indicators here, in some cases, perform much better than the moving average of 10. However, not in all circumstances.

Stochastic performs similar to the MA while BB and RSI both beat the buy and hold. It might be faster to pick up on changes and maybe more valuable and flexible than the MA, given its construction is based on other price properties such as high and low other than just the close. We usually see RSI or BB studied in the literature. PK is not often mentioned.

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Development of Optimal Stock Portfolio Selection Model in the Tehran Stock Exchange by Employing Markowitz Mean-Semivariance Model

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Abstract

In an increasingly complex financial market, selecting the optimal stock portfolio has become a subject of intense debate. This study aims to develop a model for optimal stock portfolio selection. We apply Markowitz's mean-semivariance approach to determine the downside risk of portfolios, which reflects investors' intuitive perception of risk. In the first stage, the combination of the Analytic Hierarchy Process (AHP) and Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) with interval data is employed to identify and rank good quality stocks according to the recommended criteria by experts. After selecting qualified stocks, in the second phase, we create portfolios, and the weight invested in each stock is determined. Then, three portfolios are created for three groups of risk-averse, neutral to risk, and risk-taker investors. The mean-semivariance optimization model is used in this phase. The proposed approach in the paper is implemented in a real case study of the Tehran stock exchange (TSE). Three portfolios for three groups of investors were evaluated and compared to the market performance using sharp criteria. All three portfolios outperformed the market portfolio both in terms of risk and return. The proposed model of this study can be utilized as a decision support tool when forming an optimal stock portfolio by considering both experts' opinions on stock evaluation and investor risk preferences simultaneously.

Keywords: Stock Portfolio, Interval TOPSIS, Optimization, Markowitz mean-semivariance model

I. Introduction

In an increasingly complex financial market, selecting the optimal stock portfolio has become a subject of intense debate. People around the world make investment decisions every day. It is often difficult to make such decisions due to the dynamic nature of capital markets. In developing countries, deciding about investing in financial markets, assessing portfolios, and selecting portfolios have become widespread and significant issues. Thus, portfolio selection needs to be studied in more detail (Li et al., 2017; Maykao & Yanpiranat, 2012; Wu et al., 2019). Iranian capital markets are more uncertain and riskier than other investment avenues due in part to the complex environment of global markets, rapid technological progress and globalization, and the convergence of industries. Due to the fact that investing can have a significant impact on a person's life, sophisticated mechanisms are usually required to guide the selection of a portfolio.

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II. Literature review

An optimal investment portfolio selection is the cornerstone of any investment project. What investors hope to achieve through portfolio optimization is to maximize portfolio return and minimize portfolio risk. Harry Markowitz and his groundbreaking research in 1952 formulated a revolutionary portfolio risk-return model known as mean-variance optimization. Since that time, it has been extensively used to determine optimal portfolio allocation. In their study, Chen et al. (2017) proposed a multi-objective uncertain portfolio optimization model for maximizing expected return while minimizing the risk associated with removing skewness in portfolio returns. They developed the model in order to maximize expected return coupled with skewness and therefore minimize risk and increase returns. In the study by Sun et al. (2016), an optimal analytical solution was developed to overcome the problem of choosing multiperiod portfolios. Mehlawat (2016) developed two multi-objective multi-period fuzzy portfolio selection models based on credibility, in which a multi-choice goal programming approach was used to solve the problem. Zhang and Liu (2015) presented a novel genetic algorithm aiming to deal with a multi-period fuzzy portfolio selection problem based on ranking parameters as well as returns. Zhang and Li (2021) considered risky asset returns uncertain variables that experts estimated. By considering the return, risk, liquidity, and diversification degree of portfolios, they used a mean-variance-entropy model for uncertain portfolio optimization. Their model quantified the investment return as uncertain expected value, the investment risk as uncertain variance, and the diversification level of a portfolio by entropy.

Even though the Markowitz Mean-Variance framework is theoretically sound, there has always been criticism of how risk is measured in it (Michaud & Michaud, 2008; Zhang et al., 2018). The risk in Markowitz's mean-variance framework is measured in terms of the variance of expected portfolio returns. The underlying assumption of using variance as the appropriate measure for risk is that investors weigh the probability of negative returns equally against the probability of positive returns (Boasson et al., 2011; Grootveld & Hallerbach, 1999). Variance treats the favorable upside dispersion of investment returns over the mean as part of the risk and penalizes it as much as the unfavorable downside deviations from the mean; therefore, it is an inappropriate measure of risk (Leland, 1999). An incorrect risk measure makes it possible to construct an efficient frontier that yields a nonsensical result when optimizing a portfolio (Lam, 2016).

Markowitz advocated the use of semivariance rather than variance as a measure of risk in 1959 since semivariance was perceived to reflect downside losses rather than upside gains as a risk factor. In his view, since investors are more concerned with downside risks than overall volatility, assessing risk by semivariance produces more efficient portfolios. Eftekhari and Satchell (1996) defined semivariance as the weighted sum of square deviations from a certain threshold, considering only values below the expected value of returns (see section B for more detail). In this paper, the mean-semivariance model is employed in order to construct efficient portfolios.

We should also consider that Markowitz's portfolio theory only provides a solution to an optimization problem among predetermined stocks regardless of the quality of stocks. Hence, it is not recommended to use portfolio theory blindly to assess the value of low-quality stocks without first determining the quality of such stocks. In order to build a portfolio that is as valuable as possible, it is best to select a few high-quality stocks first and then use optimization techniques to develop the portfolio. Although it appears simple initially, the process of choosing stocks is very complex because there are so many different objectives to take into consideration, sometimes with conflicting objectives (Hilborn & Walters, 2013). The key question here is how to rank the stocks based on the factors that affect their performance. In order to make sound decisions about portfolio selections, it is important to consult experts and review financial reports (Zhang et al., 2018). It is, however, extremely difficult to choose the optimal alternative when there are a large number of possible solutions to a given problem. A problem may be exacerbated in instances where some criteria have an effect on certain problems, but in order to reach a satisfactory solution, the alternatives must all be based on the same criteria, which inevitably leads to a more informed and better decision. Multi-Criteria Decision Making (MCDM) pertains to structure and solving decision and planning problems involving multiple criteria (Aruldoss et al., 2013). The MCDM process evaluates alternatives by calculating weights for each criterion and then ranking them based on the weight of the criteria (Zeleny & Cochrane, 1973).

In recent decades, MCDM methods have been increasingly popular and employed across a wide spectrum of applications (Chakraborty & Chakraborty, 2021; Malik et al., 2021; Popović, 2021; Tan et al., 2021; Velasquez & Hester, 2013). In particular, MCDM has been employed in the ranking and selection of stocks. Jamei (2020) applied the MCDM model to pharmaceutical firms listed on the Tehran Stock Exchange (TSE). In order to rank the firms, he utilized the SAW and TOPSIS MCDM models. Ece and Oguzhan (2017) conducted Fuzzy TOPSIS to determine the optimal portfolio, indicating that the portfolio had positive returns, risks, as well as favorable performance. In Galankashi et al. 's (2020) study, the criteria were derived by using a Likert-type questionnaire, and the FANP was used to rank the TSE companies.

The TOPSIS technique employed in this study is considered to be one of the most popular MCDM approaches due to its simplicity, consistency, and reliability (Al-Aomar, 2010; Formisano et al., 2017; Ince et al., 2017; Roshandel et al., 2013; Sotoudeh-Anvari & Sadi-Nezhad, 2015). Furthermore, a rigorous evaluation of several MCDM methods showed that combining these methodologies eliminates some of the weaknesses of specific MCDM methods (Velasquez & Hester, 2013), in addition to providing a novel approach to decision making. This paper aims to address these issues by combining two MCDM methods, namely AHP and Interval TOPSIS, to produce a reliable ranking of companies.

