Academy of Finance

# Journal of Finance Issues

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**To the Members of the Academy of Finance and Readers of the Journal of Finance Issues,**

I am pleased to announce the publication of Volume 22, Number 3 of the *Journal of Finance Issues*. This issue features four insightful articles that address diverse and critical topics in finance, providing valuable contributions to both academia and practice.

1. **"Analyzing Changing "Investor Exuberance": The Determinants of S&P Composite Index Total Return CAPE Changes" by C.N.V. Krishnan, Xiyao Tan, and Jiemin Yang**
   This paper delves into the factors influencing changes in investor exuberance, as measured by the S&P Composite Index Total Return Cyclically Adjusted Price-to-Earnings ratio (TR CAPE). The authors employ a variety of econometric techniques, including linear regression using Principal Component Analysis (PCA), Lasso Regression, and Ridge Regression, to identify key variables associated with shifts in investor sentiment. Their findings highlight the significant role of consumer sentiment, as measured by the University of Michigan Consumer Sentiment Index, in driving changes in investor exuberance.

2. **"Individual Stock Returns Volatility and Equity Anomalies" by Sunghan Bae and Keshav R. Bhattarai**
   This paper investigates the relationship between individual stock return volatility and equity anomalies. The authors find that volatility, estimated using the EGARCH model, plays a significant role in explaining equity anomalies such as size, value, liquidity, momentum, and short-term reversals. Their findings suggest that incorporating volatility measures into asset pricing models can enhance their explanatory power and provide a more nuanced understanding of stock return behavior.

3. **"An Update on Sector Rotation in the "Sell in May and Go Away" Strategy" by Steven Dolvin and Bryan Foltice**
   This paper revisits the well-known "Sell in May and Go Away" (SMGA) strategy, which posits that market returns tend to be higher during the November to April period compared to the May to October period. The authors find that this seasonal pattern persists in more recent periods, and they explore whether incorporating sector rotation into the strategy can further enhance returns. Their results suggest that while sector rotation alone may not improve risk-adjusted performance, combining it with a short-selling strategy can generate positive alpha.

4. **"A Stochastic Analysis of Buy and Hold Versus Annual Rebalancing Portfolio Strategies" by Michael Mattei**
   This paper examines the long-debated question of whether buy-and-hold or annual rebalancing strategies yield superior risk-adjusted portfolio returns. The author employs a stochastic simulation approach to generate a wide range of possible future asset pricing scenarios. The findings indicate that while rebalancing is generally favored, buy-and-hold can outperform in certain instances. This research provides valuable insights into

the nuances of portfolio management and highlights the importance of considering various factors when selecting an investment strategy.

The publication of this issue would not have been possible without the invaluable contributions of our dedicated reviewers. Their insightful feedback and evaluation have been instrumental in ensuring the quality and integrity of the research presented in the Journal of Finance Issues. I extend my sincere gratitude to all the anonymous reviewers for their commitment to advancing financial knowledge.

I also want to express my appreciation to the associate editors, Larry, Seongsu (David), and Won, for their exceptional work in managing the review process. Their expertise and diligence have been essential in maintaining the high standards of the journal.

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**Call for Papers: Special Issue on Artificial Intelligence in Finance**

Dear Colleagues,

We are pleased to announce a special issue of the Journal of Financial Issues dedicated to exploring the dynamic and rapidly evolving field of **Artificial Intelligence in Finance**.

In response to a recent survey, we have expanded the scope beyond investment to encompass the broader implications of AI across the financial sector. We invite you to submit papers on the following suggested topics or any other relevant area:

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- AI and economic forecasting
- AI and fraud detection
- AI and financial inclusion
- AI and ESG issues
- AI and CSR issues
- AI and financial stability
- AI and pedagogical issues
- AI and case studies

We are honored to have **Dr. Hoje Jo** from Santa Clara University serve as the Guest Editor for this special issue. Dr. Jo is the Gerald and Bonita Wilkinson Professor of Finance at the Leavey School of Business and holds a Ph.D. in Finance from the University of Florida. His research interests span corporate, international, venture capital, entrepreneurial finance, and corporate social responsibility, and his work has been published in top-tier journals, including the Journal of Finance, Journal of Financial Economics, Journal of Financial and Quantitative Analysis, Review of Accounting Studies, Journal of Business Venturing, and Journal of Business Ethics. His contributions have garnered numerous accolades, notably the 2009 Moskowitz Prize for his research in socially responsible investing.

Submissions are welcome until **April 30th, 2025**, via our online portal at https://jfi-aof.org/. Please indicate your submission is for the ”AI in Finance Special Issue“ in the "Comments for the Editor" section. We plan to publish the special issue during the second half of 2025.

If you are interested in joining the special issue's editorial board, please contact us at jfi-slu@slu.edu.

We look forward to receiving your valuable contributions!

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# Analyzing Changing "Investor Exuberance": The Determinants of S&P Composite Index Total Return CAPE Changes

C. N. V. Krishnan, Xiyao Tan, and Jiemin Yang *

**Abstract**

We analyze the determinants of changes in S&P Composite Index Total Return Cyclically Adjusted Price-to-Earnings ratio (TR CAPE), to better understand changing "investor exuberance". We use three different methods - linear regression using PCA, Lasso, and Ridge regression techniques, as well as ElasticNet method – and a large number of explanatory variables, to explain changing investor exuberance. Across all methods, we find that monthly changes in Michigan sentiment index is significantly associated with monthly changes in TR CAPE. When we cross check the results using annual changes (rather than monthly changes), across all methods, annual changes in Michigan sentiment index and changes in core inflation are significantly associated with annual changes in TR CAPE.

**Keywords**: TR CAPE, Cyclically Adjusted Price-to-Earnings ratio, investor exuberance, PCA, Principal Components Analysis, Lasso regression, Ridge regression, determinants, stock market variables, economy-wide variables, fixed income variables, commodity variables.

**JEL Code**: G10

## I. INTRODUCTION

Well-known valuation indicators such as the CAPE (Cyclically Adjusted Price-to-Earnings) ratio have been studied for their predictive power in financial markets. For example, Siegel (2016) discusses how CAPE can be used to predict future stock returns over long horizons, providing insights into market valuation and investment strategy. Mauboussin and Callahan (2014) examine the relationship between CAPE and economic cycles, highlighting its utility in signaling economic downturns and recoveries. Market sentiment analysis also incorporates CAPE to gauge investor behavior. Baker and Wurgler (2007), for example, discuss how sentiment-driven market fluctuations may be linked to understanding CAPE trends, providing insights into investor behavior and market dynamics.

Our objective in this paper is to explain the contemporaneous relation between changes in various key variables and changing investor exuberance, captured by changes in S&P Composite Index Total Return Cyclically Adjusted Price-to-Earnings ratio (TR CAPE) (Shiller, 2005). In other words, we use an explanatory model, not a predictive model (see Shmueli, 2010). Any change in any explanatory variable should affect price contemporaneously (the numerator in P/E ratio) as it changes investor perception (Fama, 1970). A high TR CAPE ratio be a sign of exuberance or speculative bubbles. Conversely, a low TR CAPE ratio can indicate pessimism about stock performances.

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There are many possible explanatory variables, including economy wide variables, market variables, fixed income variables, and commodity variables. To analyze this, we use different methods and compare results. We compare the results of linear regression using Principal Component Analysis (PCA), Lasso Regression, and Ridge Regression. Integrating PCA, Lasso, and Ridge regression techniques can enhance financial research by leveraging the strengths of each method. Hastie, Tibshirani, and Wainwright (2015), for example, provide a comprehensive guide on combining these methods to improve model robustness and predictive accuracy, illustrating the benefits of an integrated approach in financial econometrics. The significance of selecting the appropriate methodology must take into account the fact that different techniques can yield varying results in terms of variable significance, as highlighted by Zou and Hastie (2005). This enables us to compare and contrast the results.

Principal Component Analysis (PCA) has been extensively used to reduce dimensionality in large datasets by identifying key factors that drive movements. Jolliffe (2002) elaborates on the mathematical foundations and applications of PCA, emphasizing its role in reducing model complexity while retaining essential information. Baker and Wurgler (2006) explore the impact of investor sentiment on stock returns using PCA regression, demonstrating the importance of accounting for psychological factors in financial modeling. Stock and Watson (2002) applied PCA analysis for macroeconomic forecasting, to identifying the key factors from many possible predictors. Connor and Korajczyk (1986) demonstrate how PCA analysis improves understanding of market dynamics by isolating significant variables from noise. Litterman and Scheinkman (1991) applied PCA to the term structure of interest rates, showing its utility in identifying principal components that explain the variance in bond returns.

Lasso regression, introduced by Tibshirani (1996), performs variable selection and regularization by imposing an L1 penalty on regression coefficients (a penalty on the sum of the absolute values of the regression coefficients), effectively shrinking some coefficients to zero. This means that the Lasso method can identify and keep only the most significant variables in the model, while excluding others that are less important. This method is particularly useful in financial modeling, where multicollinearity is a common issue. Studies by Fan and Li (2001) have demonstrated the effectiveness of Lasso in improving financial econometric models by selecting relevant predictors and reducing the influence of less significant variables.

Ridge regression addresses multicollinearity by adding an L2 penalty to the regression equation, stabilizing coefficient estimates without reducing them to zero. The L2 penalty, also known as the Ridge penalty, is on the sum of the squared values of the regression coefficients. Unlike the L1 penalty used in Lasso regression, the L2 penalty does not set any coefficients to zero. Instead, it shrinks all coefficients toward zero. Hoerl and Kennard (1970) initially proposed Ridge regression, which has since been applied in various financial contexts. This technique has been shown to enhance the stability and predictive accuracy of economic indicators and financial forecasting models (Elliott & Timmermann, 2008). Hoerl and Kennard (1970) demonstrate the utility of Ridge regression in dealing with multicollinearity and improving the accuracy of economic predictions. Stock and Watson (2002) applied Ridge regression in macroeconomic forecasting, illustrating its effectiveness in handling high-dimensional economic data.

The comparative analysis of PCA, Lasso, and Ridge regression techniques has been a focal point in recent financial research. Hastie, Tibshirani, and Friedman (2009) provide a detailed comparison, emphasizing the unique advantages and drawbacks of each method. James, Witten, Hastie, and Tibshirani (2013) provide insights into the applications of Lasso and Ridge regression in modern financial econometrics, highlighting their effectiveness in selecting significant predictors and improving model performance.

We also use ElasticNet, introduced by Zou and Hastie (2005), as a combination of Lasso and Ridge regressions. While Lasso tends to select a few key variables and forces the rest to zero, Ridge regression includes all variables but shrinks coefficients uniformly. ElasticNet creates a balance by blending the penalties of both methods, allowing for variable selection like Lasso while retaining the regularization properties of Ridge. This makes it particularly effective when dealing with correlated predictors, which is often the case in financial datasets and the case here as well. By using ElasticNet, researchers can achieve a more robust and interpretable model that integrates the strengths of Lasso's feature selection and Ridge's handling of multicollinearity, and represents a comprehensive methodology that captures both predictive accuracy and variable significance in financial econometrics.

We use all these methods in this paper. PCA based linear regression results show that monthly change in Michigan sentiment index and one commodity (Zinc) price change are significantly associated with monthly change in Index TR CAPE. Lasso regression (using 3 different methodologies) shows that monthly change in Michigan sentiment index and change in the 5-year Treasury yield are significantly associated with monthly change in Index TR CAPE, consistently across the 3 methods. Ridge and ElasticNet regressions show that monthly change in Michigan sentiment index, change in 5-year Treasury yield and change in 10-Year High-Quality-Market Corporate-Bond Spot Rate are significantly associated with monthly change in TR CAPE, across the 2 methods.

Therefore, across all methods, monthly changes in Michigan sentiment index is significantly associated with monthly changes in TR CAPE. Consumer sentiment is seen as a leading indicator of economic activity. Carroll, Fuhrer, and Wilcox (1994) and Ludvigson (2004), for example, argue that consumer sentiment can significantly influence stock market valuations. When consumers feel confident about the economy, they may be more likely to invest in the stock market, driving up stock valuations and increasing the market-index TR CAPE ratio (Brown and Cliff, 2004; and Fisher and Statman, 2000). Conversely, a decline in sentiment might indicate a future drop in stock prices (Baker and Wurgler, 2007). The Michigan Sentiment Index (based on survey of consumers) is a widely recognized measure of consumer confidence, which can have a significant impact on economic expectations and investor behavior, and hence on TR CAPE (also see, for example, Lemmon and Portniaguina, 2006).

When we cross check the results using annual changes in Index TR CAPE (rather than monthly changes), we find, using linear regression using PCA, that the annual change in Michigan sentiment index, a commodity (Zinc) price change, and change in core inflation are significantly associated with the annual change in TR CAPE. From LASSO regression (using three different methodologies), we find that the annual change in GDP index, change in core inflation, change in money supply, change in Michigan sentiment index, and change in 5-year Treasury yield are consistently associated with the annual change in TR CAPE across all three methods. From Ridge and ElasticNet regressions, we find that the annual change in core inflation, change in Michigan sentiment index, change in 5-year Treasury yield, change in Moody's BAA rate, Dow Jones return, Nikkei 225 (Japan) and Hang Seng Index (Hong Kong) are associated with annual change in TR CAPE, across the 2 methods.

Therefore, across all methods, annual changes in Michigan sentiment index and changes in core inflation are significantly associated with annual changes in TR CAPE.

We have already discussed the importance of consumer confidence above. Core inflation, which excludes food and energy prices, tends to be more stable as it reflects the underlying trend of inflation without the noise from volatile components. This makes it a more stable indicator, providing a clearer view of long-term inflation trends without the noise of short-term price changes. Higher core inflation can lead to increased interest rates, which raises the discount rate, reducing the present value of future cash flows and, in turn, lowering stock market valuations, including the TR CAPE ratio. Campbell and Shiller (1998) demonstrated that inflationary pressures, especially core inflation, directly affect the discount rates applied to future earnings, thereby potentially influencing valuation metrics like the TR CAPE ratio. Fama and Schwert (1977) also argue that core inflation impacts the discount rates used to value future earnings, thereby impacting valuation metrics.