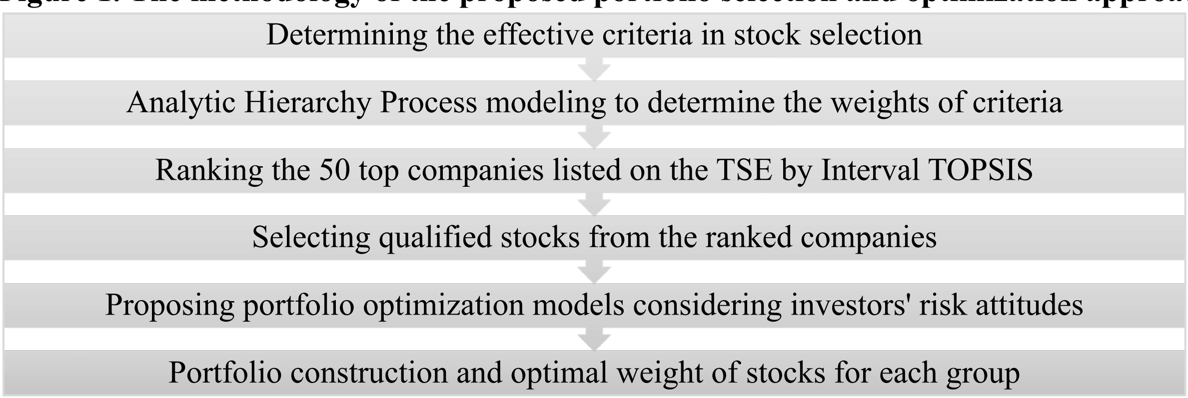
We also took into consideration in our proposed approach to portfolio construction the fact that different investors have different levels of risk tolerance. Different investors may see events very differently. For example, aggressive investors may be inclined to take risks, while conservative investors focus on the security of investment returns and the protection of their principal. A portfolio's optimal composition is determined by the investor's risk and return preferences. Taking into account the fact that investment returns vary with risk, investors need to find a balance between risk and return. In general, there is no single optimal portfolio for all investors. Thus, different decision-makers possessing different risk attitudes will require different portfolios (Deng et al., 2021; Makui & Mohammadi, 2019l Mansini et al., 2015). Investors in finance can be classified into three broad categories: risk-averse investors, risk seekers, and risk-neutral investors (Setiawan et al., 2022; Singh et al., 2020). Due to this, we construct three different portfolio models for each investment group.

In this paper, we present a robust and applicable optimal portfolio construction model using interval data in the TOPSIS matrix as opposed to definite numbers for ranking stocks, using semivariance instead of mean-variance as the risk factor, and presenting customized portfolios for different risk inclinations. In the first stage, the integration of AHP with Interval TOPSIS is used to identify and rank good quality stocks in accordance with the criteria recommended by experts. After selecting qualified stocks, we then proceed to create stock portfolios. Three portfolios are created for different investor types: risk-averse, risk-neutral, and risk-takers. The mean-semivariance is used in this step of the optimization process. Through these steps, the proposed model is applied to a real-life case study of the Tehran Stock Exchange (TSE). We will utilize sharp criteria to evaluate the performance of the portfolios in comparison with the performance of the stock market. The model developed in this study can be used in the selection of an optimal stock portfolio that takes both expert opinions in stock selection and investors' risk preferences into consideration simultaneously.

III. Methodology

The developed model of this study consists of a few steps. First, we identify quantitative and qualitative criteria in stock selection and create a hierarchical structure of those criteria. After analyzing the questionnaire, we select the final criteria. Using AHP, we assign weights to the criteria. Then, a decision matrix is developed, and the top 50 companies will be ranked by interval TOPSIS. We then provide mathematical models of linear programming for optimizing the stock portfolio for each group of investors based on their level of risk-taking. To optimize the selected stock portfolio, the Colonial Competitive Algorithm (CCA) in Matrix Laboratory (MATLAB) will be used. The monthly returns and risks of companies are used as input data, and MATLAB determines the weight of each stock in a portfolio. The methodology is shown in Figure 1:

Figure 1. The methodology of the proposed portfolio selection and optimization approach



A. Multi-Criterion Decision-Making

MCDM is a sub-discipline of operations research (OR) and a method of making complex decisions involving multiple criteria (quantitative and qualitative, conflicting criteria, etc.) (Xidonas et al., 2011). MCDM assists in evaluating and ranking top companies on TSE by considering important financial and non-financial criteria in uncertainty and determining the weight of criteria.

We analyzed and extracted factors affecting stock portfolio selection and developed a hierarchical chart of criteria and sub-criteria based on literature reviews and expert surveys. Afterward, AHP will be utilized to specify the importance of qualitative and quantitative criteria. In the following step, an interval TOPSIS will be employed to evaluate the stocks.

A.1. Analytic Hierarchy Process modeling

AHP is a methodology to determine the weight and importance of different criteria in a given situation, and it begins by providing a hierarchical tree (Mehregan, 2004). After collecting pairwise comparison matrices, weights were calculated by using the AHP technique and the Expert Choice software.

A.2. TOPSIS technique

In order to apply the TOPSIS method, information related to the decision-making process should be presented in the form of a decision matrix, as shown in Equation (1):

$$X = \left[X_{ij}\right]_{m*n} = \begin{array}{c} \\ A_1 \\ A_2 \\ . \\ . \\ . \\ A_m \end{array} \begin{array}{c} \begin{array}{cccccc} C_1 & C_2 & . & . & . & C_n \end{array} \\ \begin{bmatrix} X_{11} & X_{12} & . & . & . & X_{1n} \\ X_{21} & X_{22} & . & . & . & X_{2n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ X_{m1} & X_{m2} & . & . & . & X_{mn} \end{bmatrix} \end{array} \tag{1}$$

Where A_i is the alternatives that experts should evaluate, C_j is the decision criteria, and X_{ij} is the values for each option.

A.3. Interval TOPSIS technique

The interval TOPSIS method is a new extension of the classical multicriteria TOPSIS method. In a world where the values of the criteria for evaluating each alternative can be characterized by uncertainty, interval analysis can prove to be a powerful tool for dealing with complex decision problems. Using interval analysis to introduce data uncertainty is a simple and intuitive way to deal with complex decision problems and can be used for a wide range of practical applications (Give, 2002; Jahanshahloo et al., 2009; Wentao & Han; 2010). In this model, unlike the classic TOPSIS model, interval data were used instead of definite and accurate numbers, which might be misleading in the real world and cannot provide plausible explanations. Additionally, under certain conditions, it is difficult to determine with certainty the values of matrix elements; therefore, values might be in the range (minimum-maximum) of the decision matrix.

An overview of the interval data is presented in Equation (2).

$$\left[a^{L}{}_{ij}, b_{ij}^{U}\right] \ , \forall i = 1, 2, 3, \ldots m, \ j = 1, 2, 3, \ldots n \tag{2}$$

In this regard, a_{ij}^{L} represents the low limit or the minimum amount, and the b_{ij}^{U} is for the high limit or the maximum amount available for the variable. The steps for using the interval TOPSIS method are as follows:

Step 1: Construct a decision matrix

$$X_{ij} = \begin{bmatrix} [a_{11} \quad ,b_{11}] & [a_{12} \quad ,b_{12}] \ldots & [a_{1m} \quad ,b_{1m}] \\ [a_{21} \quad ,b_{21}] & [a_{22} \quad ,b_{22}] \ldots & [a_{2m} \quad ,b_{2m}] \\ \vdots & \vdots & \vdots \end{bmatrix} \tag{3}$$

Step 2: Weighting the criteria: In this step weights assign to the criteria based on the weighting method or the decision maker's experience or discretion. It should be noted that the weights of the criteria are definite and accurate. The linear matrix of the weights of the criteria can be seen in Equation (4):

$$W = [W_1 \quad W_2 \ . \ . \ . \ W_n] \tag{4}$$

Step 3: Construct a normalized decision matrix in Equation (5):

$$r_{ij}^{l} = \frac{a_{ij}}{\sqrt{\sum a_{ij}^2 + \sum_{i=1}^{n} b_{ij}^2}} \ ;$$

$$r_{ij}^{u} = \frac{b_{ij}}{\sqrt{\sum a_{ij}^2 + \sum_{i=1}^{n} b_{ij}^2}} \tag{5}$$

$$v_{ij}^{l} = w_i r_{ij}^{l} \ ;$$

$$v_{ij}^{u} = w_i r_{ij}^{u} ;$$

Where r_{ij}^{l} and r_{ij}^{u} is the normalized values for the lower and upper limits of the interval data, and w_i is the weight value of i-th criterion obtained from the weighting methods.

Step 4: Determining the ideal solutions of positive and negative intervals for each option. Equations (6) and (7) are used to find positive ideal solutions, and Equations (8) and (9) are used to find negative ideal solutions.

$$A_k^{+u} = \{(v_1^{+u}, v_2^{+u}, \dots, v_n^{+u})\} = \{(\max v_{ij}^{u} | i \in O), (\min v_{ij}^{u} | i \in I)\} \quad (6)$$

$$A_k^{+l} = \{(v_1^{+l}, v_2^{+l}, \dots, v_n^{+l})\} = \{(\max v_{ij}^{l} | i \in O), (\min v_{ij}^{l} | i \in I)\} \quad (7)$$

$$A_k^{-u} = \{(v_1^{-u}, v_2^{-u}, \dots, v_n^{-u})\} = \{(\min v_{ij}^{u} | i \in O), (\max v_{ij}^{u} | i \in I)\} \quad (8)$$

$$A_k^{-l} = \{(v_1^{-l}, v_2^{-l}, \dots, v_n^{-l})\} = \{(\min v_{ij}^{l} | i \in O), (\max v_{ij}^{l} | i \in I)\} \quad (9)$$

Where O and I are the profit and cost criteria, respectively.

Step 5: Calculate the positive and negative distances of the options using the Euclidean distance of each of the options from the ideal positive and negative solutions according to the Equations (10)-(13).

$$d_k^{+u} = \sqrt{\sum_{i \in I} (v_i^{+u} - v_{ij}^{u})^2 + \sum_{i \in O} (v_i^{+u} - v_{ij}^{l})^2} \quad (10)$$

$$d_k^{+l} = \sqrt{\sum_{i \in I} (v_i^{+l} - v_{ij}^{l})^2 + \sum_{i \in O} (v_i^{+l} - v_{ij}^{u})^2} \quad (11)$$

$$d_k^{-u} = \sqrt{\sum_{i \in I} (v_i^{-u} - v_{ij}^{l})^2 + \sum_{i \in O} (v_i^{-u} - v_{ij}^{u})^2} \quad (12)$$

$$d_k^{-l} = \sqrt{\sum_{i \in I} (v_i^{-l} - v_{ij}^{u})^2 + \sum_{i \in O} (v_i^{+l} - v_{ij}^{l})^2} \quad (13)$$

$$where\ d_k^{+1} \le d_k^{+u}\ and\ d_k^{-1} \le d_k^{-u}$$

Step 6: Calculating interval efficiency as follow:

$$\frac{d_k^{-l}}{d_k^{-u} + d_k^{+u}} \le C_i \le \frac{d_k^{-u}}{d_k^{-l} + d_k^{+l}} \quad (14)$$

Step 7: The next step is calculating the mid-point (m(E)) and half-width (HW(E)) of interval efficiency using equations (15) and (16):

$$m(E) = \frac{1}{2}(e^l + e^u) \quad (15)$$

$$HW(E) = \frac{1}{2}(e^u - e^l) \quad (16)$$

Step 8: We are now going to rank the alternatives in order to find the best alternative using the mid-point values from Equation (15). When the midpoints were equal, then one needed to calculate the half-width values (Equation (16)) and determine the rank accordingly. If the midpoints are not equal, the mid-point values will be used as the basis for comparison and ranking (Jahanshahloo et al., 2009). Interval data were used only for quantitative data in this study.

B. Optimization and development of a model for optimal stock portfolio selection

In 1952, Markowitz showed that portfolios could be accurately defined through the use of optimization programs and by adjusting weights when sufficient information was available. Basically, optimization aims to improve performance in order to achieve the optimum point or points. There are two parts to this definition: 1- Search for ways to improve, and 2- Reaching the optimal point. Optimization can be defined as seeking an optimal solution to a problem.

Markowitz's mean-variance basic model does not take into account some important factors. In this study, we also applied behavioral biases (risk-taking level) to investment decision-making.

B.1. Portfolio optimization and Markowitz's mean-semivariance optimization model

The modern theory of portfolio defines risk as the variability of total return around the average return, which is calculated based on variance, and is then classified as a symmetric risk criterion. Positive fluctuations, however, are more appealing to investors with short-term objectives, while negative fluctuations are deemed risky.

In 1959, Markowitz advocated the use of semivariance as a measure of risk rather than the variance in Post-modern portfolio theory. According to him, semi-variance-based analyses produce better stock portfolios than those that are based on variance. The assumption is that the target rate of return should be interpreted as the Minimum Required Return (MRR), which indicates the minimum rate of return that must be achieved in order to avoid losing certain important financial goals. According to Markowitz, there are two methods for calculating downside risk:

Method 1: The semivariance method, which is obtained from the average of the squared deviations (deviations less than the average rate of return) around the mean rate of return (below-mean semivariance)

Method 2: Semivariance is determined by the sum of undesirable deviations (deviations less than the target rate of return) relative to the target rate of return (below-target semivariance).

Equation (17) shows this point:

$$\sum\nolimits_{\rho B}^{2} = \frac{1}{T}\sum_{t=1}^{T} Min\ [(R_{\rho t} - B), 0]^{2} \quad (17)$$

Where T is the number of asset returns *R (return)* and a benchmark *B (expected return)* set by the investor, B is the average rate of return or target rate, $R_{\rho t}$ stands for the portfolio returns, and $\sum^2{}_{\rho B}$ is portfolio semivariance.

Portfolio risk is calculated through Equation (18) (Estrada, 2008).

$$\sum\nolimits_{\rho B}^{2} = \sum\nolimits_{i=1}^{n}\sum\nolimits_{j=1}^{n} X_i X_j S_{ijB}, \quad (18)$$

$$S_{ijB} = (1/T).\sum\nolimits_{t=1}^{n}(R_{it} - B)(R_{jt} - B)$$

Where X_i and X_j denote the proportions of the portfolio invested in assets i and j, respectively, with returns Rit and Rjt for the expected assets, and n is the number of assets in the portfolio.

We used Markowitz's model to determine the optimal allocation of investors' capital to different stocks, which is based on two important parameters, risk and expected returns, with semivariance as the primary risk factor. A complex but realistic mathematical model in Equation (19) is formed in order to reflect the characteristics of the real stock market and consider amateurs' risk-taking levels:

$$Min\ ((1-\lambda)*R_p - \lambda * Var\ (R_p)) \tag{19}$$

In this pattern, R_p is portfolio return, $Var\ (R_p)$ is portfolio risk, and λ is a parameter between 0 and 1, so that by $\lambda = 0$, the total value of the weighting factor is assigned to the return and regardless of the risk-taking factor, the stock portfolio will have the lowest risk value. In the interval between 0 and 1, portfolios were optimized considering risk and optimal return factors. In other words, by adding λ, risk-taking reduction became more important; in this case, since $(1 - \lambda)$ was reduced, maximizing return became less important.

B.2. Determining the weights of risk and return criteria for three investor groups

In step 1, the weights of each return and risk criteria used in Equation (19) were determined by asking individuals to give the criteria a weight between 1 and 9, with 1 showing equal importance and 9 showing the greatest importance.

C. Performance evaluation of the formed portfolios

A number of criteria exist for evaluating the performance of stock portfolios, such as Sharpe, Treynor, Jensen, and Sortino. There are a number of traditional measures of stock portfolio performance that are widely used, including the Sharpe ratio and the Treynor index.

The Sharpe index is one of the most commonly used traditional metrics for evaluating performance relative to modified risk. This index measures the excess return that the portfolio has achieved over the return on a risk-free asset.

This index is calculated from Equation (20), which is also known as RVAR.

$$RVAR = \frac{\overline{TR_P} - \overline{R_F}}{SD_P} = \frac{Excess\, return}{Risk} \tag{20}$$

Where $\overline{TR_P}$ is the average return of the stock portfolio over a period of time, $\overline{R_f}$ is the average risk-free rate of return during the period under review, and SD_P is the standard deviation of weekly returns of the portfolio throughout the study.

IV. Model Analysis and Results

In this section, we perform the proposed model and provide optimized portfolios for three investment groups. Finally, we will evaluate the performance of three portfolios for three groups of investors using sharp criteria.

A. Data

Every quarter, the Tehran Stock Exchange publishes a list of the top 50 companies, mainly based on financial criteria, that is, the liquidity of stocks, the extent to which the company has an impact on the market, as well as the company's financial performance. The primary criticism of this method is that it does not determine the top companies but rather the largest ones. As a result, the list provided by the stock exchange is not representative of the most important companies (Mirdamadian et al., 2016).

We selected the top 50 companies listed on the Tehran Stock Exchange in the first quarter of 2012, based on last year's data. The study period covered three years, from 2009 to 2012. In this study, our sample population and sample is divided into three levels:

Level 1: The top 50 companies listed on the TSE are used to select and evaluate stocks and rankings.

Level 2: Chief executive officers of brokerage firms, investment companies, and experts, including 190 individuals (faculty of financial studies at universities in Tehran that are familiar with stock markets).

Level 3: A random sample of 96 nonprofessional investors in the TSE comprising accounting and financial management graduate students and nonprofessional investors in the TSE.

B. Determining the effective criteria in stock selection

Using the literature review, we identified quantitative and qualitative criteria in stock selection and created a hierarchical structure of those criteria. A questionnaire was used to refine and limit the criteria extracted from prior studies, with ten-point Likert scale questions. Experts determined the numerical value of each criterion, and the criteria with mean values higher or equal to seven were chosen as the final criteria. The specific details are presented in Table I.

According to the results shown in Table I, the criterion of *The impact of government decisions* had the highest importance, and *Dividend per Share (DPS)* and *Stock price fluctuations* had the lowest importance in stock selection and evaluation, respectively.