## II. DATA AND VARIABLES

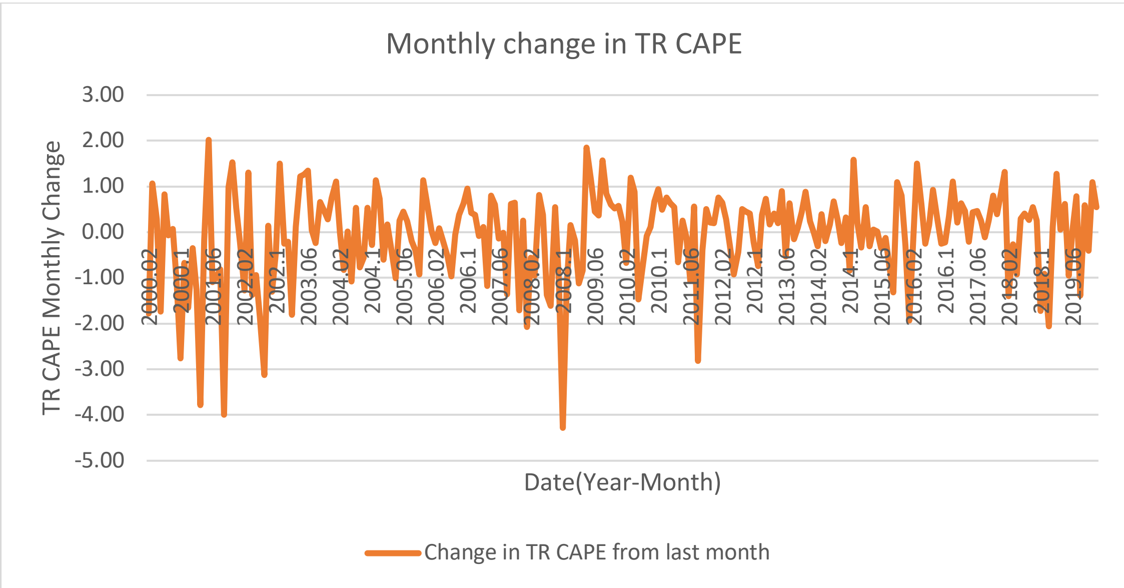
The variable we examine is the monthly Cyclically Adjusted S&P Composite Index Total Return Price Earnings Ratio (TR CAPE) from February 2000 to December 2019 taken from Robert Shiller's website (this data, updated, was used in "Irrational Exuberance" by Robert Shiller, Princeton University Press, 2000, 2005, 2015). Cyclically Adjusted Price Earnings Ratio" (CAPE), also known as P/E-10, and is a valuation measure that divides the current price of a stock or index by the average of ten years of earnings, adjusted for inflation. This ratio is used to assess whether a stock or market is over- or undervalued by comparing the current CAPE ratio to historical averages. TR CAPE (or TR P/E10) is similar to CAPE ratio but adjusted to include total return, which incorporates dividends and capital gains. It divides the current price of a stock or index by the average of ten years of earnings, adjusted for inflation and total returns. This ratio provides a more comprehensive view of the market's valuation by considering the full return profile. Therefore, we use this to specifically examine the determinants of the monthly change TR CAPE, to examine what determines changes in investor exuberance from month to month. As an additional check, we examine yearly changes in TR CAPE. We collect 42 explanatory variables from Bloomberg, FRED database of Federal Reserve Bank of St Louis and WRDS (Wharton Research Database System) as possible determinants to changes in Index TR CAPE.

We first examine monthly changes in the S&P Index TR CAPE. From Figure 1, the average monthly change in Index TR CAPE over 239 months in the database is approximately -0.060, with a standard deviation of 0.976 and a median of 0.150. The negative mean suggests a general downward trend in TR CAPE from the previous month. The standard deviation is moderate, indicating that the changes in TR CAPE can vary from month to month. The positive median, slightly higher than the average, suggests that while the average change is negative, the distribution of changes is such that the central tendency (median) remains positive, indicating that there are more positive changes than the negative mean might imply.

The first set of explanatory variables we consider that can influence changes in Index TR CAPE are the economy-wide explanatory variables. We consider monthly changes as in (This month - Last month) for US GDP index, US CPI index, Core inflation, Federal Funds Rate, Unemployment rate, Industrial production, M2 money supply, Personal income index, Producer price index, and Michigan sentiment index. Some of the key ones are US GDP index, Core inflation, M2 money supply, and Michigan sentiment index. Fama (1990) highlighted the critical role of GDP growth as a driver of stock market valuations. An increase in GDP is typically indicative of economic expansion, leading to higher corporate earnings and, consequently, higher stock prices. Higher core inflation generally leads to increased interest rates, which raises the discount rate, reducing the present value of future cash flows and, in turn, lowering stock market valuations. Friedman and Schwartz (1963) explored the relationship between monetary aggregates, such as the M2 money supply, and stock market valuations. Their findings suggest that an increase in the M2 money supply, often a result of expansionary monetary policy, enhances liquidity in financial markets, thereby pushing up asset prices as more money chases the same number of assets. As explained earlier, the Michigan Sentiment Index (based on survey of consumers) is a widely recognized measure of consumer confidence, which can have a significant impact on economic expectations and investor behavior. Figure 2 shows the plots of a few key economic indicators, to get a sense of the variabilities, while Table 1 shows the descriptive statistics.

**Figure 1: Plot of Monthly change in TR CAPE**

*This figure shows the plot of monthly changes in S&P Composite Index Total Return CAPE from February 2000 to December 2019, the data for which taken from Robert Shiller's website.*




From Figure 2, the change in the US GDP index shows significant variability over the period, particularly during the 2008 financial crisis. The GDP index reflects the overall economic output, and sharp movements can be observed during periods of economic expansion or contraction. The most pronounced fluctuations occur around the periods of economic recessions and recoveries, such as in 2008-2009, where GDP changes are larger and more erratic. The change in core inflation is less variable compared to the other variables. Core inflation, which excludes food and energy prices, tends to be more stable as it reflects the underlying trend of inflation without the noise from volatile components. The relative stability in core inflation suggests that underlying inflationary pressures remained moderate throughout the period, even during economic downturns. The change in M2 money supply is one of the more variable indicators, particularly showing large spikes around periods of economic intervention, such as during the financial crisis and subsequent quantitative easing programs. The Federal

Reserve's actions to increase liquidity in the financial system are evident in the large positive changes in M2. This variability reflects the monetary policy actions aimed at stabilizing the economy during periods of financial stress. The Michigan Sentiment Index, which measures consumer confidence, also exhibits notable variability, especially during economic downturns like the 2008 financial crisis. Changes in consumer sentiment are typically more volatile as they reflect consumer reactions to economic news, labor market conditions, and overall economic health. The sentiment index tends to react quickly to economic conditions, making it a likely candidate for a leading indicator of economic activity. Thus, the variability in these economic indicators highlights different aspects of the economic environment.

**Figure 2: Monthly Changes in Key Economy Wide Explanatory Variables**

*This figure shows the plots (in different colors) of monthly changes in 4 key economic indicators from February 2000 to December 2019, taken from different sources: Bloomberg, FRED, and WRDS.*

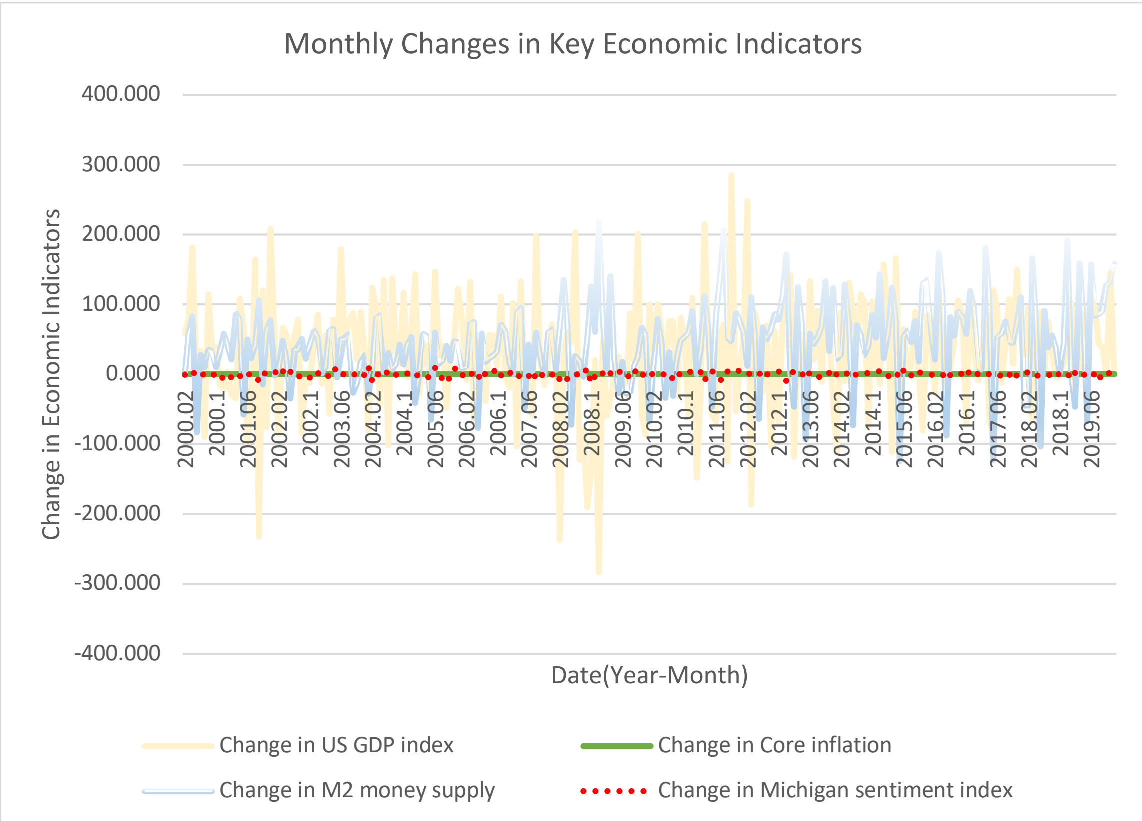



Table 1 presents the descriptive statistics of monthly changes in various key explanatory variables. The means of most variables show small deviations from zero, except for the change in the M2 money supply, personal income index, and US GDP index, which have positive means, indicating a general trend of increase over the observed period. The standard deviations vary, with the M2 money supply and personal income index showing the highest variability, reflecting significant fluctuations in these indicators. The change in the Federal Funds Rate and unemployment rate have medians close to zero, suggesting general stability in these measures during the observed period. As mentioned above, the Michigan Sentiment Index also exhibits notable variability.

**Table 1: Descriptive Statistics of Monthly changes in key Economic Variables**

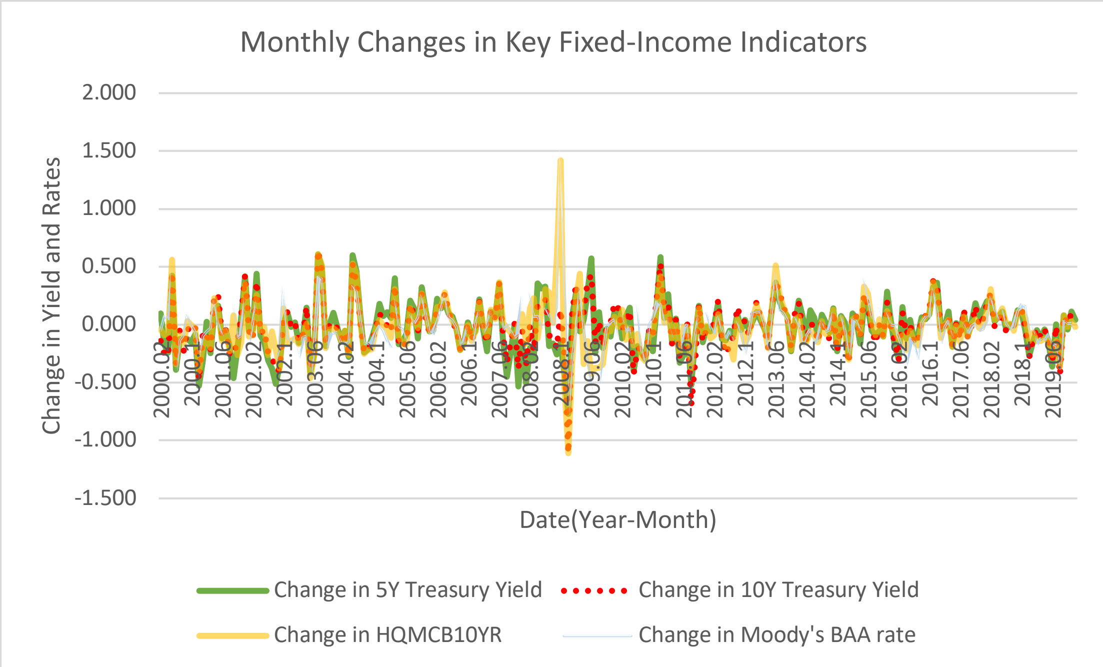
*This table shows the mean, median, standard deviation (SD) of monthly changes, as well as the number of months of observations (N), of all the economy-wide explanatory variables we collect. The sources of data are Bloomberg, WRDS and FRED databases*

| | N | Mean | SD | Median |
|---|---|---|---|---|
| Change in US GDP index | 239 | 30.071 | 84.260 | 35.645 |
| Change in US CPI index | 239 | 0.374 | 0.613 | 0.406 |
| Change in Core inflation | 239 | 0.363 | 0.168 | 0.393 |
| Change in Federal Funds Rate | 239 | -0.016 | 0.166 | 0.002 |
| Change in Unemployment rate | 239 | -0.002 | 0.160 | 0.000 |
| Change in Industrial production | 239 | 0.044 | 0.623 | 0.082 |
| Change in M2 money supply | 239 | 44.864 | 57.705 | 48.840 |
| Change in Personal income index | 239 | 38.518 | 85.332 | 41.200 |
| Change in Producer price index | 239 | 0.296 | 2.047 | 0.400 |
| Change in Michigan sentiment index | 239 | -0.053 | 4.069 | -0.200 |

The next set of explanatory variables we consider that can influence changes in Index TR CAPE are the fixed-income explanatory variables. We consider monthly changes, computed as (This month - Last month) for 1M Treasury Yield, 6M Treasury Yield, 1Y Treasury Yield, 5Y Treasury Yield, 10Y Treasury Yield, 30Y Treasury Yield, HQMCB10YR (10-Year High Quality Market Corporate Bond Spot Rate), Moody's AAA rate, Moody's BAA rate, and TED spread. Some of the key ones are 5-year (5Y) Treasury Yield, 10-year (10Y) Treasury Yield, HQMCB10YR, and Moody's BAA rate. Campbell and Viceira (2002) found that changes in intermediate-term Treasury yields, such as the 5-year Treasury yield, have a direct impact on equity valuations. As Treasury yields rise, the discount rate applied to future earnings increases, leading to lower present values of expected cash flows and a reduction in the TR CAPE ratio. Conversely, lower yields can support higher equity valuations by reducing the discount rate. Shiller (2005) analyzed the effect of long-term Treasury yields on stock market valuations. He concluded that the 10-year Treasury yield, often used as a benchmark for long-term interest rates, influences investor expectations about future inflation and economic growth. Higher 10-year yields may lead to lower TR CAPE ratios as they signal higher discount rates and potential economic slowing. Fama and French (1989) explored the relationship between high-quality corporate bond yields and equity valuations. They found that changes in these yields reflect the credit risk perceptions of high-quality corporate bonds, which can signal broader economic conditions. Rising corporate bond yields often correlate with increased risk aversion among investors, leading to lower equity valuations. Chen, Roll, and Ross (1986) investigated the impact of credit spreads, (e.g., Moody's BAA rate), on stock market valuations. They demonstrated that wider BAA spreads indicate higher credit risk and economic uncertainty, leading investors to demand higher returns on equities. This can depress stock prices and can reduce the TR CAPE ratio. Conversely, narrowing spreads suggest improving economic conditions, which can boost equity valuations. Figure 3 shows the plots of a few key fixed income indicators, to get a sense of the variabilities, while Table 2 shows the descriptive statistics.