It should be noted that by considering any number below seven as the base average, the number of criteria examined would be at least 31, making the research process very complicated.

C. Analytic Hierarchy Process modeling

To determine the importance weight of each criterion, pairwise comparisons and analyses based on the AHP model must be conducted. Modeling based on this method relies on the AHP hierarchical tree, which represents the problem under study. Stock rankings are the objective of this study. The hierarchy tree in Figure 2 represents this. As can be seen, the factors influencing the objective of stock ranking can be divided into four main categories, each of which includes sub-criteria.

Figure 2. Criteria of the AHP model to rank shares

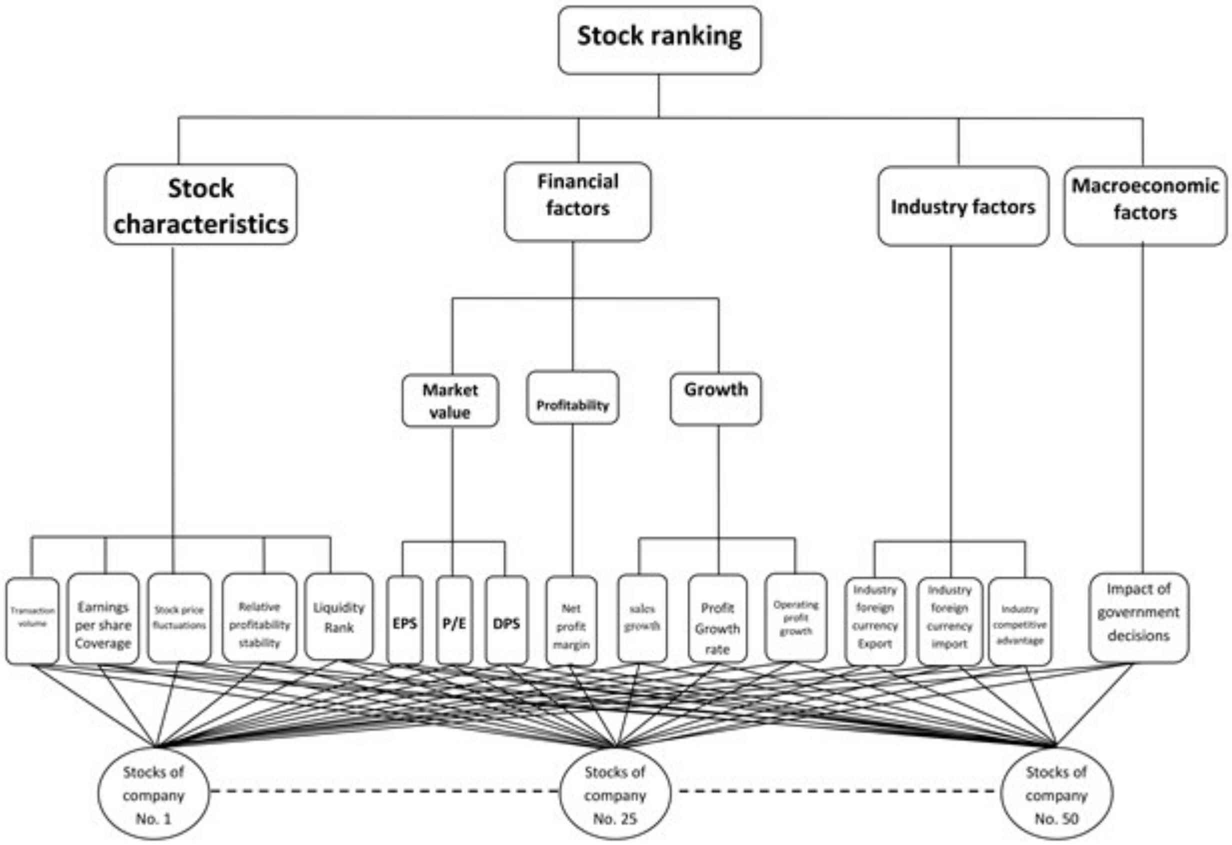


Table I. Effective criteria in-stock selection

Row	Effective criteria in-stock selection	Description	Score
1	The impact of government policies	When the government's decisions impact a company, investors' behavior in stock selection and ranking is also affected.	8.5
2	The competitive advantage of the industry	An industry with a better perspective is preferred over other industries.	8.1
3	Foreign currency export of the company	By purchasing the required machinery and technology transfer, import companies need to source their products abroad. As a result, they need foreign currency, and any fluctuation in rates impacts their production rate.	7.7
4	Foreign currency import of the company	An increase in exchange rates increases the profitability of export companies. Increased foreign exchange rates have	8

		positive consequences for export companies.	
5	Net profit margin	The ratio of net profit to net sales	7.1
6	Earnings per share (EPS)	The ratio of common share profit (loss) to the number of common shares	8.7
7	Price/earnings ratio (P/E)	The ratio of the current share price to its earnings per share (EPS)	8.6
8	Divided per share (DPS)	A part of the profit is the share of the company that is divided among shareholders: The ratio of dividend per share (the sum of declared dividends issued by a company) to the number of shares	7.3
9	Sales growth	Growth in sales is defined as the proportion of net sales that grow from one fiscal period to another.	7.7
10	Net profit growth rate	Growth in net profit is defined as the proportion of net profit that grows from one fiscal period to another.	7.2
11	Operational profit growth	Operating profit growth is defined as the percentage of net operating profit that grows from one fiscal period to another.	7.1
12	Liquidity rate	A number that shows the liquidity of a share in the market	7.3
13	Transaction volume percentage	The ratio of transacted shares of a company in a day relative to its total number of shares	7.3
14	Stock price fluctuations	Stock trading price changes on the stock market. Stock price standard deviation is used for each company to determine stock price fluctuations.	7.1
15	Coverage of earnings per share	The ratio of real earnings per share in a period to predicted earnings per share based on the actual performance in the period	7.4
16	The relative stability of profitability	This ratio indicates the extent to which profitability deviates from the average profit or expected profit—the more stable the profitability, the lower the risk, and the higher the share value.	7.8

D. Determining the weights of criteria in stock assessment

Generally, to determine the importance or weight of criteria, the AHP technique is used. Table II provides the descending order of the qualitative-quantitative scale, nature, and criteria weight.

Table II. Criteria and their importance weights

Row	Criteria affecting stock evaluation	Scale	Nature	Criteria weight (In descending order)
1	The impact of government decisions	Qualitative [1]	Min	0.4866
2	The competitive advantage of the industry	Qualitative	Max	0.1135
3	Foreign currency export	Qualitative	Min	0.0733
4	Earnings Per Share (EPS)	Quantitative	Max	0.0483
5	Net profit margin	Quantitative	Max	0.0401
6	Foreign currency import	Qualitative	Max	0.0355
7	Profit coverage of each share	Quantitative	Min	0.0331
8	Sales growth	Quantitative	Max	0.0288
9	Price/earnings ratio (P/E)	Quantitative	Max	0.0261
10	The relative stability of profitability	Quantitative	Max	0.0231
11	The net profit growth rate	Quantitative	Max	0.0203
12	Liquidity rank	Quantitative	Min	0.0194
13	Transaction volume	Quantitative	Max	0.0151
14	Operational profit growth	Quantitative	Max	0.0124
15	Dividend per Share (DPS)	Quantitative	Max	0.0122
16	Stock price fluctuations	Quantitative	Min [2]	0.0121

[1] To score qualitative criteria, a five-point Likert scale was used (from 1: very low to 5: very high).

[2] This criterion is equal to stock price changes and is of the cost (Min) type.

E. Ranking the 50 top companies listed on the TSE

The mid-point and half-width values obtained from the interval TOPSIS technique were used to rank the 50 companies listed on the TSE during the first three months of 2012. The results are shown in Table III.

Table III. The final ranking of the 50 top companies listed on TSE over the first three months of 2012

Company's name	Mid-point	Rank from Interval TOPSIS	Company's name	Mid-point	Rank from Interval TOPSIS
National Iranian Copper Industries	0.506090	1	Khazar Cement	0.377461	26
Sina Bank	0.504921	2	Leasing Ryan Saipa	0.377134	27
Iran Transfo	0.472663	3	Sahand Tire Industries	0.375610	28
Foolad Mobarake	0.472347	4	Sepah Investment	0.375157	29
Qadir Investment (Holding)	0.443557	5	Boali Investment	0.374201	30
Post Bank	0.433875	6	Pardis Investment	0.373955	31
Mellat Bank	0.432990	7	Razak Pharmaceutical	0.373905	32
Behshahr Industries Development	0.432857	8	Iranian Sanitary	0.371906	33
Petrochemical Industries Development (Holding)	0.424957	9	Mellat Investment	0.367060	34
Fars & Khuzestan Cement	0.423784	10	Techinco Inspection & Corrosion Control	0.366143	35
North Drilling	0.423181	11	Shahid Bahonar Copper Industries	0.363306	36
International Building Development	0.422680	12	Ardakan Industrial Ceramics	0.362984	37
Karafarin Bank	0.422387	13	Industry and Mining Industry	0.353977	38
Tehran Renovation and Building	0.419879	14	Jaber Ebne Hayyan Pharmaceutical	0.352828	39
Eghtesad Novin Bank	0.418180	15	National Investment of Iran	0.346785	40
Iranian Building Investment	0.417487	16	Ghadir Car Leasing	0.332768	41
Behceram Granite Production	0.416685	17	Bahman Investment	0.337120	42

Pars Tooshe Investment	0.411592	18	National Iranian Lead and Zinc	0.316934	43
Shahrood Cement	0.404805	19	Saipa	0.301276	44
Parsian Bank	0.404086	20	Insurance Industry Investment	0.298237	45
Bafgh Mines	0.402689	21	Iran Khodro Investment Expansion	0.295394	46
West Cement	0.402458	22	Pars Khodro	0.293155	47
Atiehe Damavand Investment	0.389025	23	Azarab Industry	0.281667	48
Informatics Services	0.384706	24	Piazer Agro-Industry	0.279548	49
Kowsar Pharmacology	0.378872	25	Gaz Looleh	0.260488	50

F. Stock portfolio selection from the ranked companies

Investors can determine the investment amount in each company's share based on the mean values obtained from the interval TOPSIS. However, mean-semivariance optimization with objective function and constraints is used to specify a more accurate weight for each share in the optimal stock portfolio.

Using the mean values obtained from interval TOPSIS, we created a portfolio of 24 companies whose mean values exceeded the total mean value (0.383515). The removal method is based on the method of interest described in previous studies. To determine the weight and number of the chosen stocks, an optimization method is conducted with certain constraints.

By analyzing existing databases, we collected monthly returns from companies over 36 months (March 2009 to February 2012), calculated semivariance and semi-covariance values from the returns, and then utilized these values as inputs to our model.

F.1. Determining the weights of risk and return criteria for three investor groups

We determined the weights of return and risk criteria for each group of investors using the AHP method in steps 1 and 2.

Step 1: Calculating the geometric mean: The geometric mean of the matrices obtained from the completed questionnaires is shown in Table IV.

Table IV. The geometric mean of criteria comparison by different investor groups

	Risk-averse investors		Neutral investors relative to risk		Risk-taker investors	
	Return	**Risk**	**Return**	**Risk**	**Return**	**Risk**
Return	1.000	2.106	1.000	4.846	1.000	6.299
Risk	0.475	1.000	0.206	1.000	0.159	1.000

Step 2: Identifying the relative weight of criteria by different investor groups

Table V summarizes the number of investors in each group and the importance of each criterion to each group of investors.

Table V. The importance of the return and risk criteria from the perspectives of different investors

The investor group	The number of investors (number)	The number of investors (%)	The importance of return (%)	The importance of risk (%)
Risk-averse	38	36.5	0.322	0.678
Neutral towards risk	35	33.7	0.546	0.454
Risk-taker	31	29.8	0.863	0.137

F.2. Multi-objective optimization model for portfolio selection for risk-averse investors:

By using the return and risk coefficients from Table V, we present a model in Equation (21) for the optimal selection of stocks portfolios with an objective function and economic constraints for risk averse investors:

$$\begin{aligned}
&Min\ (0.678 * Var\ (R_p) - 0.322 * R_p) \\
&1)\ \sum W_i . R_i = R_p \\
&2) \sum W_i = 100\% \\
&3)\ W_i \leq \alpha_i\ (6.67\%) \\
&4) W_i \geq 0
\end{aligned} \tag{21}$$

Objective Function: $Min\ (0.678 * Var\ (R_p) - 0.322 * R_p)$
Constraint Functions: 1, 2, 3, 4
R_i= Expected return of stock i
R_p= Minimum expected portfolio returns by investor
W_i = Weight of stock i
α_i = Maximum weight of each stock in portfolio
Objective function constraints for optimization were expressed as follows:

Constraint 1: The sum of the weight multiplication or the allocable stock ratio in the expected return must equal the return the investor considered to be satisfactory.

Constraint 2: Due to the fact that short transactions or exceeding the budget were not allowed, the total of the select stock weights had to be one (100 percent). This constraint is well suited to the real conditions of most shareholders.

Constraint 3: Stock weights had not to exceed 6.67 percent: $\frac{100}{15} = 6.67\%$

According to Evans and Archer (1968), unsystematic risk can be reduced by holding between 10 and 15 stocks. Accordingly, if there are 15 shares in a portfolio, we can accept the restriction that no single stock may exceed 6.67 percent in weight. Their study found that portfolio diversification with more than 15 or 16 stocks was not negative, but it was also not beneficial.

Constraint 4: Short selling is prohibited, so stock weights could not exceed 0. This assumption is logical and supported by market realities.

The results of performing the provided model for risk-averse investors are shown in Table VI. The portfolio's return level is 23.42 percent, and the risk level is 7.08 percent.

Table VI. Optimal stock portfolio for risk-averse investors

Row	Company's name	Companies' stock weight	Row	Company's name	Companies' stock weight
1	National Iranian Copper Industries	6.67	13	Karafarin Bank	6.67
2	Sina Bank	6.67	14	Tehran Renovation and Building	6.67
3	Iran Transfo	0	15	Eghtesad Novin Bank	6.67
4	Foolad Mobarake	0	16	Iranian Building Investment	5.58
5	Ghadir Investment (Holding)	6.67	17	Behceram Granite Production	0
6	Post Bank	0	18	Pars Tooshe Investment	1.1
7	Mellat Bank	6.67	19	Shahrood Cement	6.67
8	Behshahr Industrial Development (Holding)	6.67	20	Parsian Bank	6.67
9	Petrochemical Industries Development	0	21	Bafgh Mines	0
10	Fars & Khuzestan Cement	6.67	22	West Cement	0
11	North Drilling	0	23	Atieh Damavand Investment	6.67
12	International Building Development	6.67	24	Informatics Services	6.61

F.3. Multi-objective optimization model for portfolio selection for investors neutral to risk:

A model for selecting the optimal portfolio of stocks presented to investors that are neutral to risk using the return and risk coefficients from Table V and with the objective function and constraints defined by Equation (22):

$$Min\ (0.454 * Var\ (R_p) - 0.546 * R_p)$$

$$1)\ \sum W_i . R_i = R_p \qquad (22)$$

$$2) \sum W_i = 100\%$$

$$3)\ W_i \leq \alpha_i\ (6.67\%)$$

$$4) W_i \geq 0$$

Table VII shows the results of performing the model with a rate of return of 27.42 percent and a level of risk of 8.75 percent for investors neutral to risk.

Table VII. Optimal stock portfolio for investors neutral to risk

Row	Company's name	Companies' stock weight	Row	Company's name	Companies' stock weight
1	National Iranian Copper Industries	6.67	13	Karafarin Bank	6.67
2	Sina Bank	6.67	14	Tehran Renovation and Building	6.67
3	Iran Transfo	0	15	Eghtesad Novin Bank	6.67
4	Foolad Mobarake	0	16	Iranian Building Investment	0
5	Ghadir Investment (Holding)	6.67	17	Behceram Granite Production	0
6	Post Bank	0	18	Pars Tooshe Investment	6.67
7	Mellat Bank	6.67	19	Shahrood Cement	6.67
8	Behshahr Industrial Development (Holding)	6.67	20	Parsian Bank	6.67
9	Petrochemical Industries Development	0	21	Bafgh Mines	6.67
10	Fars & Khuzestan Cement	6.67	22	West Cement	0
11	North Drilling	0	23	Atieh Damavand Investment	6.67
12	International Building Development	0	24	Informatics Services	6.67

F.4. Multi-objective optimization model for portfolio selection for risk-taking investors:

Equation (23) represents an optimal stock portfolio selection model with an objective function for risk-taking investors:

$$
\begin{aligned}
&Min\ (0.137 * Var\ (R_p) - 0.863\ * R_p) \\
&1)\ \sum W_i . R_i = R_p \\
&2) \sum W_i = 100\% \\
&3)\ W_i \leq \alpha_i\ (6.67\%) \\
&4) W_i \geq 0
\end{aligned}
\tag{23}
$$

After establishing mathematical models in this section, we implemented the models in MATLAB software and determined the weight of stocks in the portfolios for each group of investors.

The results of performing a risk-taker investor model with a return level of 29.32 percent and a risk level of 10.8 percent can be seen in Table VIII.