**Figure 3: Monthly changes in Fixed Income Variables.**

*This figure shows the plots (in different colors) of monthly changes in 4 key fixed-income indicators from February 2000 to December 2019, taken from different sources: Bloomberg, FRED, and WRDS.*




From Figure 3, the 5Y Treasury Yield exhibits moderate variability over the period, influenced by short- to medium-term economic outlooks and monetary policy decisions. It tends to react strongly to changes in Federal Reserve policy and shifts in investor sentiment regarding inflation and economic growth. The 10 year Treasury Yield also shows moderate variability but is generally more stable than the 5Y Treasury Yield. This is because the 10-year yield represents long-term interest rates and is less sensitive to short-term economic fluctuations. However, it does respond to long-term economic expectations, inflation forecasts, and major policy announcements. Notable fluctuations are observed during periods of economic stress, such as the financial crisis of 2008. HQMCB10YR, which reflects high-quality corporate bond yields over a 10-year period, shows significant variability, particularly during the 2008 financial crisis. The spikes in this yield reflect periods of increased risk aversion among investors, where demand for safe assets like government bonds increases, and corporate bond yields rise due to perceived higher credit risk. This yield is sensitive to changes in economic conditions that affect corporate creditworthiness. Moody's BAA rate, which tracks yields on lower investment-grade corporate bonds, exhibits the highest variability among these variables, especially during the 2008 crisis. The pronounced spikes in the BAA rate reflect periods of heightened credit risk, where investors demand higher yields for taking on additional risk associated with lower-rated bonds. This rate is highly sensitive to economic downturns and periods of financial instability. Thus, this plot illustrates how different fixed income variables react to economic conditions. Treasury yields (5Y and 10Y) generally reflect broader economic expectations and are influenced by monetary policy, while corporate bond yields (HQMCB10YR and Moody's BAA rate) are more sensitive to credit risk and investor sentiment.

**Table 2: Descriptive Statistics of Fixed Income Variables**

*This table shows the mean, median, standard deviation (SD) of monthly changes, as well as the number of months of observations (N), of all the fixed-income explanatory variables we collect. The sources of data are Bloomberg, WRDS and FRED databases.*

| | N | Mean | SD | Median |
|---|---|---|---|---|
| Change in 1M Treasury Yield | 221 | 0.007 | 0.303 | 0.003 |
| Change in 6M Treasury Yield | 239 | -0.017 | 0.171 | -0.001 |
| Change in 1Y Treasury Yield | 239 | -0.019 | 0.171 | -0.002 |
| Change in 5Y Treasury Yield | 239 | -0.02 | 0.217 | -0.028 |
| Change in 10Y Treasury Yield | 239 | -0.02 | 0.21 | -0.023 |
| Change in 30Y Treasury Yield | 239 | -0.018 | 0.187 | -0.03 |
| Change in HQMCB10YR | 239 | -0.021 | 0.225 | -0.04 |
| Change in Moody's AAA rate | 239 | -0.02 | 0.174 | -0.026 |
| Change in Moody's BAA rate | 239 | -0.019 | 0.205 | -0.022 |
| Change in TED spread | 239 | -0.001 | 0.193 | -0.001 |

Table 2 presents the descriptive statistics for various fixed income variables. The means of most variables are slightly negative, indicating a slight downward trend in these fixed income rates over the observed period. The standard deviations vary, with the highest observed in the changes in 1 month Treasury Yield and HQMCB10YR, reflecting higher volatility in these indicators. The medians also display a mixture of positive and negative values, with the majority being negative, suggesting a general downward central tendency among the fixed income variables, though with some slight upward tendencies in certain cases, such as the 1M Treasury Yield.

The stock market explanatory variables that we consider are monthly returns computed as (Percentage change) = ((This month - Last month)/Last month)×100 for Dow Jones Index, S&P 500 Index, Russell 5000 Index, Value-weighted Center for Research in Security prices (VWCRSP) index, Equally weighted CRSP (EWCRSP) index, FTSE 100 (UK) Index, DAX (Germany) Index, CAC 40 (France) Index, Nikkei 225 (Japan) Index, Hang Seng (Hong Kong) Index, S&P/ASX 200 (Australia) index, and TSX Composite Index (Canada) Index. Some of the key ones are Dow Jones return (a blue-chip index that includes 30 large, well-established U.S. companies), S&P 500 return, Nikkei 225 (Japan) and Hang Seng Index (Hong Kong). Asness (2000) finds that positive returns in the Dow Jones Industrial Average often reflect strong economic fundamentals and investor optimism, leading to higher stock valuations. This upward pressure on valuations can result in an increased TR CAPE ratio as stock prices rise relative to long-term earnings. Shiller (1981) argues that strong performance in the S&P 500, which represents a broad measure of the U.S. stock market, is typically associated with increased investor confidence and expectations of future earnings growth. This can drive the TR CAPE ratio higher, as investors are willing to pay more for stocks in anticipation of continued growth. Froot and Ramadorai (2008) find that the performance of the Nikkei 225 can influence investor sentiment globally, especially in markets with close economic ties to Japan. Strong returns in the Nikkei 225 can boost confidence in Asian markets and contribute to higher TR CAPE ratios globally. Bekaert and Harvey (1997) find that fluctuations in the Hang Seng Index, which reflects the economic and financial conditions in Hong Kong and China, can impact global investor sentiment. Positive returns in the Hang Seng Index can lead to increased valuations in other markets, pushing up the TR CAPE ratio as investors become more optimistic about global economic prospects. Figure 4 shows the plots of a few key stock market returns, to get a sense of the variabilities, while Table 3 shows the descriptive statistics.

**Figure 4: Monthly returns in Stock Market Variables**

*This figure shows the plots (in different colors) of monthly changes in 4 key stock market returns from February 2000 to December 2019, taken from different sources: Bloomberg, FRED, and WRDS.*

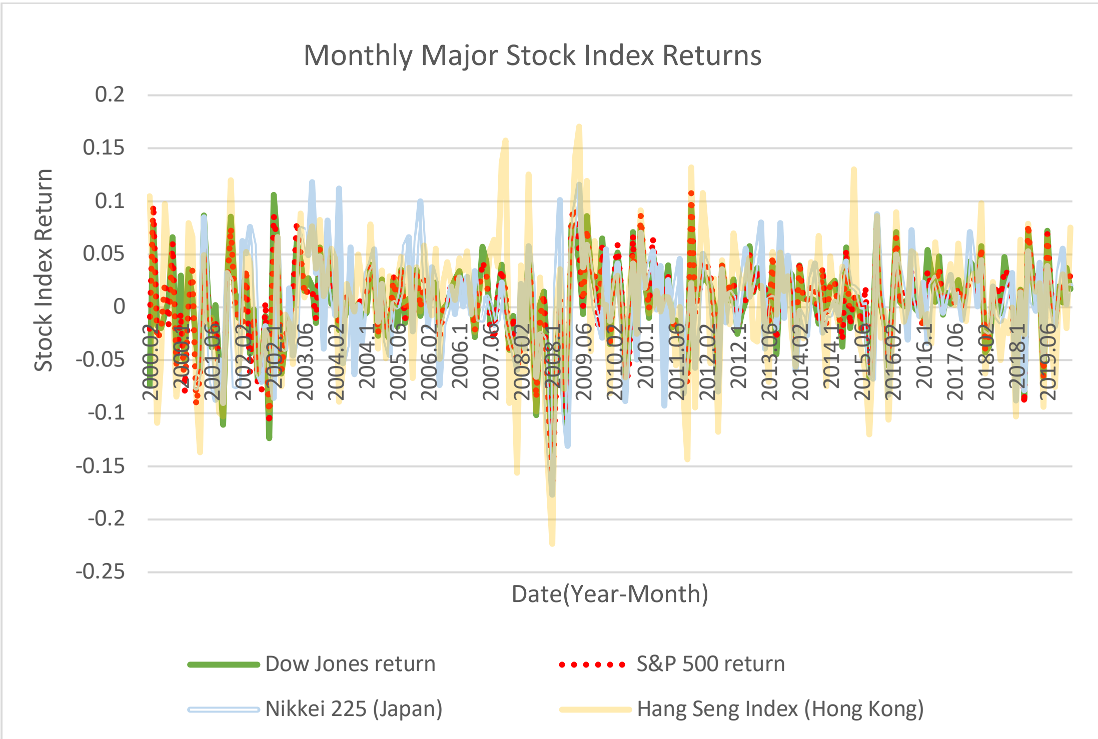



Figure 4 shows that the Dow Jones return is relatively stable compared to the other indices but still exhibits significant fluctuations, particularly during periods of economic stress such as the 2008 financial crisis. It tends to reflect broader economic conditions in the U.S. The S&P 500 return is like the Dow Jones in terms of variability but slightly more volatile due to its broader composition of 500 companies across various sectors. The S&P 500 is often seen as a better representation of the overall U.S. stock market, and its returns show a clear reaction to major economic events, such as the 2008 financial crisis, where significant dips are observed. The Nikkei 225 return exhibits moderate variability, reflecting the performance of the Japanese stock market. The index is sensitive to both domestic and international economic conditions, as Japan's economy is highly integrated into global trade and finance. Notable volatility can be seen during global economic downturns, such as in 2008, where the Nikkei experienced significant swings. The Hang Seng Index return is the most volatile among the four indices. This high variability is due to the Hang Seng Index's sensitivity to regional and global economic conditions, particularly those affecting China and the broader Asia-Pacific region. The index shows large swings during periods of financial instability, such as the 2008 financial crisis and subsequent global market events. The Hang Seng's high volatility reflects the dynamic nature of the Hong Kong market, which is influenced by both local factors and broader geopolitical and economic developments. The variability in these stock market indices highlights their sensitivity to different economic environments.

**Table 3: Descriptive Statistics of Stock Market Variables**

*This table shows the mean, median, standard deviation (SD) of monthly returns, as well as the number of months of observations (N), of all the stock market explanatory variables we collect. The sources of data are Bloomberg, WRDS and FRED databases.*

| | N | Mean | SD | Median |
|---|---|---|---|---|
| Dow Jones return | 239 | 0.005 | 0.04 | 0.008 |
| S&P 500 return | 239 | 0.004 | 0.042 | 0.01 |
| Russell 5000 return | 186 | 0.007 | 0.041 | 0.012 |
| VWCRSP index return | 239 | 0.004 | 0.042 | 0.01 |
| EWCRSP index return | 239 | -0.052 | 0.921 | 0.01 |
| FTSE 100 (UK) return | 239 | 0.001 | 0.046 | 0.003 |
| DAX (Germany) return | 239 | 0.006 | 0.067 | 0.007 |
| CAC 40 (France) return | 239 | 0.003 | 0.059 | 0.005 |
| Nikkei 225 (Japan) return | 227 | 0.004 | 0.048 | 0.009 |
| Hang Seng Index (Hong Kong) return | 239 | 0.004 | 0.06 | 0.01 |
| S&P/ASX 200 (Australia) return | 239 | 0.005 | 0.059 | 0.01 |
| TSX Composite Index (Canada) return | 239 | 0.005 | 0.056 | 0.008 |

The table shows that the means of the stock market variables are slightly positive, indicating small average monthly returns across the observed indices. The standard deviations are relatively low, with the highest observed for the EWCRSP index.[†] The medians are all positive, suggesting a general upward tendency in returns. However, the EWCRSP index shows a negative mean, one reason for which is the vulnerability of small-cap stocks to economic downturns (Fama & French, 1989). The equal-weighting method amplifies the impact of these losses since smaller firms are given the same weight as larger ones, further driving down this index's overall performance (Bekaert & Harvey, 1997).

The commodity prices explanatory variables that we consider are changes in prices computed as (This month price - Last month price) for sugar, coffee, soybean,

† The high SD of EWCRSP index is attributable to several factors. First, small-cap stocks, which play an equal role in equal-weighted indices like EWCRSP, tend to be more sensitive to changes in investor sentiment, leading to greater volatility compared to large-cap stocks (Baker & Wurgler, 2006). Additionally, small cap firms typically face lower liquidity, which amplifies price swings and contributes to higher variability in returns (Fama & French, 1989). Finally, small cap firms tend to experience higher overall volatility due to increased market risks, further explaining the larger standard deviation observed in the EWCRSP index (Bekaert & Harvey, 1997).

tobacco, crude oil, natural gas, gold, zinc, and rice. Some of the key ones are soybean price change, crude oil price change, gold price change and zinc price change. Bessler and Kling (1989) find that changes in soybean prices can influence the earnings of companies in the agricultural sector, as well as companies dependent on agricultural products. Additionally, soybean prices can impact inflation expectations, which in turn influence equity valuations. Hamilton (2009) find that fluctuations in oil prices significantly affect corporate earnings across various sectors, especially energy-intensive industries. High oil prices can increase production costs, reduce profit margins, and ultimately lower stock valuations, leading to a decrease in TR CAPE. Baur and McDermott (2010) find that rising gold prices often signal investor concerns about inflation or financial instability, which can lead to lower stock valuations and a decrease in TR CAPE as investors shift their portfolios toward safer assets. Erten and Ocampo (2013) find that changes in zinc prices can reflect broader economic conditions, particularly in industries such as construction and manufacturing. An increase in zinc prices can indicate higher demand in these sectors, potentially leading to higher earnings and stock prices, which could increase TR CAPE. Figure 5 shows the plots of a few key commodity price changes, to get a sense of the variabilities, while Table 4 shows the descriptive statistics.

**Figure 5: Monthly Changes in Commodity Prices Variables**

*This figure shows the plots (in different colors) of monthly changes in 4 key commodity price changes from February 2000 to December 2019, taken from different sources: Bloomberg, FRED, and WRDS.*

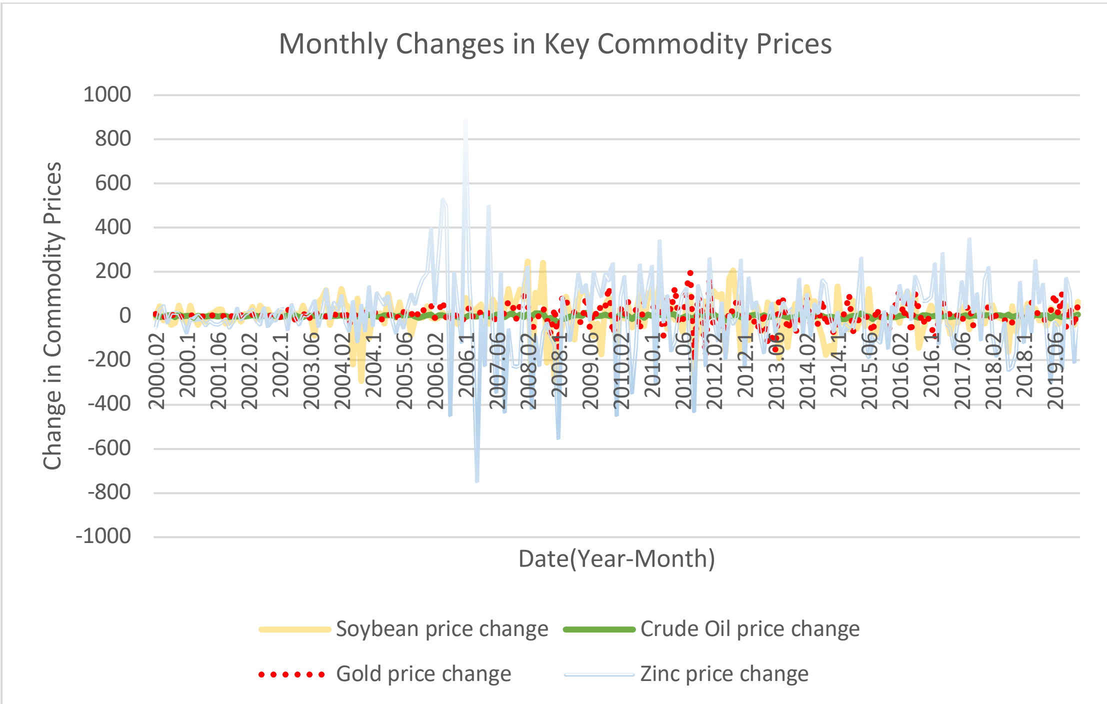



Figure 5 shows that the Soybean price changes show moderate variability. Soybean prices are influenced by factors such as weather conditions, crop yields, global demand, and trade policies. Crude oil price changes are more variable, with significant fluctuations throughout the period, especially during the mid-2000s and the 2008 financial crisis. Oil prices are highly sensitive to geopolitical events, changes in supply and demand dynamics, and OPEC's production decisions. Gold price change demonstrates relatively moderate variability, with notable spikes during times of economic uncertainty, such as the 2008 financial crisis. Gold is often considered a safe-haven asset, so its price tends to rise during periods of economic instability or when inflation expectations increase. Zinc price change is the most variable among the four commodities, with significant spikes and drops, particularly noticeable around 2006-2008. Zinc prices are closely tied to industrial demand, especially in construction and manufacturing. The high variability reflects the cyclicality of these industries and the impact of global economic conditions on demand for base metals like zinc, showing zinc's strong correlation with industrial demand.

**Table 4: Descriptive Statistics of Commodity Price Changes**

*This table shows the mean, median, standard deviation (SD) of monthly changes, as well as the number of months of observations (N), of all the stock market explanatory variables we collect. The sources of data are Bloomberg, WRDS and FRED databases.*

| | N | Mean | SD | Median |
|---|---|---|---|---|
| Sugar price change | 239 | 0.033 | 1.564 | 0.010 |
| Wheat price change | 239 | 1.266 | 51.627 | 1.250 |
| Coffee price change | 239 | 0.078 | 12.078 | -0.900 |
| Soybean price change | 239 | 1.820 | 81.130 | 5.250 |
| Tobacco price change | 239 | 3.207 | 8.599 | 1.800 |
| Crude Oil price change | 239 | 0.140 | 5.978 | 0.670 |
| Natural Gas price change | 239 | -0.002 | 0.903 | 0.015 |
| Gold price change | 239 | 5.162 | 52.085 | 4.550 |
| Zinc price change | 239 | 4.732 | 174.889 | 6.000 |
| Rice price change | 239 | 0.031 | 0.926 | 0.040 |

Table 4 presents the descriptive statistics for various commodity price changes over the observed period. The mean values for most commodities are positive, indicating that, on average, there were slight increases in these commodity prices. However, the standard deviations vary significantly across the different commodities, with some like zinc and soybean showing high variability, reflecting greater price fluctuations. Soybean, gold, and zinc price changes have notably high positive medians, suggesting a general upward trend in their prices. On the other hand, coffee and natural gas exhibit negative and near-zero median values, indicating a tendency for price stability or slight declines.

## III. RESULTS

### *Linear Regression using PCA*

We use a standard method of determining the principal components (PCs) that capture the bulk of the variability of the underlying. First, we compute the z-scores of monthly changes in all the explanatory variables, to remove dimensionality. When variables are grouped, PCA focuses on explaining the variance within each group, meaning the principal components reflect the dominant patterns within those specific groups. However, when all variables are taken together, the principal components prioritize the overall variance in the dataset, leading to shifts in the principal component structure and revealing relationships that may not be representative of each specific group (Kritzman, Li, Page, & Rigobon, 2011). In other words, variables that may not capture the variation of each group of explanatory variables can become significant contributors in explaining the overall variance (Adrian, Moench, & Shin, 2010). Therefore, following standard practice, we have determined the PCs by the groups – economy-wide, fixed income, stock market and commodity variables.

We then calculate the Explained Variance Ratio for each principal component. We examine the cumulative variance and select the number of principal components required to achieve 75% cumulative interpretation variance (see Jolliffe, 2002). For example, the cumulative variance from PC1 to PC6 is 0.780598 for economy-wide variables, which exceeds 75%, so we choose the first 6 principal components. The same approach applies to other groups.

**Table 5: Variance Explained by each Principal Component**

*This table shows the variation of the underlying variables explained by each PC (Principal component) and the cumulative variance explained, for economy-wide variables (Panel A), fixed-income variables (Panel B), stock market variables (Panel C), and commodity variables (Panel D). In bold are shown the PC, up to which explain at least 75% of the variability of the underlying variables.*

**Panel A:** Monthly changes in Economy-wide Variables PC Explained Variance

| PC | Explained Variance | Cumulative Variance |
|---|---|---|
| PC1 | 0.218533 | 0.218533 |
| PC2 | 0.152287 | 0.37082 |
| PC3 | 0.118664 | 0.489484 |
| PC4 | 0.108884 | 0.598368 |
| PC5 | 0.098247 | 0.696615 |
| **PC6** | **0.085739** | **0.782354** |
| PC7 | 0.075755 | 0.858109 |
| PC8 | 0.066458 | 0.924568 |
| PC9 | 0.05685 | 0.981418 |
| PC10 | 0.018582 | 1 |

**Panel B:** Monthly changes in Fixed Income Variables PC Explained Variance

| PC | Explained Variance | Cumulative Variance |
|---|---|---|
| PC1 | 0.512412 | 0.512412 |
| **PC2** | **0.240501** | **0.752913** |
| PC3 | 0.098469 | 0.851382 |
| PC4 | 0.073137 | 0.924519 |
| PC5 | 0.038981 | 0.9635 |
| PC6 | 0.018625 | 0.982125 |
| PC7 | 0.007523 | 0.989648 |
| PC8 | 0.006996 | 0.996644 |
| PC9 | 0.002348 | 0.998992 |
| PC10 | 0.001008 | 1 |

**Panel C:** Monthly Stock Market Returns PC Explained Variance

| PC | Explained Variance | Cumulative Variance |
|---|---|---|
| **PC1** | **0.818558** | **0.818558** |
| PC2 | 0.059178 | 0.877736 |
| PC3 | 0.035813 | 0.913549 |
| PC4 | 0.029085 | 0.942634 |
| PC5 | 0.021458 | 0.964092 |
| PC6 | 0.014734 | 0.978826 |
| PC7 | 0.009985 | 0.988812 |
| PC8 | 0.00504 | 0.993851 |
| PC9 | 0.004048 | 0.9979 |
| PC10 | 0.001934 | 0.999834 |
| PC11 | 0.000155 | 0.999989 |
| PC12 | 0.000011 | 1 |

**Panel D:** Monthly changes in Commodity Prices PC Explained Variance

| PC | Explained Variance | Cumulative Variance |
|---|---|---|
| PC1 | 0.265532 | 0.265532 |
| PC2 | 0.118514 | 0.384046 |
| PC3 | 0.114201 | 0.498247 |
| PC4 | 0.102295 | 0.600542 |
| PC5 | 0.092274 | 0.692816 |
| **PC6** | **0.078743** | **0.771559** |
| PC7 | 0.069366 | 0.840926 |
| PC8 | 0.064938 | 0.905863 |
| PC9 | 0.051243 | 0.957106 |
| PC10 | 0.042894 | 1 |

As Table 5 shows, 6 PCs are needed to explain at least 75% of variability in monthly changes in economy-wide variables and monthly changes in commodity prices, only 2 for monthly changes in fixed income variables (which arguably move together), only 1 for monthly stock market returns. Using the important PC's from Table 5 (that explain at least 75% of the variability), we regress monthly change in TR CAPE on these important PC's to find the significant ones.‡ We find monthly changes in economy-wide PC2 and PC3, monthly changes in Commodity Prices PC1, and PC6 are the only significant ones, as shown in Table 6 below.

To find the loadings of the original variables for each significant PC (from Table 6), the significant PC loading information is first extracted from PCA results. We then calculate the loading of each significant PC on the original variable in its group, and select variables with an absolute value greater than 0.5, indicating that these variables

‡ If we use only the first two PCs from each group, for example, we find fewer PC's were significant in explaining changes in TR CAPE, which leads to the result (not consistent with the other methods) that none of the original explanatory variables associated with these principal components were found to be significant for explaining changes in TR CAPE. Thus, using all PCs that capture the bulk of the variability in each group of explanatory variables is important.

have a significant contribution to the principal component (see Yang, Florescu, and Islam, 2020).[§]

**Table 6: Important PCs for changes in monthly TR CAPE**

*This table shows the regression coefficients and the associated t statistics (in parenthesis) of only the significant PCs (at the 1% level) from out of all the explanatory variables that are significantly associated with monthly changes in all explanatory variables.*

| Significant PCs | Monthly change in TR CAPE |
|---|---|
| Monthly changes in economy-wide variables PC2 | -0.1320 (-2.639) |
| Monthly changes in economy-wide variables PC3 | -0.2574 (-4.426) |
| Monthly changes in commodity prices PC1 | -0.1421 (-3.801) |
| Monthly changes in commodity prices PC6 | -0.2498 (-3.494) |

**Table 7: Loadings of Original Variables on PCs**

*The two panels of this table show the loading of the significant PCs on the original explanatory variables. In bold are the ones with "significant" loadings.*

**Panel A:** Loadings of Original Variables on economy-wide variables PC2 and PC3

| Original explanatory variable | PC2 | PC3 |
|---|---|---|
| Change in US GDP index | -0.427662 | 0.354378 |
| Change in US CPI index | 0.416444 | 0.041977 |
| Change in Core inflation | 0.321924 | -0.057046 |
| Change in Federal Funds Rate | -0.258503 | -0.279961 |
| Change in Unemployment rate | 0.363341 | 0.190894 |
| Change in Industrial production | -0.459959 | 0.287033 |
| Change in M2 money supply | 0.005516 | 0.302857 |
| Change in Personal income index | -0.061184 | -0.014348 |
| Change in Producer price index | 0.307141 | 0.0161 |
| **Change in Michigan sentiment index** | **-0.177574** | **-0.761581** |

§ Sticking to a 0.5 cutoff point, as highlighted in the literature by Yang, Florescu, and Islam (2020), is consistent with prior studies that advocate for selecting variables with significant loadings (i.e., above 0.5) to reduce model complexity while retaining essential variability. Using a standardized cutoff like 0.5 ensures comparability with other studies and reduces overfitting, as supported by Jolliffe (2002) in the context of PCA.

**Panel B:** Loadings of Original Variables on Commodity Prices PC1 and PC6

| Original explanatory variable | PC1 | PC6 |
|---|---|---|
| Sugar price change | -0.216772 | 0.185001 |
| Wheat price change | -0.426898 | 0.017523 |
| Coffee price change | -0.407641 | 0.382389 |
| Soybean price change | -0.435825 | -0.029871 |
| Tobacco price change | -0.037542 | 0.003803 |
| Crude Oil price change | -0.31942 | -0.153091 |
| Natural Gas price change | -0.165806 | 0.360197 |
| Gold price change | -0.320895 | 0.281196 |
| **Zinc price change** | **-0.290882** | **-0.71722** |
| Rice price change | -0.310061 | -0.26774 |

Finally, we regress monthly change in TR CAPE on these important original variables.

**Table 8: Significant Original Variables for Monthly Change in TR CAPE**

*This table shows the regression coefficients and the associated t statistics (in parenthesis) of significant original variables (from Table 7) on monthly Change in TR CAPE. In bold are the significant original variables (at 1% level).*

| Original explanatory variable | Monthly change in TR CAPE |
|---|---|
| Change in Michigan sentiment index | **0.3058**<br>**(5.770)** |
| Zinc price change | **0.2268**<br>**(4.279)** |

From Table 8, we find that both consumer sentiment and commodity prices can influence investor expectations about future economic conditions. We find that monthly change in Michigan sentiment index and Zinc price change are significantly associated with monthly changes in TR CAPE. As discussed earlier, a rise (drop) in the Michigan Sentiment Index can lead to higher (lower) stock prices and an increase (decrease) in the TR CAPE ratio. Changes in zinc prices can be indicative of global economic activity. A rise in zinc prices may suggest increased industrial demand, which could be associated with economic growth and higher corporate earnings, potentially driving up stock prices and the TR CAPE ratio (Fattouh, 2011).

As a check, we examine the determinants of annual change in TR CAPE next. We just report only the final results in Table 9 (that corresponds to the results in Table 8 for monthly changes in TR CAPE).

**Table 9: Significant Original Variables for Annual Changes in TR CAPE**

*This table shows the regression coefficients and the associated t statistics (in parenthesis) of significant original variables (on annual changes in TR CAPE. In bold are the significant original variables (at 1% level).*

| Original explanatory variable | Annual change in TR CAPE |
|---|---|
| Change in Michigan sentiment index | **2.1875** |
| | **(8.008)** |
| Zinc price change | **1.3037** |
| | **(4.960)** |
| Sugar price change | 0.0110 |
| | (0.046) |
| Change in M2 money supply | 0.4457 |
| | (1.732) |
| Change in Core inflation | **-0.8024** |
| | **(-3.077)** |

Consistent with the results for the monthly changes in TR CAPE, annual change in Michigan sentiment index, and Zinc prices are significantly associated annual change sin TR CAPE. The additional explanatory variable is change in core inflation. Higher core inflation can lead to increased interest rates, which can lower stock market valuations, and affect TR CAPE.