Table VIII. Optimal stock portfolio for risk-taker investors

Row	Company's name	Companies' stock weight	Row	Company's name	Companies' stock weight
1	National Iranian Copper Industries	6.67	13	Karafarin Bank	6.67
2	Sina Bank	6.67	14	Tehran Renovation and Building	6.67
3	Iran Transfo	6.67	15	Eghtesad Novin Bank	6.67
4	Foolad Mobarake	6.67	16	Iranian Building Investment	00
5	Ghadir Investment (Holding)	6.67	17	Behceram Granite Production	0
6	Post Bank	0	18	Pars Tooshe Investment	1.1
7	Mellat Bank	6.62	19	Shahrood Cement	6.67
8	Behshahr Industrial Development (Holding)	6.67	20	Parsian Bank	6.67
9	Petrochemical Industries Development	0	21	Bafgh Mines	6.67
10	Fars & Khuzestan Cement	0	22	West Cement	0
11	North Drilling	0	23	Atieh Damavand Investment	0
12	International Building Development	0	24	Informatics Services	6.67

F.5. Optimal stock portfolio for investors in general

The risk shown in Table IX is in line with the expected returns for each group of investors. In other words, the stock portfolio risk increased in response to the expected returns. Investors had to tolerate more risk in order to achieve a higher return.

Table IX. Expected return and risk in general

	Risk-averse	Neutral to risk	Risk-taker
Return	23.42	27.42	29.32
Risk	7.08	8.75	10.8

G. Performance evaluation of the formed portfolios

We utilized sharp criteria to evaluate and assess the performance of the portfolios in comparison with the performance of the stock market. As can be seen from Table X, this index for the formed stock portfolios was higher than the market portfolio; therefore, these portfolios outperformed the market in terms of risk and return.

Table X. Stock portfolio performance evaluation criterion using Sharp index

Stock portfolio	Sharp index
A portfolio offered to risk-averse investors	0.9
A portfolio offered to neutral investors to risk	1.2
A portfolio offered to risk-taker investors	1.1
Market portfolio	-3.8

V. Conclusion and Future Work

Taking into account uncertain conditions associated with investing in the TSE and the risk tolerance of investors, this study attempted to determine the optimal stock portfolio among fifty top companies. In selecting and evaluating stocks, we used both qualitative and quantitative criteria.

Portfolio selection was influenced by macroeconomic/political factors, such as government decisions affecting the industry, and industry-specific factors, such as competitive advantage and foreign exchange rate. Moreover, financial factors, such as growth factors (i.e., operational profit growth, net growth rate, and sales growth), profitability (net profit margin), and stock market value (i.e., DPS, EPS, P/E), were among the most important criteria. We also investigated the characteristics of stocks (e.g., liquidity rank, transaction volume percentage, stock price fluctuations, earnings coverage, and relative stability of profitability) and a variety of important factors influencing stock selection. The criterion of *The impact of government decisions* had the greatest importance, and *Dividend per Share (DPS)* and *Stock price fluctuations* had the lowest importance in stock selection and evaluation. Afterward, the companies were ranked using a technique called interval TOPSIS, which considers interval values for financial information. We should also note that the ranking of companies based on extracted criteria differed from the ranking of the 50 top-ranking companies on the TSE. Having selected qualified stocks, the next step consists of creating portfolios and determining the weight invested in each stock. Afterward, three portfolios were designed, one for an investor who was risk-averse, one who was neutral, and one for an investor who was risk-taking. At this stage of the optimization process, the mean-semivariance optimization model is used. Three portfolios for three groups of investors were evaluated and compared to the market performance using sharp criteria. On both a risk and return basis, each of the three portfolios has outperformed the market portfolio. This study proposes a model that can be utilized as a decision support tool for forming an optimal stock portfolio when considering both the expertise of experts as well as the preferences of investors when assessing stocks simultaneously.

Globalization, the growing economy, and the need to handle large amounts of data from various organizations around the globe necessitate the development of optimized algorithms to manage large amounts of data efficiently and timely (Cormen et al., 2022; Sepahyar et al., 2019). A variety of sorting algorithms can be employed in data processing, including quicksort, shell sort, and many others. Furthermore, machine learning and data mining algorithms (Qu et al., 2017; Pahwa & Agarwal, 2019) can be used to process large amounts of data in a very short period of time with a more efficient method. However, further research and study are required, and we will continue to study this in the future.

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China's CEO Pay Reform: An Analysis of the Financial Impact on Central State-Owned Enterprises (CSOEs)

Xiaochuan Tong and Robert A. Kunkel

Abstract

China's Central State-Owned Enterprises (CSOEs) are considered to be inefficient with major agency problems where the interests of management and shareholders are poorly aligned. Rather than making decisions that make the CSOE more profitable and to help grow the economy, management will choose to solve government problems such as social stability and low employment (retain redundant workers). As China grows its socialist market economy and lists its stocks on markets in New York and London, it has become increasingly more important to reduce agency problems to better align the interests of management and shareholders. *China's CEO pay reform* is passed to reform executive compensation at CSOEs. This reform is driven by President Xi and the Central Politburo of the Communist Party of China. Using an event study methodology, we find CSOEs, on average, experience a 3.05 percent increase in market capitalization from *China's CEO pay reform*. In dollar terms, the mean market capitalization increase was $89 million, and cumulatively, the twenty-four CSOEs in our sample gained $2.136 billion in market capitalization. We conclude that *China's CEO pay reform* was highly successful, and the gains were a result of lower executive compensation expenses without increased perk consumption and tunneling activities.

Keywords: Event Study, Financial Analysis, Reform, CEO Compensation, State Ownership, China

I. Introduction

The People's Republic of China (PRC) was founded on October 1, 1949, under Chairman Mao Zendong and the Communist Party of China (CPC). The goals of the CPC included industrialization, raising the living standard, improving income equality, and creating a modern military. The CPC chose to adopt the Soviet Union's centrally planned economy, where state-owned enterprises (SOEs) would be used to industrialize the economy, versus a market-based economy with private ownership. In 1950, the Chinese stock markets closed and China entered a period of isolation from the West. While the economies of the Western nations saw continued growth and increases in standards of living with their market-based economies, Chinese living standards improved less so under the centrally planned economy resulting in China reforming and opening up their economy in 1978 (Zhong, 2006).

1949-1978: While state-owned enterprises (SOEs) produced the goods and services needed in the economy, SOEs also played a major role in providing societal benefits such as employment, education, healthcare, and retirement benefits. The government would own

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the SOE, hire managers to operate the SOE, set price and production quota for the SOE, and collect/cover all profits/losses of the SOE. The SOE structure created agency problems where managers must decide whether to run a more profitable business (e.g., lay off redundant workers) or help solve government problems such as social stability and low employment (e.g., retain redundant workers). Employment incentives typically led managers to choose solving government problems, which makes business inefficient and limits China's economic growth (Zhong, 2006).

1978-1990: China started a massive undertaking of *reform and opening-up* the economy in 1978. To better incentivize managers, some SOEs were allowed to keep a portion of their profits and pay employee bonuses when goals were achieved. The Chinese government further incentivized managers by awarding *operating rights* to managers along with three-to-five-year contracts. This *operating rights* structure enabled managers to resemble residual claimants, which helped reduce agency problems, as it now was in management's interest to generate profits. Unfortunately, this *operating rights* structure incentivized short-term profits at the expense of long-term profits. Managers had little incentive to maintain equipment and buildings or to undertake new capital budgeting projects. Agency problems remained, and SOEs still were inefficient, with around four out of ten losing money (Lin, Lu, Zhang, and Zheng, 2020).

1990-2002: The Chinese government introduced a socialist market economy. The Chinese government had reformed *operating rights* from 1978 to 1990 and now the Chinese government reformed *ownership rights*. The Shanghai and Shenzhen Stock Exchanges opened in 1990, and the Company Law was passed in 1994. The Company Law created a corporate governance structure with three main bodies: general meeting of shareholders, board of directors, and board of supervisors. The Company Law allowed SOEs to convert to limited liability companies, where the government and private investors jointly owned the SOE. Even with these changes, SOEs struggled to turn a profit, and taken together, industrial SOEs provided no net revenues for the government, but instead absorbed fiscal resources estimated at five percent of the gross domestic product. Agency problems remained, and SOEs continued to be inefficient, but there were SOE stock corporations (Gang and Hope, 2013).

2003-2014: The Chinese government expanded the socialist market economy by reforming the largest and most important SOEs. The government also worked on *ownership rights* by identifying the owners of an SOE who would assume the responsibilities of shareholders, on behalf of the state. By 2011, there were 1,000+ SOEs listed on the Shanghai and Shenzhen stock markets. Some SOEs even were listed on other stock markets such as Hong Kong, New York, London, and Singapore. Typically, the Chinese government would hold a majority ownership in the listed SOE company while selling a minority interest to the public in the form of an initial public offering (IPO). These reforms of *ownership rights* improved firm performance, but agency problems still existed, as executives at SOEs still were much less likely to be replaced by their boards than executives at private companies (Gang and Hope, 2013).