**Lasso Regression**

LASSO (Least Absolute Shrinkage and Selection Operator) performs variable selection and regularization by imposing L1 penalties on the regression coefficients. LASSO regression reduces some regression coefficients to zero, effectively selecting the variables that are most important to the response variable. We change the penalty function and only report the non-zero coefficient variables in different penalty function scenarios for comparative analysis with Lasso. Recent advancements have integrated non-convex penalties like smoothly clipped absolute deviation (SCAD) (Fan and Li, 2012), and minimax concave penalty (MCP) (Breheny and Huang, 2015) to enhance variable selection consistency and reduce bias. Zou (2006) introduces the adaptive Lasso method, which improves feature selection by applying different penalties to different coefficients, thereby addressing challenges posed by high-dimensional data. This method enhances the Lasso's ability to select significant features by adapting the penalty weights based on the importance of the features, often informed by prior information such as correlation or other criteria. Therefore, using adaptive Lasso, SCAD and MCP, we get the following results.

From Table 10, we find that monthly changes in Michigan sentiment index and the 5-year Treasury Yield are associated with monthly changes in TR CAPE, consistently across the 3 methods. We have already discussed the importance of consumer sentiment. Interest rates, particularly the 5-year Treasury yield, can be another critical factor in stock valuations. This rate is often used as a benchmark for discounting future earnings in stock valuation models. Asness (2000) discusses how changes in interest rates affect the equity risk premium. Campbell and Shiller (1988) also explored the relationship between bond yields and equity prices, noting that rising bond yields can make bonds more attractive relative to stocks.

**Table 10: Non-Zero Coefficients for monthly Change in TR CAPE**

*This table shows the non-zero coefficients using three different LASSO regression met hods – adaptive LASSO, SCA and MCP. In bold are the coefficients that are non-zero across all 3 methods.*

| | Adaptive Lasso | SCAD | MCP |
|---|---|---|---|
| Change in Core inflation | | -0.000756 | |
| Change in Michigan sentiment index | **0.0728** | **0.111027** | **0.076238** |
| Change in 5Y Treasury Yield | **0.140772** | **0.292441** | **0.266117** |
| Change in HQMCB10YR | | -0.208182 | -0.20448 |
| Change in Moody's BAA rate | -0.069698 | | |
| Dow Jones return | | 0.386654 | 0.386594 |
| Russell 5000 return | 0.357357 | | |
| Nikkei 225 (Japan) | 0.093038 | 0.064351 | |
| TSX Composite Index (Canada) | 0.015752 | | |
| Tobacco price change | | -0.036195 | |
| Change in TED spread | -0.009251 | | |
| FTSE 100 (UK) | 0.014113 | | |

We check results with annual changes in TR CAPE. From Table 11, we find that, in addition to the explanatory variables found important for monthly changes in TR CAPE – change in Michigan sentiment index and the 5-year Treasury Yield - annual change in US GDP index, change in Core inflation and M2 money supply are also significantly associated with annual change in TR CAPE, consistently across the 3 methods. Barro (1990) demonstrated that the growth rate in GDP is a significant determinant of stock market valuations via increased corporate earnings. Christiano, Motto, and Rostagno (2003) explored the relationship between monetary aggregates like M2 and stock market valuations via liquidity.

## Ridge and ElasticNet Regressions

The optimization of Ridge regression using techniques such as matrix differential calculus, as initially explored by Hoerl and Kennard (1970), highlighting the potential of this method to improve model performance, particularly in addressing multicollinearity issues in regression analysis.In Ridge regression, to determine which variable is the primary determinant, we can look at the absolute value of the coefficient. Gelman and Hill (2007) suggest that coefficients with absolute values greater than 0.1 (when predictors are standardized) may be considered large enough to be practically significant. We can use a combination of statistical significance (p-values) and practical significance (magnitude of standardized coefficients) to define a threshold for "large enough" coefficients.

**Table 11: Non-Zero Coefficients for annual changes in TR CAPE.**

*This table shows the non-zero coefficients using three different LASSO regression met hods – adaptive LASSO, SCA and MCP. In bold are the coefficients that are non-zero across all 3 methods.*

| Variables | **Adaptive Lasso** | **SCAD** | **MCP** |
|---|---|---|---|
| **Change in US GDP index** | **0.062228** | **0.859216** | **0.886479** |
| Change in US CPI index | -0.062286 | | |
| **Change in Core inflation** | **-0.50784** | **-0.603869** | **-0.597813** |
| **Change in M2 money supply** | **0.022026** | **0.292416** | **0.309301** |
| **Change in Michigan sentiment index** | **0.286443** | **0.773844** | **0.910871** |
| **Change in 5Y Treasury Yield** | **0.570288** | **0.758615** | **1.547698** |
| Change in 10Y Treasury Yield | 0.212297 | 0.613633 | |
| Change in Moody's BAA rate | -0.521703 | -0.551752 | |
| Dow Jones return | 2.081543 | | |
| Nikkei 225 (Japan) | 0.109781 | | |
| Hang Seng Index (Hong Kong) | 0.161715 | 0.163172 | |
| Coffee price change | 0.032653 | 0.054182 | |
| Gold price change | -0.080118 | | |
| EWCRSP index return | | 0.073658 | |
| DAX (Germany) | | 0.605722 | 1.147329 |
| Change in Unemployment rate | | -0.197706 | |
| Change in Producer price index | | 0.256226 | 0.364755 |
| Change in 1M Treasury Yield | | -0.306875 | -0.452185 |
| Change in 30Y Treasury Yield | | 0.18989 | |
| Change in HQMCB10YR | | -0.463175 | -0.739327 |

ElasticNet is chosen for its ability to combine the advantages of LASSO and Ridge regression, providing a robust approach to handling high-dimensional data and multicollinearity. It offers a balanced solution by performing variable selection and ensuring model stability, leading to improved predictive performance. The integration of both L1 and L2 penalties allows ElasticNet to address the limitations of each method individually, making it a preferred choice certain financial data analyses. ElasticNet can perform automatic variable selection, setting some coefficients to zero and thus simplifying the model (Zou and Hastie, 2005). The L2 penalty in Ridge regression helps ensure model stability and reduces sensitivity to small changes in the data, effectively addressing the limitations of purely L1 penalization in LASSO, particularly in situations involving multicollinearity (Hoerl and Kennard, 1970).

From Table 12, we find that monthly change in Michigan sentiment index, and 5Y Treasury Yield (consistent with Lasso regression methods) and the change in HQMCB10YR are associate with monthly change in TR CAPE, across the 2 methods. HQMCB10YR, which represents the yield on high-quality corporate bonds, impacts corporate financing costs and the relative attractiveness of stocks versus bonds. When the yield on these bonds increases, it can make bonds more attractive relative to equities, potentially leading to lower stock prices (see, for example, Adrian, Moench, and Shin (2010)).

**Table 12: Significant Coefficients for monthly change in TR CAPE**

*This table shows the coefficients using 2 different regression methods –Ridge and ElasticNet. In bold are the coefficients that are non-zero across both methods.*

| Variables | RIDGE | ElasticNet |
|---|---|---|
| Change in Michigan sentiment index | **0.128087** | **0.115114** |
| Change in 5Y Treasury Yield | **0.152981** | **0.120158** |
| Change in 10Y Treasury Yield | 0.146129 | |
| Change in 30Y Treasury Yield | 0.122688 | |
| Change in HQMCB10YR | **-0.191198** | **-0.102890** |
| Change in Moody's AAA rate | -0.138462 | |
| Dow Jones return | 0.164454 | |
| DAX (Germany) | 0.169977 | |
| CAC 40 (France) | -0.203325 | |

We check results with annual changes in TR CAPE.

**Table 13: Significant Coefficients for annual change in TR CAPE**

*This table shows the coefficients using 2 different regression methods –Ridge and ElasticNet. In bold are the coefficients that are non-zero across both methods.*

| Variables | RIDGE | ElasticNet |
|---|---|---|
| Change in US GDP index | 0.176222 | |
| Change in US CPI index | -0.154916 | |
| **Change in Core inflation** | **-0.306288** | **-0.531418** |
| Change in Unemployment rate | -0.114378 | |
| Change in Industrial production | 0.107554 | |
| Change in M2 money supply | 0.134871 | |
| **Change in Michigan sentiment index** | **0.320523** | **0.298494** |
| **Change in 5Y Treasury Yield** | **0.228477** | **0.628361** |
| Change in 10Y Treasury Yield | 0.220049 | |
| Change in 30Y Treasury Yield | 0.164942 | |
| Change in HQMCB10YR | -0.116616 | |
| **Change in Moody's BAA rate** | **-0.278571** | **-0.498345** |
| **Dow Jones return** | **0.386095** | **2.089984** |
| S&P 500 return | 0.278705 | |
| Russell 5000 return | 0.272973 | |
| VWCRSP index return | 0.280822 | |
| EWCRSP index return | 0.22005 | |
| DAX (Germany) | 0.157595 | |
| **Nikkei 225 (Japan)** | **0.207744** | **0.127994** |
| **Hang Seng Index (Hong Kong)** | **0.214298** | **0.176161** |
| Sugar price change | -0.101073 | |
| Coffee price change | 0.132685 | |

From Table 13, we find that, in addition to the explanatory variables found important for monthly changes in TR CAPE – change in Michigan sentiment index and the 5-year Treasury Yield - we find that annual change in Core inflation, Moody's BAA rate, Dow Jones return, Nikkei 225 (Japan) and Hang Seng Index (Hong Kong) are all also associated with annual change in TR CAPE, across the 2 methods.

## IV. Conclusion

Recent advances in data science and machine learning have further enhanced the use of regression techniques in financial studies. Gu, Kelly, and Xiu (2020), for example, have explored empirical asset pricing using machine learning, demonstrating significant improvements in predictive accuracy and robustness. These advancements illustrate the ongoing evolution of regression techniques, driven by the need to handle increasingly complex financial datasets and improve the reliability of financial models. We do the same in this paper. We analyze the determinants of changes in S&P Composite Index Total Return Cyclically Adjusted Price-to-Earnings ratio (TR CAPE), to better understand changing "investor exuberance". We use three different methods - linear regression using PCA, Lasso, and Ridge regression techniques – and a large number of explanatory variables, to compare and contract significant determinants.

The results differ with the method used. But, across all the methods we use, monthly changes in Michigan sentiment index is significantly associated with monthly changes in TR CAPE. Cross-checking these results using annual changes in TR CAPE and annual changes in explanatory variables, across all methods, we find that annual changes in Michigan sentiment index and in core inflation are significantly associated with annual changes in TR CAPE.

Overall, changes in the Michigan Sentiment Index appears to have significant association with changes in investor exuberance. This is a measure of consumer sentiment, when high, typically reflects optimism about future economic growth, leading to increased consumer spending and higher corporate earnings expectations. This positive outlook can boost investor confidence, driving up stock prices and, consequently, increasing the TR CAPE ratio as markets anticipate stronger future earnings. In contrast, the financial crisis of 2008, for example, led to a sharp decline in the Michigan Sentiment Index as consumer confidence plummeted due to fears of a prolonged recession.

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# Individual Stock Returns Volatility and Equity Anomalies

Sunghan Bae and Keshav R. Bhattarai*


## Abstract

We examine the explanatory power of stock return volatility on well-documented equity anomalies. We estimate the time-varying volatility of stock returns using the EGARCH model. Using the estimated stock return volatility, we first form a volatility factor and incorporate it into a wide range of asset pricing models. Second, we adjust the loadings of risk factors in these asset pricing models. We then assess the impact of these two volatility-associated model modifications in mitigating equity anomalies. Our findings indicate that the volatility modifications enhance the explanatory power of asset pricing models. Notably, momentum effects are reduced when the volatility factor is applied alone. Size, value, and short-term reversal anomalies are mitigated when factor loadings are adjusted for volatility where the impacts are more pronounced when both volatility adjustments are applied. Regarding liquidity anomaly, we observe positive effects when both volatility adjustments are incorporated into more sophisticated models. Our results remain robust across two sub-sample periods—covering the latest 20 years and periods of economic recession.


**Keywords:** Stock Returns Volatility, Equity Anomalies, Momentum Effects, EGARCH Model

**JEL Classifications:** G12, G14

## I. Introduction

The capital asset pricing model (CAPM) developed by Sharpe (1964), Lintner (1965), and Black (1972) has long been fundamental in asset pricing literature. However, subsequent research has challenged the notion that average stock returns can be fully explained by the market risk of CAPM (β) alone. As a result, numerous studies have explored additional risk factors and non-risk firm-specific factors to enhance the explanatory power of CAPM.[1] Fama and French, in their studies from 1992, 1993, and 1996, argue that firm-specific variables essentially scale stock prices and that well-specified risk factors should account for their effects. They introduced two market-wide risk factors, size (ME) and book-to-market equity (BE/ME), resulting in the SMB (small minus big) and HML (high minus low) factors, respectively. Fama and French claimed that their three-factor model, which includes market beta, captures most equity market anomalies overlooked by CAPM, except for the momentum effects identified by Jegadeesh et al. (1990, 1993).[2] However,

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[1] Such as size (Banz, 1981), leverage (Bhandari, 1988), book value of common equity (Rosenberg et al., 1985), book to market equity ratio (Chan et al., 1991), earnings-price ratios (Basu, 1977; Ball, 1978).

[2] Behavior explanations have been more devoted to explaining the momentum effects. See, for example, Barberis et al. (1998, conservatism bias), Daniel et al. (1998, self-attribution biased overconfidence), Grinblatt et al. (2005,

subsequent research has reported persistent market anomalies even after incorporating additional or alternative risk factors.

This paper is part of ongoing efforts to better explain stock return movements and mitigate existing equity market anomalies. Instead of searching for indirect variables linked to stock return movements, we focus directly on the individual stock return. Specifically, we examine the standard deviation as a direct risk measure of individual stock returns and investigate how its predictive power can mitigate well-documented equity market anomalies. Under the efficient market hypothesis, all publicly available information is reflected in stock prices, meaning the degree of stock return fluctuation should incorporate all firm-specific information and investor behavior biases, which are critical sources contributing to equity market anomalies.

Considerable research has explored why stock return volatility matters. Levy (1978) argued that stock return volatility could be a key factor within the CAPM framework, while Merton (1987) demonstrated that under imperfect information, volatility affects expected returns due to idiosyncratic risk. Campbell et al. (1993) and Campbell (1996) showed within the Intertemporal Capital Asset Pricing Model (ICAPM) framework that stock return volatility is significant because risk-averse investors hedge against changes in volatility, as it positively affects future expected stock returns. Ang et al. (2006) examined the pricing of aggregate volatility risk in the cross-section of stock returns and concluded that securities with high sensitivities to volatility risk hedge against substantial market declines. More recent studies by Cremers et al. (2015), Jordan et al. (2015), Daniel and Moskowitz (2016), Moreira et al. (2017), and Baltussen et al. (2018) further supported volatility risk as a priced factor and explored its implications on stock return movements. Additionally, several studies have examined the direct impact of individual stock return volatility on equity anomalies. For instance, Campbell et al. (2018) show that incorporating stochastic volatility into the ICAPM helps to explain some equity anomalies.[3]

We investigate the explanatory power of the standard deviation of stock returns on the cross-sectional differences in average stock returns. To capture the time-varying nature of stock return volatility, we use the exponential generalized autoregressive conditional heteroskedasticity (EGARCH) model, specifically EGARCH (1,1) with ARCH (1,0) as the mean equation. The estimated volatility is then applied to a range of asset pricing models in two ways. First, a volatility factor is developed using the Fama and French procedure of sorting individual stocks into 2x3 portfolios based on size and volatility, and it is added to tested asset pricing models: CAPM, the Fama and French three-factor model, the Fama and French three-factor plus liquidity and momentum factors, and the Fama and French five-factor model. Second, we condition and allow the factor loadings of asset pricing models to vary according to individual stock return volatility, thus individualizing market-wide risk factors to better incorporate stock-specific return variations, which helps alleviate equity anomalies. We then examine whether the volatility factor and the conditional beta loadings can enhance the explanatory power in addressing well-documented equity anomalies, specifically size, value, liquidity, and past return-initiated anomalies (momentum and short-term reversal).