2014-2015: *China's CEO pay reform* took place to reform executive compensation at central state-owned enterprises (CSOEs), which were the large SOEs controlled by the central government (Hao and Lu, 2018). There were two main reasons for government intervention into executive compensation at CSOEs. One, executive compensation at CSOEs were much greater than those of non-CSOEs. Two, there was a huge disparity in compensation among executives at CSOEs. This reform was called the "2015 Reform Plan for Salary System for Centrally

Administered State-owned Enterprises," also known as a "pay restriction policy." *China's CEO pay reform* introduced a three-component executive compensation plan. The first component was *basic salary* that should not exceed twice the average salary of employees. The second component was *performance wages* and cannot exceed twice the *basic salary*. The third component was *term incentive income,* which directly tied executive compensation to firm performance to align manager interests with shareholder interest.

Government intervention into executive compensation is debated widely on whether or not it is effective. Governments have tried new rules, more disclosure requirements, and tax reforms. One argument concludes restricting executive compensation is ineffective, as it leads to unintended consequences such as greater perk consumption and tunneling activities, all of which have a negative impact on shareholders. Another argument concludes restricting executive compensation can be done effectively, making the firm more efficient. To our knowledge, this is the first paper that examines the immediate financial impact of *China's CEO pay reform* on the stockholders of CSOEs. This study provides valuable information on *China's CEO pay reform* to policy makers and investors.

II. Research Objective and Data

We use an event study methodology (Brown and Warner, 1985; Peterson, 1989; Schweitzer, 1989; Wells, 2004) to examine the immediate financial impact of *China's CEO pay reform* on CSOEs. Event studies have been used to evaluate many regulatory events. For example, prior research finds investment companies suffered under President Obama's Fiduciary Rule reform of 2015, which required a fiduciary relationship on retirement accounts. Evidence shows that variety stores and restaurants benefitted from the Durbin Amendment of 2010, which reformed debit card fees. Research shows large retailers who issued credit cards suffered from the Credit Card Act of 2009. Hoag (2002) finds that the Cable Communications Policy Act of 1984 benefitted cable companies.

We can isolate the impact of *China's CEO pay reform* because of two distinct stock price characteristics. One, a stock price is driven by the expected future earnings of the CSOE. Two, the stock market is efficient such that a stock price reacts quickly and efficiently when there is an event announcement that will affect the CSOE's expected future earnings. Thus, if investors believe *China's CEO pay reform* will successfully align manager interests with shareholders, thereby increasing future earnings, then the stock prices of CSOEs will increase. However, if investors perceive the reform will be ineffective, then stock prices will not increase. Therefore, policy makers and investors can evaluate the expected economic impact of China's pay restriction policy on CSOEs.

Our event study methodology separates a CSOE's stock return into two unique components. The first component is the normal return, which is the change in stock return that occurs from the overall stock market movement. The second component is the abnormal return, which is the change in stock return attributed to *China's CEO pay reform*. In this paper, we examine those abnormal returns.

II.1 Event Windows

We identify two events for *China’s CEO pay reform*. Additionally, we combine the two events to determine the overall impact of *China’s CEO pay reform*.

President Xi Effect: The first event is August 18, 2014, which is when China’s President Xi announced executive salaries of major SOEs were unreasonably high and “must be regulated.” We believe this announcement provided significant information to the markets. To capture the event’s impact on stock prices, we use a four-day event window where day zero, (t = 0), is defined as August 18, 2014, which is the announcement date. Day minus one, (t = -1), is defined as one trading day before President Xi’s announcement; day plus one, (t = +1), is one trading day after the announcement date; and so forth, where day plus two, (t = +2), is two trading days after the announcement. The event window is shown in Table 1.

Politburo Effect: The second event is August 29, 2014, which is when the Central Politburo of the Communist Party of China promulgated *China’s CEO pay reform.* Once again, we use a four-day event window, where day zero is defined as August 29, 2014. The event window is shown in Table 1.

Combined Effect: The combined event is the combination of both events: (i) *President Xi Effect,* and (ii) *Central Politburo Effect*. This provides the overall impact of *China’s CEO pay reform.*

Table 1. Event Windows for *China’s CEO pay reform*

President Xi Event	President Xi announced that the executive salaries of CSOEs were unreasonably high and “must be regulated.”			
Politburo Event	The Central Politburo of the Communist Party of China promulgated *China’s CEO pay reform*.			
Event Day	-1	0	+1	+2
President Xi Event	8/15/2014	8/18/2014	8/19/2014	8/20/2014
Politburo Event	8/28/2014	8/29/2014	9/1/2014	9/2/2014

II.2 Data

The sample is comprised of CSOEs collected from CSMAR (China Stock Market & Accounting Research Database). To isolate the stock price reaction due only to *China’s CEO pay reform*, it is crucial that CSOEs do not have any major news announcement around the event window. Thus, each CSOE must have no major news announcement around the event window, and CSMAR is used to determine whether there are any other major news announcements. When there is another major announcement affecting the CSOE around the event window, the effect of *China’s CEO pay reform* cannot be isolated, and the observation is removed from the sample. We find that 24 CSOEs do not have a major announcement around the two event windows.

The final sample of 24 CSOEs is listed in Table 2, and includes 17 industrial, 3 utility, and 4 property companies. The 24 CSOEs have mean and median market capitalizations of $1.84 billion and $925 million, respectively. The largest and smallest companies are CRRC Corporation and Linhai with market capitalizations of $9.94 billion and $289 million, respectively.

Table 2. Chinese Central State-Owned Enterprises (CSEOs), Ticker, Revenue, Industry, and Market Capitalization (MC) in US Dollars (Millions).

	CSOE	Ticker	Revenue	Industry	MC
1.	Aeolus Tyre	600469	$1,316	Industrial	$624
2.	Antong Holdings	600179	$174	Utility	$350
3.	CRRC Corporation	601766	$19,289	Industrial	$9,943
4.	Cangzhou Dahua	600230	$502	Industrial	$531
5.	Changchun FAW Fuwei Automobile Parts	600742	$1,816	Industrial	$832
6.	China XD Electric	601179	$2,267	Industrial	$3,317
7.	Cinda Real Estate	600657	$781	Property	$898
8.	COSCO SHIPPING Technology	002401	$89	Utility	$775
9.	Greatwall Information Industry	000748	$275	Industrial	$1,506
10.	Grinm Advanced Materials	600206	$390	Industrial	$1,787
11.	Guizhou Aerospace Electronics	002025	$258	Industrial	$1,030
12.	Guizhou Qianyuan Power	002039	$346	Industrial	$501
13.	Huludao Zinc Industry	000751	$702	Industrial	$1,735
14.	Kangxin New Material	600076	$3	Industrial	$339
15.	Linhai	600099	$58	Industrial	$289
16.	NARI Technology	600406	$1,435	Industrial	$5,745
17.	SPIC Dongfang New Energy Corporation	000958	$125	Industrial	$806
18.	Shanghai Tongji Science & Technology Industrial	600846	$664	Property	$958
19.	Shenzhen Overseas Chinese Town	000069	$4,949	Property	$6,520
20.	Sichuan Mingxing Electric Power	600101	$201	Industrial	$442
21.	Sichuan Minjiang Hydropower	600131	$134	Industrial	$394
22.	Sinoma International Engineering	600970	$3,684	Property	$1,349
23.	Unisplendour Corporation	000938	$1,796	Industrial	$953
24	Xinxing Ductile Iron Pipes	000778	$9,795	Industrial	$2,530
	Mean		$2,127		$1,840
	Median		$583		$925

Notes: Revenue is calculated as of December 31, 2014. MC is calculated as the average market capitalization between August 15 and August 20, 2014. All companies are Co., Ltd. or limited liability company.

II.3 Research Questions and Hypotheses

The research question is whether *China's CEO pay reform* was effective in aligning the interests of executives and shareholders, such that CSOEs experienced a significant stock price change during the event window. To answer this question, the following hypotheses are considered in the alternative form.

H_{a1}: The stock returns (cumulative abnormal returns) of the CSOEs attributed to the *President Xi Effect* are different from zero.

H_{a2}: The percent of positive stock returns (cumulative abnormal returns) of the CSOEs attributed to the *President Xi Effect* are different from fifty percent.

H_{a3}: The stock returns (cumulative abnormal returns) of the CSOEs attributed to the *Politburo Effect* are different from zero.

H_{a4}: The percent of positive stock returns (cumulative abnormal returns) of the CSOEs attributed to the *Politburo Effect* are different from fifty percent.