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disposition effect), and Hirshleifer et al. (2009, investors' limited ability to attention).

[3] Campbell et al. (2018) estimated individual stock return volatility using a Vector Auto Regression (VAR) model, which is designed to capture low-frequency movements in volatility. They then integrated this stochastic volatility into the ICAPM framework. Their findings indicate that the three-beta ICAPM, incorporating stochastic volatility, outperforms the traditional two-beta ICAPM in explaining equity anomalies, such as growth versus value stocks and the low returns associated with certain beta exposures.

We focus on individual stocks rather than portfolios and options because this approach allows market-based risk factors to vary with individual stock-level features. Ang et al. (2020) showed that using portfolios to mitigate estimation errors in risk factor loadings does not necessarily result in smaller standard errors of cross-sectional coefficient estimates, as grouping stocks can destroy information contained in individual stock factor loadings.[4] Studying individual stocks also addresses the information loss inherent in portfolio-based asset pricing tests (Litzenberger and Ramaswamy, 1979) and accounts for data-snooping biases (Lo and MacKinlay, 1990). Additionally, using individual stock returns rather than options for volatility estimation enables the study to investigate sensitivities to market-wide volatility factors and introduce various risk factors within the same model.

We observe offsetting relationships between individual stock return volatility and all tested anomaly constituents, suggesting that volatility may play a role in mitigating equity anomalies. Examining the impact of the two volatility-related modifications, we find that when the volatility factor is applied alone, it effectively reduces momentum effects and improves the size anomaly. However, it also exacerbates liquidity and short-term reversal anomalies, while the results for the value anomaly remain inconsistent. Beta conditioning enhances the models' explanatory power for size, value, and short-term reversal anomalies, but does little to address liquidity and momentum anomalies. When both volatility adjustments are applied together, the models show the most substantial improvements in mitigating size, value, and short-term reversal anomalies. However, in most cases, momentum effects worsen. For the liquidity anomaly, we observe positive effects when both volatility adjustments are incorporated into more sophisticated models—such as the Fama and French three-factor model with added liquidity and momentum factors, as well as the five-factor model.

We contribute to the asset pricing literature by exploring methods to mitigate equity anomalies, building on prior research that has expanded the foundational CAPM model. Since the introduction of the CAPM, numerous studies have shown that incorporating additional risk factors enhances its explanatory power (e.g., Fama and French, 1992, 1993, 1996; Jegadeesh et al., 1990, 1993; Pastor and Stambaugh, 2003). Several studies have also examined how stock return volatility can help address equity anomalies (Levy, 1978; Merton, 1987; Campbell et al., 1993, 1996; Ang et al., 2006; Cremers et al., 2015; Jordan et al., 2015; Daniel and Moskowitz, 2016; Baltussen et al., 2018; Campbell et al., 2018). In line with this body of research, we apply two volatility-related modifications to various asset pricing models and find that these adjustments effectively mitigate equity anomalies, depending on the application. Specifically, adding a volatility factor is most effective in addressing the momentum anomaly, while beta conditioning proves useful in mitigating size, value, and short-term reversal anomalies. Liquidity anomaly is effectively mitigated when both volatility adjustments are applied to more sophisticated models. Moreover, we show that these adjustments in volatility are effective across different market segments and under varying economic conditions, which highlights the importance of including volatility adjustments when analyzing stock returns.

The rest of this paper is organized as follows. Section II describes the methodology, Section III details the sampling process and provides summary statistics, Section IV analyzes the data and reports empirical findings, Section V provides robustness tests, and Section VI concludes.

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[4] See Jegadeesh et al. (2019) for further arguments on the shortcomings of using portfolios to address the errors-in-variables (EIV) problem.

## II. Methodology

The asset pricing tests in this study are based on the monthly absolute alphas from various equity anomaly-initiated investment strategies. Zero-investment portfolios are constructed by longing the top quintile of stocks and shorting the bottom quintile within a given month. If the asset pricing models adjusted with stock return volatility measures are effective, we should observe a decrease in these absolute alphas. To estimate the key variable, individual stock return volatility, we use the EGARCH model, which accounts for the time-varying and asymmetric properties of stock return volatility.[5] This volatility characteristic of individual stock returns is applied to the asset pricing models in two ways: first, by developing a volatility factor using the Fama and French procedure, and second, by allowing the beta loadings of risk factors in each asset pricing model to vary with individual stock return volatility.

We then examine whether the volatility factor and the conditional beta loadings can enhance the explanatory power in addressing well-documented equity anomalies. In addition, previous literature suggests that returns of NASDAQ stocks differ from returns of NYSE-AMEX stocks, we conduct our analyses separately for a sample that only contains NASDAQ stocks (Avramov and Chordia, 2006).

### Time series monthly absolute alphas and beta loadings

To compute time series monthly absolute alphas, we run time-series regressions of individual stocks' excess returns on various asset pricing models, recording alphas (realized excess returns minus model-predicted excess returns) for each stock. Each month, we sort individual stocks by equity anomaly constituents—size, value, liquidity, momentum, and short-term reversals—into five portfolios with equal numbers of stocks. The top portfolio contains the 20% of stocks with the highest anomaly attributes, and the bottom portfolio contains the 20% with the lowest. We then calculate time series average absolute alphas by constructing zero-investment portfolios that long the top portfolio and short the bottom portfolio.

Beta loadings of each risk factor are estimated on a rolling basis over 60 months. Specifically, from July 1963 to June 2018, we use the previous 60 months of stock return data to estimate beta loadings for each risk factor, following the methodology of Fama and French (1992). For stocks with less than 60 months of prior data, we require at least 24 return observations, starting from the 25th month. For instance, for a stock with return data starting in July 1963, the rolling estimation process begins in July 1965 with 24 return observations. In August 1965, factor loadings are estimated using 25 observations, and this continues until the rolling window reaches 60 months, after which the duration remains constant at 60 months.

### Volatility estimation: EGARCH model

Due to the time-varying properties of stock return variance—such as clustering, mean-reversion, and leptokurtic distribution—we estimate stock return variance using the EGARCH model rather than relying solely on realized past return variance.

After Engle (1982) introduced the autoregressive conditional heteroskedasticity (ARCH) model to capture time-varying volatility, Bollerslev (1986) extended it to the generalized ARCH

[5] We calculate individual stock return volatility using the previous 60 months of monthly returns.

(GARCH) model, which incorporates a more dynamic structure of conditional variances, providing a more flexible framework. Nelson (1991) further advanced this by developing the Exponential GARCH (EGARCH) model, designed to capture the asymmetric effects of shocks (e.g., policy changes, newly announced information) in the stock market by differentiating the impacts of good and bad news. EGARCH model posits that good news tends to stabilize stock returns and reduce volatility, while bad news tends to destabilize returns and increase volatility. Pagan and Schwert (1990) found Nelson's EGARCH model to be the best fit for estimating monthly U.S. stock return volatility, which is further supported by the evidence of Engle and Ng (1991). We employ a straightforward EGARCH framework: EGARCH (1,1) with ARCH (1,0) as the mean equation. The functional forms are as follows:

$$R_{i,t} = \alpha_1 R_{i,t-1} + \mu_{i,t}; \mu_{i,t} \sim N(0, \sigma_{i,t}^2) \tag{1}$$

$$\ln \sigma_{i,t}^2 = \alpha_i + \sum_{i=1}^{q} \omega_{i,t} \left| \frac{\mu_{i,t-1}}{\sqrt{\sigma_{i,t-1}^2}} \right| + \sum_{i=1}^{q} \gamma_{i,t} \left| \frac{\mu_{i,t-1}}{\sqrt{\sigma_{i,t-1}^2}} \right| + \sum_{k=1}^{p} \theta_k \ln \sigma_{i,t-k}^2 \tag{2}$$

where $\omega_{i,t}$ represents the ARCH effects, $\gamma_{i,t}$ captures the asymmetric effects, $\theta_k$ denotes the GARCH effects, and the logarithm of variance, $\ln \sigma_{it}^2$, makes the leverage effect exponential rather than the quadratic as in the standard GARCH model.

**Volatility factor**

To construct a volatility factor, we follow the Fama and French procedure. First, we sort all individual stocks for a given month into two size groups and three volatility groups, with breakpoints calculated using NYSE stocks. The volatility breakpoints are set at the 30th and 70th percentiles of the individual stocks' one-month lagged standard deviation. This 2x3 sorting on size and volatility results in six portfolios. The volatility factor is then computed as the equal-weighted average of the returns of the two high-volatility portfolios minus the equal-weighted average of the returns of the two low-volatility portfolios.

$$\begin{aligned} &Volatility\ factor \\ &\quad = \frac{1}{2}(small\ size\ high\ volatility + big\ size\ high\ volatility) \\ &\quad - \frac{1}{2}(small\ size\ low\ volatility + big\ size\ high\ volatility) \end{aligned} \tag{3}$$

**Conditional beta framework: varying individual stock level beta loadings**

Conditioning the beta loadings of risk factors in asset pricing models involves tailoring market-wide risk factors to better reflect individual stock-level characteristics. This approach allows asset pricing models to more effectively incorporate fluctuations in stock returns, potentially reducing deviations between realized and model-predicted excess returns. Equity anomalies—systematic deviations documented in stock returns that correlate with certain firm characteristics or past returns—are among these discrepancies. By adjusting risk factor loadings to account for key stock-level features, modified asset pricing models can improve their ability to explain these anomalies.

We follow the beta-varying methodology outlined by Avramov and Chordia (2006), allowing the factor loadings in asset pricing models to vary based on individual stocks' one-month lagged standard deviation.[6] The beta scaling framework is illustrated as follows:

$$r_{j,t} = \alpha_j + \sum_{i=1}^{N} \beta_{j,(2i-1)} F_{k,t} + \sum_{i=1}^{N} \beta_{j,(2i-1)} F_{k,t} Sd_{j,(t-1)} + \mu_{j,t} \tag{4}$$

Where $r_{j,t} = R_{i,t} - R_{F,t}$ the excess return of the individual stocks, $F_{k,t}$ denotes various risk factors, and $Sd_{i,(t-1)}$ is the one-month lagged standard deviation of individual stocks. For example, applying this conditional beta modeling to the Fama-French three-factor model, the time-varying factor loadings can be specified as follows:

$$\begin{aligned} r_{j,t} = \alpha_j + \left(\beta_{j1} + \beta_{j2} Sd_{j,(t-1)}\right) \times r_{m,t} + \left(\beta_{j3} + \beta_{j4} Sd_{j,(t-1)}\right) \times SMB_t \\ + \left(\beta_{j5} + \beta_{j6} Sd_{k,(t-1)}\right) \times HML_t \end{aligned} \tag{5}$$

Alternatively, this can be expressed as:

$$\begin{aligned} r_{j,t} = \alpha_j + \beta_{j1} r_{m,t} + \beta_{j2} r_{m,t} \times Sd_{j,(t-1)} + \beta_{j3} SMB_t + \beta_{j4} SMB_t \times Sd_{j,(t-1)} \\ + \beta_{j5} HML_t + \beta_{j6} HML_t \times Sd_{j,(t-1)} \end{aligned} \tag{6}$$

where $r_{m,t}$ is the excess return on the value-weighted market index, SMB and HML are size and value factors in the Fama-French three factor model, respectively.

## III. Data

We collect monthly stock returns for NYSE, AMEX, and NASDAQ stocks from the Center for Research in Security Prices (CRSP) and firm-specific variables—such as size (market capitalization), book-to-market ratio (book equity divided by market capitalization), and turnover (trading volume divided by total shares outstanding)—from COMPUSTAT. Our sample period spans from July 1963 to June 2019, totaling 672 months.[7] To be included in the sample for a given

[6] Avramov and Chordia (2006) allow beta loadings to change as a function of firm-specific variables, such as size and book-to-market ratio, thus extending the framework of Brennan et al. (1998), where rolling regressions enable beta loadings to evolve over time. Avramov and Chordia's (2006) approach also differ from that of Shanken (1990), Ferson and Harvey (1999), and Lettau and Ludvigson (2001) in that the economic quantity obtained by multiplying firm-specific variables by factors does not constitute an additional risk factor.

[7] Trading volume data of NASDAQ stocks, a variable in computing firm turnover rate, is available after November 1982. Thus, our sample for NASDAQ stocks is from July 1983. We exclude COVID 19 period to mitigate the

month, stocks must meet the following criteria. Stock returns data must be available for at least 100 consecutive months to ensure stable convergence in the EGARCH estimation of individual stock returns volatility. Firm-specific data necessary to calculate key variables, such as size and turnover ratio from CRSP and the book-to-market ratio as of December of the previous year from COMPUSTAT, must be available. After applying these criteria, our sample consists of 7,939 different firms (including 4,556 NASDAQ firms) over the 672-month period, with an average of 2,666 firms (1,883 NASDAQ firms) included per month.

Given the substantial negative skew in our key variables, we use logarithmic transformations for firm-specific variables such as size, book-to-market ratio, and turnover ratio, while stock returns data—including lagged returns for momentum and short-term reversal effects—remain untransformed. Following the methodology of Fama and French (1992), we calculate book equity for July of year t−1 to June of year t using accounting data from the end of year t−2. Following Brennan et al. (1998) and Avramov and Chordia (2006), firm characteristics and stock return volatility are lagged one month with respect to excess returns. Risk factors—including market portfolio return, SMB, HML, liquidity, momentum, profitability, and investment factors—are obtained from Kenneth R. French's data library.[8] To mitigate the impact of extreme observations, stock return standard deviation estimates, and stock return measures (such as excess returns and past returns for momentum and short-term reversals) are winsorized at the 0.995 and 0.005 percentiles.

**Table 1: Summary Statistics**

| | **Mean** | **Median** | **Standard Deviation** | **Skewness** |
|---|---|---|---|---|
| Panel A: NYSE, AMEX, and NADAQ stocks | | | | |
| Excess return (%) | 0.14 | -0.27 | 10.07 | 0.03 |
| Ln(Firm size) ($ million) | 2.25 | 2.18 | 0.96 | 0.32 |
| Ln(Book-to-market ratio) | -0.23 | -0.20 | 0.39 | -0.73 |
| Ln(Turnover) (%) | 1.66 | 1.67 | 0.57 | -0.25 |
| Ret2_12 (%) | 7.44 | 4.82 | 36.99 | 0.18 |
| Ret1 (%) | 0.47 | 0.00 | 10.05 | 0.02 |
| Standard Deviation (of returns) | 12.41 | 11.38 | 5.27 | 0.49 |
| Panel B: NASDAQ stocks | | | | |
| Excess return (%) | 0.08 | -0.31 | 10.64 | 0.02 |

impact of unusual observations.

[8] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/index.html

| | | | | |
|---|---|---|---|---|
| Ln(Firm size) ($ million) | 2.05 | 2.00 | 0.83 | 0.35 |
| Ln(Book-to-market ratio) | -0.27 | -0.23 | 0.41 | -0.73 |
| Ln(Turnover) (%) | 1.73 | 1.77 | 0.60 | -0.45 |
| Ret2_12 (%) | 6.64 | 3.75 | 39.51 | 0.16 |
| Ret1 (%) | 0.35 | 0.00 | 10.63 | 0.02 |
| Standard Deviation (of returns) | 13.67 | 13.05 | 5.68 | 0.16 |

This table presents the descriptive statistics of stocks listed on the NYSE, AMEX, and NASDAQ. Panel A includes all stocks in the sample, averaging 2,666 firms per month from July 1963 to June 2019, while Panel B focuses on NASDAQ stocks, averaging 1,883 firms per month from July 1982 to June 2019. Excess return is defined as the monthly raw stock return minus the one-month T-bill rate. Firm size is measured by the market value of equity in millions of dollars. The book-to-market ratio is calculated as the fiscal year-end book value of common equity divided by market equity. Turnover is defined as the monthly share trading volume divided by shares outstanding. Due to the substantial negative skewness of these firm-specific variables, all firm characteristics are presented as natural logarithms. Ret2_12 represents cumulative returns over the second through twelfth months, while Ret1 denotes the previous month's stock return, capturing short-term reversals. Standard deviations of individual stock returns are estimated using the EGARCH model. To mitigate the influence of extreme observations, standard deviation estimates, excess return, Ret2_12, and Ret1 are winsorized at the 0.005 and 0.995 percentiles.

Table 1 provides key descriptive statistics for the entire sample. It highlights differences in firm-specific information and stock return characteristics between NASDAQ stocks and those listed on NYSE and AMEX, supporting the need for separate analyses. Additionally, focusing on individual stocks rather than portfolios reveals that return measures, including standard deviation estimates, exhibit a broader range around the average.

## IV. Empirical analysis

This section evaluates the effectiveness of existing asset pricing models by integrating individual stock return volatility, measured as standard deviation, into the analysis. The goal is to assess whether incorporating this direct risk measure helps mitigate well-known equity anomalies. The asset pricing models we examine include: (i) the Capital Asset Pricing Model (CAPM), (ii) the Fama and French three-factor model (1992), (iii) the Fama and French three-factor model augmented with the liquidity factor of Pastor and Stambaugh (2003), (iv) the Fama and French three-factor model augmented with the momentum factor of Jegadeesh and Titman (1993), (v) the Fama and French three-factor model incorporating both liquidity and momentum factors, and (vi) the Fama and French five-factor model (2015). The equity anomalies considered are size (market capitalization), value (book-to-market ratio), liquidity (turnover ratio), momentum (past two-to-twelve-month returns), and short-term reversals (past one-month returns).

Our approach involves estimating individual stock return volatility using the EGARCH model, with these volatility measures incorporated into the asset pricing models either as an additional factor or by conditioning factor loadings on volatility. We then compare the absolute alphas from zero-investment strategies—going long on the top quintile and short on the bottom quintile of stocks sorted by equity anomaly characteristics—against those from the baseline models. A reduction in absolute alphas would indicate that the volatility-related modifications improve the models' ability to explain the returns associated with equity anomalies.

**Table 2: Portfolios Sorted by Volatility**

| Rank | Size | Value | Liquidity | Momentum | Short-term Reversal |
|---|---|---|---|---|---|
| Panel A: NYSE, AMEX, and NADAQ stocks | | | | | |
| 1 Low | 2.73 | -0.20 | 1.34 | 11.01 | 0.57 |
| 2 | 2.65 | -0.22 | 1.48 | 12.99 | 0.78 |
| 3 | 2.44 | -0.20 | 1.56 | 13.05 | 0.92 |
| 4 | 2.17 | -0.19 | 1.63 | 13.58 | 1.05 |
| 5 High | 1.71 | -0.21 | 1.70 | 13.87 | 1.66 |
| Panel B: NASDAQ stocks | | | | | |
| 1 Low | 2.28 | -0.17 | 1.40 | 13.57 | 0.76 |
| 2 | 2.30 | -0.24 | 1.67 | 15.71 | 0.89 |
| 3 | 2.14 | -0.28 | 1.81 | 15.60 | 0.98 |
| 4 | 1.96 | -0.32 | 1.87 | 14.31 | 1.13 |
| 5 High | 1.67 | -0.36 | 1.89 | 14.63 | 2.43 |

This table presents equal-weighted quintile portfolios for each equity anomaly constituent, sorted by individual stock return volatility (standard deviation). Panel A includes all stocks in the sample from July 1963 to June 2019, while Panel B focuses on NASDAQ stocks from July 1982 to June 2019. Individual stock return volatility is estimated using the EGARCH model. Portfolios are rebalanced monthly, and the statistics in each column represent the time-series averages over the entire sample period. Rank 1 corresponds to portfolios of stocks with the lowest volatilities, while Rank 5 corresponds to those with the highest volatilities. Size is measured by the log of market capitalization; Value represents the log of the book-to-market ratio; Liquidity is defined as the monthly turnover ratio; Momentum denotes cumulative returns over the last twelve months, excluding the previous month; and Short-term Reversal refers to the stock return from the past month.

Table 2 presents time-series averages of equal-weighted quintile portfolios sorted by individual stock return volatility. The results show that stock return volatility significantly reduces the magnitude of equity anomalies. In Panel A, Column 2 reveals a negative relationship between firm size and volatility, where higher volatility is associated with a smaller average firm size, implying that strategies targeting the size anomaly may generate positive alphas but come with higher risk. Similarly, Column 4 shows that higher momentum (past two-to-twelve-month returns) is linked to increased stock return volatility, indicating that momentum-based strategies, while potentially profitable, carry higher risk. This trade-off between alpha generation and risk is observed across all anomaly factors. Panel B, which focuses on NASDAQ stocks, supports these findings with only minor variations in the magnitude of the relationships. The results suggest incorporating individual stock return volatility into asset pricing models substantially mitigates equity anomalies. In the following subsections, we will explore how individual stock return volatility interacts with each asset pricing model in more detail.

**The capital asset pricing model (CAPM)**

Table 3 presents the average monthly absolute alphas from zero-investment strategies based on the CAPM for various equity anomalies. The first column reports results using the baseline CAPM, which assumes fixed betas without any volatility adjustments. The second column incorporates a volatility factor into the CAPM. In the third column, the CAPM allows betas to vary with the standard deviation of stock returns. The fourth column shows the results from a CAPM model that includes both a volatility factor and scaled factor loadings based on stock return volatility.

Column 1 of Table 3 reveals that the average monthly absolute alphas from the baseline CAPM are significantly different from zero, suggesting that all the tested equity anomalies can generate considerable excess returns when stock returns are priced solely on market beta. However, introducing a volatility factor in Column 2 leads to noticeable improvements in the model's ability to capture some of the anomalies. For instance, the absolute alpha for the size anomaly decreases from 0.7023 to 0.4348, for the value anomaly from 0.9809 to 0.8796, and for the momentum anomaly from 0.4928 to 0.4012. On the other hand, the volatility factor exacerbates certain anomalies, particularly liquidity and short-term reversal, with the absolute alphas increasing from 0.0178 to 0.1467 for liquidity and from 1.4959 to 1.6928 for short-term reversal.

Conditioning beta loadings, as shown in Column 3 of Table 3, helps reduce the absolute alphas for the size and value anomalies to 0.6742 and 0.8579, respectively, with a noticeable decrease in the short-term reversal anomaly to 1.4385. However, this approach worsens the momentum anomaly, increasing the absolute alpha to 0.6163, and the liquidity anomaly to 0.0527. The improvements in size and value anomalies are more pronounced when both volatility modifications are applied. Column 4 of Table 3 shows a further reduction in absolute alphas, with the size anomaly dropping to 0.3664 and the value anomaly decreasing to 0.6807. Nevertheless, the model still struggles with liquidity, momentum, and short-term reversal anomalies, as these absolute alphas remain elevated.

Panel B, which focuses on NASDAQ stocks, reveals similar patterns. While the baseline approach shows higher average monthly alphas across all anomaly strategies (except for momentum), the effects of the two volatility modifications remain consistent. Adding a volatility factor improves the model's explanatory power for size, value, and momentum anomalies but worsens its performance for liquidity and short-term reversal anomalies. Beta conditioning provides slight improvements for size, value, and short-term reversal anomalies, though it further exacerbates the liquidity and momentum anomalies. When both volatility modifications are applied, the model demonstrates more pronounced positive impacts on size and value anomalies, but continues to exhibit negative effects on liquidity, momentum, and short-term reversal anomalies.

**Table 3: Time Series Average Absolute Alphas: the CAPM**

| | Baseline | Volatility Factor | Conditional Betas | Volatility Factor & Conditional Betas |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| Panel A: NYSE-AMEX and NASDAQ Stocks | | | | |
| Size | 0.7023<br>[-21.14] | 0.4348<br>[-20.47] | 0.6742<br>[-22.18] | 0.3664<br>[-20.00] |
| Value | 0.9809<br>[31.69] | 0.8796<br>[44.86] | 0.8579<br>[30.27] | 0.6807<br>[39.39] |
| Liquidity | 0.0178<br>[0.57] | 0.1467<br>[-12.4] | 0.0527<br>[1.66] | 0.1216<br>[-10.81] |
| Momentum | 0.4928<br>[16.51] | 0.4012<br>[28.05] | 0.6163<br>[27.08] | 0.5816<br>[54.78] |
| Short-term Reversal | 1.4959<br>[-53.59] | 1.6928<br>[-66.02] | 1.4385<br>[-52.51] | 1.5661<br>[-73.72] |
| Panel B: NASDAQ stocks | | | | |
| Size | 1.3834<br>[-21.91] | 1.3567<br>[-25.15] | 1.2804<br>[-20.64] | 1.0490<br>[-21.76] |
| Value | 1.5706<br>[19.5] | 1.4362<br>[27.81] | 1.3543<br>[17.25] | 1.0759<br>[19.46] |
| Liquidity | 0.0451<br>[-0.51] | 0.1767<br>[-4.48] | 0.0746<br>[-0.85] | 0.1796<br>[-5.09] |
| Momentum | 0.0386<br>[0.37] | 0.0111<br>[0.15] | 0.3067<br>[3.25] | 0.3449<br>[5.89] |
| Short-term Reversal | 1.697<br>[-22.74] | 1.8932<br>[-32.13] | 1.6309<br>[-20.34] | 1.8067<br>[-35.75] |

This table reports the average monthly absolute alphas from well-documented equity anomaly strategies derived from the Capital Asset Pricing Model (CAPM). Panel A includes all stocks from July 1963 to June 2019, while Panel B focuses on NASDAQ stocks from July 1982 to June 2019. The second column ("Baseline") reports time-series absolute alphas using the CAPM; the third column adds a volatility factor; the fourth column allows the CAPM beta to vary with individual stock return standard deviation; and the final column augments the CAPM with a volatility factor while also allowing the beta to be conditional on volatility. The Newey-West (1987) corrected t-statistics are reported in the bracket.

## The Fama and French three factor model

Panel A of Table 4 presents the average monthly absolute alphas from zero-investment strategies based on the Fama and French three-factor model, assessing how well the model captures equity anomalies. The baseline model's performance improves with the addition of two volatility modifications, showing patterns and magnitudes similar to those seen with the CAPM, though with similar challenges.

As with the CAPM, adding a volatility factor helps mitigate size, value, and momentum anomalies, but worsens the model's ability to explain liquidity and short-term reversal anomalies. When the factor loadings for beta, SMB (small-minus-big), and HML (high-minus-low) are conditioned on individual stock return volatility, the modified model further reduces size, value, and short-term reversal anomalies, though it continues to perform worse for liquidity and momentum anomalies. A key difference is that beta conditioning has a larger impact on mitigating size and value anomalies in the Fama and French model compared to the CAPM, where the volatility factor appeared to be more effective. When both volatility modifications are applied, the improvements are amplified, with roughly half of the abnormal returns from size and value anomalies disappearing. Additionally, the model provides a better explanation for short-term reversal, though it continues to struggle with liquidity and momentum effects.

**Table 4: Time series average absolute alphas: the Fama and French 3 Factor Model**

| | Baseline | Volatility Factor | Conditional Betas | Volatility Factor & Conditional Betas |
|---|---|---|---|---|
| Panel A: NYSE-AMEX and NASDAQ Stocks | | | | |
| Size | 0.2926 | 0.2502 | 0.1618 | 0.1469 |
| | [35.64] | [33.44] | [25.70] | [21.80] |
| Value | 0.6380 | 0.6075 | 0.3446 | 0.2861 |
| | [37.27] | [60.31] | [29.14] | [32.34] |
| Liquidity | 0.0144 | 0.0981 | 0.0206 | 0.0332 |
| | [0.66] | [8.67] | [-1.16] | [3.77] |
| Momentum | 0.3931 | 0.3423 | 0.5796 | 0.5015 |
| | [18.5] | [32.66] | [44.36] | [70.02] |
| Short-term Reversal | 1.6117 | 1.7126 | 1.4800 | 1.3775 |
| | [-62.49] | [-69.9] | [-61.63] | [-77.3] |
| Panel B: NASDAQ stocks | | | | |
| Size | 1.1201 | 1.0537 | 0.7865 | 0.5696 |
| | [20.85] | [23.65] | [17.57] | [14.44] |
| Value | 1.2663 | 1.1833 | 0.8435 | 0.6485 |
| | [25.54] | [31.21] | [22.77] | [16.58] |
| Liquidity | 0.0288 | 0.1375 | 0.0148 | 0.0275 |
| | [-0.55] | [4.30] | [-0.34] | [1.06] |
| Momentum | 0.0437 | 0.0191 | 0.4622 | 0.3610 |
| | [0.49] | [-0.29] | [7.08] | [7.01] |
| Short-term Reversal | 1.7829 | 1.8934 | 1.6075 | 1.4235 |
| | [-23.2] | [-28.18] | [-21.84] | [-31.95] |

This table reports the average monthly absolute alphas from well-documented equity anomaly strategies derived from the Fama and French 3 factor model. Panel A includes all stocks from July 1963 to June 2019, while Panel B focuses on NASDAQ stocks from July 1982 to June 2019. The second column ("Baseline") reports time-series absolute alphas using the Fama and French 3 factor model; the third column adds a volatility factor; the fourth column allows the Fama and French 3 factor model betas to vary with individual stock return standard deviation; and the final column augments the Fama and French 3 factor model with a volatility factor while also allowing the beta to be conditional on volatility. The Newey-West (1987) corrected t-statistics are reported in the bracket.