H_{a5}: The stock returns (cumulative abnormal returns) of the CSOEs attributed to the *Combined Effects* are different from zero.
H_{a6}: The percent of positive stock returns (cumulative abnormal returns) of the CSOEs attributed to the *Combined Effects* are different from fifty percent.

To test the odd numbered hypotheses, that the cumulative abnormal returns are different from zero, we use both a parametric t-test and a non-parametric Wilcoxon signed rank test. To test the even numbered hypotheses, that the number of positive and negative cumulative abnormal returns are not equal to fifty percent, non-parametric sign tests are used. Non-parametric tests address issues related to non-normally distributed date and small samples.

III. Methodology

The normal return is calculated for each day in the event window for each CSOE. The normal return is what one would expect if there were no *CEO pay reform* event. Because the return on the market index is commonly used as the normal return, we use the daily market return of the Chinese CSI 300 Index as the normal return. The CSI 300 Index represents China's 300 top stocks that are traded on the Shanghai and Shenzhen stock exchanges. The CSI 300 index is considered the blue-chip index for Chinese stocks and is an excellent proxy for the market return.

The abnormal return is calculated for each CSOE for each day over the four-day event window. The abnormal return represents the return not predicted by the market index, and is an estimate of the change in the stock price on that day due to *China's CEO pay reform* event. The abnormal return, AR_{it}, for each CSOE i on day t is defined as:

$$AR_{it} = R_{it} - R_{mt} \qquad (1)$$

where R_{it} is the return on the common stock of CSOE i on day t, and R_{mt} is the return on the market index (CSI 300 Index) on day t.

The cumulative abnormal return is calculated for each CSOE because in many cases, the market reaction to the announcement of an event may linger for days. For example, it may take financial analysts and investors several days to determine the impact of *China's CEO pay reform* event upon a CSOE's expected future earnings captured by the stock price. The stock market may continue to make stock price adjustments over a couple of days. Thus, the cumulative abnormal return is an estimate of the stock return caused by the event over the four-day event window. The cumulative abnormal return, CAR_i, for each CSOE for the four-day event window beginning with day -1 through day +2 is defined as:

$$CAR_i = \sum_{t=-1}^{+2} AR_{it} \qquad (2)$$

where AR_{it} is the daily abnormal return for CSOE i on day t.

Lastly, the mean and median cumulative abnormal returns are calculated for the CSOEs in the sample. The mean cumulative abnormal return can be viewed as a diversified portfolio, which eliminates unique individual stock returns by offsetting random positive stock returns with random negative stock returns. Thus, we have a mean cumulative abnormal return that captures only the characteristics of *China's CEO pay reform* event. Furthermore, if *China's CEO pay reform* event did not impact the future earnings of CSOEs, then the mean cumulative abnormal return should not be significantly different from zero. Likewise, the median cumulative abnormal return should not be significantly different from zero if *China's CEO pay reform* event did not impact future earnings of CSOEs. Finally, we examine the percent of cumulative abnormal returns that are positive for each of the three events. If *China's CEO pay reform* event did not impact the future earnings of CSOEs, then the percent of cumulative abnormal returns that are positive should not be significantly different from fifty percent.

We employ t-tests and Wilcoxon signed rank tests to determine whether the cumulative abnormal returns are significantly different from zero. The t-tests examine the mean return, and the Wilcoxon signed rank tests examine the difference in median returns and do not assume normally distributed data. Binomial z-statistics tests are used to determine whether the proportion of positive cumulative abnormal returns is significantly greater than 50 percent under the assumption of no reaction to the event. The binomial z-statistic is appropriate for small samples with non-normal distributions because it requires neither normally distributed data nor that the population be symmetric. Under the efficient market hypothesis, the likelihood of a rise or fall in stock price should be a flip of the coin, on average.

IV. Results

Below we examine *China's CEO pay reform* via the results of the *President Xi Effect*, the *Politburo Effect*, and the *Combined Effect*.

IV.1 President Xi Effect

We evaluate the *President Xi Effect,* and find that the CSOEs experienced significant financial gains from President Xi's announcement. As shown in Table 3, the CSOE's mean and median CARs for the *President Xi Effect* are 1.58% and 1.06%, respectively. When we calculate the absolute dollar impact of the *President Xi Effect*, we find the CSOE's mean and median market capitalization gains to be $16 million and $7 million, respectively. Cumulatively, the twenty-four CSOEs gained $384 million in market capitalization from President Xi's announcement. Our results support that *China's CEO Pay Reform* will benefit CSOEs, which will incur lower executive compensation expenses without increased perk consumption and tunneling activities.

IV.2 Politburo Effect

When we evaluate the *Politburo Effect*, we find that the CSOEs benefit greatly from the Politburo promulgating *China's CEO pay reform*. As shown in Table 3, the CSOE's mean and median CARs for the *Politburo Effect* are 1.58% and 1.06%, respectively. We calculate the absolute dollar impact and find the CSOE's mean and median market capitalization gains to be $73 million and $80 million, respectively. Cumulatively, the twenty-four CSOEs gained almost $1.75 billion in market capitalization from the Politburo announcement. Our results support that *China's CEO Pay Reform* will benefit CSOEs through lower executive compensation expenses.

IV.3 Combined Effect

The *Combined Effect* is the cumulative results of the *President Xi Effect* and the *Politburo Effect*. When we add the Politburo's promulgating of *China's CEO pay reform* to President Xi's announcement of the reform, we get a compounding effect where CSOEs benefited financially from both events.

When the *Combined Effect* is evaluated, we find that the shareholders of CSOEs experienced tremendous gains from *China's CEO pay reform.* As shown in Table 3, the CSOE's mean and median CAR for the *Combined Effect* are 3.05% and 2.73%, respectively. Additionally, we find that 83% of the CSOEs experienced a positive CAR for the *Combined Effect*. When we calculate the absolute dollar impact on the CSOEs, we find the mean and median market capitalization gains to be $89 million and $87 million, respectively. Cumulatively, the twenty-four CSOEs gained $2.136 billion in market capitalization. The $2+ billion gain is driven from the fact that CSOEs will incur lower executive compensation expenses without increased perk consumption and tunneling activities. Thus, the earnings will be higher, and the CSOE is more valuable to investors. All three of our test results, which are both statistically and economically significant, clearly show that the shareholders of CSOEs benefitted financially with *China's CEO pay reform*.

Table 3: China's Central State-Owned Enterprises (CSOEs) – Cumulative Abnormal Returns (CAR) for President Xi Effect, Politburo Effect, and Combined Effect

CSOEs (n=24)	President Xi Effect	Politburo Effect	Combined Effect
Mean CAR	1.58%**	1.47%***	3.05%***
t-statistic	*2.56*	*3.12*	*3.75*
(p-value)	*(0.017)*	*(0.005)*	*(0.001)*
Median CAR	1.06%**	1.57%***	2.73%***
Wilcoxon signed rank test	*86*	*95*	*112*
(p-value)	*(0.011)*	*(0.0041)*	*(<0.001)*
Percent positive CARs	75%**	83%***	83%***
Sign test	*-2.45*	*-3.27*	*-3.27*
(p-value)	*(0.014)*	*(0.001)*	*(0.001)*
Shapiro-Wilk test for normality	Not normal	Not normal	Not normal
(p-value)	*(0.422)*	*(0.691)*	*(0.606)*

***, **, and * denote one, five, and ten percent significance levels, respectively.

V. Conclusion

The People's Republic of China founded in 1949 adopted the Soviet Union's centrally planned economy with central state-owned enterprises (CSOEs) to industrialize the economy. After almost 20 years of little success, China called for *reform and opening up* the economy in 1978, but still maintained a planned economy. Finally, in 1990, China opened the Chinese stock markets and introduced a socialist market economy. From 1990 to 2015, private ownership evolved, and China recognized the need to resolve the well-known agency problem of owners versus managers. For years, centrally state-owned enterprises (CSOEs) incentivized managers to solve social problems (e.g., retain redundant workers) rather than managing well run businesses (e.g., lay off redundant workers). CSOE manager compensation was far too high, and did not effectively utilize firm performance-based incentives. After China's President Xi called for compensation reform and the Central Politburo of the Communist Party of China promulgating for compensation reform, *China's CEO pay reform* was passed. We find that the shareholders benefitted tremendously from *China's CEO pay reform,* whereby the shareholders of CSOEs experienced a mean and median gain in market capitalization of $89 million and $87 million, respectively. Cumulatively, the twenty-four CSOEs gained over $2 billion in market capitalization. This is not surprising given that the reform included a firm performance component that had been omitted previously, thereby better aligning the interests of managers and shareholders. We conclude that *China's CEO pay reform* was highly effective in reducing agency problems.

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