Panel B shows similar results for NASDAQ stocks, with two notable distinctions. First, the impact of adding the volatility factor on momentum effects is particularly significant, reducing the alpha from 0.0437 to 0.0191. Second, the liquidity anomaly diminishes meaningfully, from 0.0288 to 0.0148 when factor loadings are conditioned on volatility, and to 0.0275 when both volatility modifications are included in the baseline model.

An important observation is the baseline performance of the Fama and French three-factor model, which captures equity anomalies more effectively than the CAPM. For example, comparing the second columns in Panel A of Tables 3 and 4, the absolute alphas for size and value strategies decline from 0.7023 to 0.2926 and from 0.9808 to 0.6380, respectively, when the SMB and HML factors are added to the CAPM. Similar improvements are observed for liquidity and momentum anomalies, with the exception of short-term reversal. Moreover, when the Fama and French model is enhanced with both volatility modifications, all absolute alphas, including for short-term reversal, are smaller than those in the CAPM with the same modifications. These findings contrast sharply with several previous studies, notably Avramov and Chordia (2006), which argued that the SMB and HML factors in the Fama and French model fail to account for the effects of firm characteristics like size and book-to-market ratio on individual stock returns. Our results suggest that the Fama and French model, particularly when combined with volatility adjustments, is more capable of explaining equity anomalies than previously thought.

**The Fama and French three factor model augmented with the liquidity factor**

Table 5 presents the average monthly absolute alphas from zero-investment strategies based on the Fama and French three-factor model augmented with a liquidity factor. The first consideration is how the addition of the liquidity risk factor affects each equity anomaly before incorporating stock return volatility adjustments. The baseline results in both Panel A and Panel B demonstrate that the inclusion of the liquidity factor markedly improves the model's explanatory power, providing a stronger ability to account for equity anomalies compared to the Fama and French three-factor model on its own.

Consistent with previous findings, incorporating a volatility factor into the baseline model results in notable improvements for the size, value, and especially momentum anomalies. However, this adjustment leads to a worsening of liquidity and short-term reversal anomalies. When beta loadings are conditioned on volatility, the model shows even greater improvements in explaining size and value anomalies, as well as noticeable reductions in the short-term reversal anomaly. On the other hand, beta conditioning exacerbates liquidity and momentum anomalies. When both volatility modifications are applied together, the model further improves its ability to explain size and value anomalies, but the liquidity and momentum anomalies are still exacerbated. The results for NASDAQ stocks, as shown in Panel B, follow similar patterns, indicating that the volatility-related modifications have consistent effects across different market segments.

**Table 5: Time series average absolute alphas: the FF 3 Factor + Liquidity Factor Model**

| | Baseline | Volatility Factor | Conditional Betas | Volatility Factor & Conditional Betas |
|---|---|---|---|---|
| Panel A: NYSE-AMEX and NASDAQ Stocks | | | | |
| Size | 0.2491 | 0.2133 | 0.1581 | 0.1090 |
| | [31.2] | [29.64] | [25.98] | [16.19] |
| Value | 0.5647 | 0.5375 | 0.2559 | 0.2065 |
| | [33.63] | [52.46] | [22.83] | [24.55] |
| Liquidity | 0.0026 | 0.0720 | 0.0382 | 0.0119 |
| | [0.12] | [6.66] | [-2.21] | [1.50] |
| Momentum | 0.4190 | 0.3709 | 0.5786 | 0.5470 |
| | [19.58] | [34.19] | [45.98] | [76.44] |
| Short-term Reversal | 1.4744 | 1.5758 | 1.1417 | 1.2172 |
| | [-61.24] | [-69.15] | [-52.12] | [-73.79] |
| Panel B: NASDAQ stocks | | | | |
| Size | 1.0435 | 0.9907 | 0.6478 | 0.4593 |
| | [19.72] | [23.38] | [14.92] | [11.75] |
| Value | 1.1931 | 1.1140 | 0.6382 | 0.4830 |
| | [24.09] | [30.25] | [16.65] | [10.46] |
| Liquidity | 0.0579 | 0.1037 | 0.0693 | 0.0229 |
| | [-1.05] | [3.35] | [-1.62] | [-0.79] |
| Momentum | 0.0738 | 0.0214 | 0.4052 | 0.2716 |
| | [0.84] | [0.32] | [5.88] | [5.97] |
| Short-term Reversal | 1.597 | 1.6912 | 1.1853 | 1.1979 |
| | [-20.97] | [-27.53] | [-18.32] | [-32.61] |

This table reports the average monthly absolute alphas from well-documented equity anomaly strategies derived from the Fama and French 3 factor model plus the liquidity factor. Panel A includes all stocks from July 1963 to June 2019, while Panel B focuses on NASDAQ stocks from July 1982 to June 2019. The second column ("Baseline") reports time-series absolute alphas using the Fama and French 3 factor model plus the liquidity factor; the third column adds a volatility factor; the fourth column allows betas in the Fama and French 3 factor model plus the liquidity factor to vary with individual stock return standard deviation; and the final column augments the Fama and French 3 factor model plus the liquidity factor with a volatility factor while also allowing the beta to be conditional on volatility. The Newey-West (1987) corrected t-statistics are reported in the bracket.

### The Fama and French three-factor model augmented with the momentum factor

Table 6 presents the average monthly absolute alphas from zero-investment strategies of anomaly constituents, based on the Fama and French three-factor model plus the momentum factor. Notably, the addition of the momentum factor does not enhance the model's explanatory power; in fact, two of the anomalies worsen. When compared to adding a liquidity factor to the Fama and French three-factor model, the momentum factor generally performs worse at capturing most equity anomalies, with the exception of the momentum effect itself. The baseline column consistently reports higher alphas across both samples (all stocks and NASDAQ stocks), except for the momentum effects. This trend persists throughout all columns, even when models are further adjusted for volatility, with the momentum effects remaining an outlier.

Regarding the two volatility adjustments, we find consistent results. Adding the volatility factor alone to the baseline model reduces size and momentum anomalies but continues to struggle with liquidity and short-term reversal. Unlike the previous sections, adding the volatility factor increases value anomaly. Conditioning on beta alone mitigates size, value, and short-term reversal anomalies, though it exacerbates momentum and liquidity anomalies. Applying both volatility adjustments has the most positive impact on size and value anomalies, with noticeable improvement in the short-term reversal anomaly. However, momentum and liquidity anomalies still increase. These patterns are also evident when examining NASDAQ stocks, as shown in Panel B.

**Table 6: Time series average absolute alphas: the FF 3 Factor + Momentum Factor Model**

| | Baseline | Volatility Factor | Conditional Betas | Volatility Factor & Conditional Betas |
|---|---|---|---|---|
| Panel A: NYSE-AMEX and NASDAQ Stocks | | | | |
| Size | 0.2897 | 0.2442 | 0.1972 | 0.1111 |
| | [33.96] | [33.36] | [32.21] | [16.82] |
| Value | 0.6559 | 0.6722 | 0.3817 | 0.3337 |
| | [40.93] | [79.4] | [38.88] | [47.27] |
| Liquidity | 0.0102 | 0.1093 | 0.0188 | 0.0521 |
| | [0.52] | [11.03] | [-1.16] | [6.70] |
| Momentum | 0.3959 | 0.3204 | 0.5061 | 0.4505 |
| | [17.7] | [36.41] | [65.29] | [84.76] |
| Short-term Reversal | 1.6172 | 1.6880 | 1.3394 | 1.3758 |
| | [-66.73] | [-55.85] | [-54.06] | [-63.52] |
| Panel B: NASDAQ stocks | | | | |
| Size | 1.1281 | 0.9650 | 0.6344 | 0.3669 |
| | [20.98] | [23.73] | [14.89] | [9.65] |
| Value | 1.2861 | 1.2114 | 0.6680 | 0.5439 |
| | [26.31] | [35.77] | [18.96] | [11.54] |
| Liquidity | 0.0217 | 0.1288 | 0.1313 | 0.0252 |
| | [-0.47] | [4.82] | [-3.64] | [-1.12] |
| Momentum | 0.0535 | 0.0136 | 0.4675 | 0.3159 |
| | [0.60] | [-0.21] | [7.82] | [7.00] |
| Short-term Reversal | 1.7942 | 1.8408 | 1.3180 | 1.3421 |
| | [-25.01] | [-24.99] | [-23.59] | [-32.2] |

This table reports the average monthly absolute alphas from well-documented equity anomaly strategies derived from the Fama and French 3 factor plus the momentum factor. Panel A includes all stocks from July 1963 to June 2019, while Panel B focuses on NASDAQ stocks from July 1982 to June 2019. The second column ("Baseline") reports time-series absolute alphas using the Fama and French 3 factor plus the momentum factor; the third column adds a volatility factor; the fourth column allows betas in the Fama and French 3 factor plus the momentum factor to vary with individual stock return standard deviation; and the final column augments the Fama and French 3 factor plus the momentum factor with a volatility factor while also allowing the beta to be conditional on volatility. The Newey-West (1987) corrected t-statistics are reported in the bracket.

**The Fama and French three-factor model plus the liquidity and momentum factors**

Table 7 presents the average monthly absolute alphas from zero-investment strategies of anomaly constituents, based on the Fama and French three-factor model augmented with both liquidity and momentum factors. The results largely align with previous findings. Baseline results indicate that adding these two risk factors enhances the explanatory power of the Fama and French three-factor model. Momentum effects are most effectively mitigated when the volatility factor alone is introduced. Size and value anomalies are significantly reduced, and short-term reversals shrink noticeably when beta loadings are conditioned. The combined volatility adjustments yield the most pronounced positive impacts. Notably, the abnormal returns associated with size are the smallest across all asset pricing models tested.

**Table 7: Time series average absolute alphas: the FF 3 factor + Liquidity factor + Momentum factor**

| | Baseline | Volatility Factor | Conditional Betas | Volatility Factor & Conditional Betas |
|---|---|---|---|---|
| Panel A: NYSE-AMEX and NASDAQ Stocks | | | | |
| Size | 0.2619 | 0.2091 | 0.135 | 0.0634 |
| | [36.39] | [29.37] | [22.65] | [10.06] |
| Value | 0.5869 | 0.5995 | 0.2725 | 0.2239 |
| | [43.38] | [70.97] | [30.08] | [31.88] |
| Liquidity | 0.0222 | 0.0829 | 0.0384 | 0.0112 |
| | [-1.07] | [8.72] | [-2.51] | [1.54] |
| Momentum | 0.4359 | 0.3447 | 0.5506 | 0.4973 |
| | [34.97] | [37.55] | [75.29] | [89.40] |
| Short-term Reversal | 1.4516 | 1.5502 | 1.1701 | 1.2056 |
| | [-56.82] | [-59.85] | [-54.72] | [-66.04] |
| Panel B: NASDAQ stocks | | | | |
| Size | 0.9991 | 0.8995 | 0.5312 | 0.2805 |
| | [20.20] | [22.63] | [12.23] | [7.65] |
| Value | 1.1360 | 1.1411 | 0.4573 | 0.3780 |
| | [24.70] | [33.58] | [10.87] | [8.89] |
| Liquidity | 0.1530 | 0.1184 | 0.1803 | 0.0207 |
| | [-2.89] | [4.51] | [-5.05] | [-0.88] |
| Momentum | 0.1183 | 0.0231 | 0.3980 | 0.3398 |
| | [1.62] | [0.36] | [8.07] | [8.24] |
| Short-term Reversal | 1.5150 | 1.6382 | 1.0712 | 1.0971 |
| | [-23.71] | [-26.50] | [-23.12] | [-28.92] |

This table reports the average monthly absolute alphas from well-documented equity anomaly strategies derived from the Fama and French 3 factor model plus liquidity and momentum factor. Panel A includes all stocks from July 1963 to June 2019, while Panel B focuses on NASDAQ stocks from July 1982 to June 2019. The second column ("Baseline") reports time-series absolute alphas using the Fama and French 3 factor model plus liquidity and momentum factor; the third column adds a volatility factor; the fourth column allows betas in the Fama and French 3 factor model plus liquidity and momentum factor to vary with individual stock return standard deviation; and the final column augments the Fama and French 3 factor model plus liquidity and momentum factor with a volatility factor while also allowing the beta to be conditional on volatility. The Newey-West (1987) corrected t-statistics are reported in the bracket.

In addition, the model with the combined volatility adjustments begins to address the liquidity anomaly. Abnormal return expectations from liquidity decrease from 0.0222 to 0.0112 for all stocks, and from 0.1530 to 0.0207 for NASDAQ stocks. These findings are significant because neither the volatility factor nor beta conditioning alone effectively mitigate the liquidity anomaly—in previous cases, they mostly exacerbate it. However, despite these improvements, the model with added volatility and beta conditioning still struggles to address the momentum anomaly.

### The Fama and French five-factor model

Table 8 presents the average monthly absolute alphas from zero-investment strategies of anomaly constituents, based on the Fama and French five-factor model. The results are largely consistent with previous findings. Adding the volatility factor alone helps mitigate size and value anomalies. However, unlike in earlier sections where the treatment had positive impacts, it exacerbates the momentum effects, but only marginally. Including beta conditioning reduces size, value, and short-term reversal anomalies but fails to address momentum and liquidity anomalies. When both volatility adjustments are applied together, we observe the most significant improvements in size, value, and short-term reversal anomalies, along with continued progress in mitigating the liquidity anomaly. Similar results are found when analyzing NASDAQ stocks.

Consistent patterns observed across the previous sections can be summarized as follows. First, adding additional risk factors generally enhances the ability of asset pricing models to capture most equity anomalies, both with and without volatility adjustments. Second, when the volatility factor is added alone, it effectively reduces momentum effects and further improves the size anomaly. However, it exacerbates liquidity and short-term reversal anomalies. The findings on the value anomaly are inconsistent. Third, beta conditioning strengthens the models' explanatory power for size, value, and short-term reversal anomalies, but fails to address liquidity and momentum anomalies. Fourth, when both volatility adjustments are applied together, the models show the most significant improvements in mitigating size, value, and short-term reversal anomalies. However, momentum effects worsen in most cases. The impact on the liquidity anomaly is mixed; however, more sophisticated models—such as the Fama and French three-factor model with added liquidity and momentum factors, as well as the five-factor model—are effective at mitigating the liquidity anomaly. Fifth, NASDAQ-listed stocks generally exhibit higher levels of anomalies, except for the momentum effect. Nonetheless, the effects of adding the volatility factor and conditioning on beta for NASDAQ stocks are broadly similar to those observed for all stocks.

**Table 8: Time series average absolute alphas: the Fama and French 5 Factor Model**

| | Baseline | Volatility Factor | Conditional Betas | Volatility Factor & Conditional Betas |
|---|---|---|---|---|
| Panel A: NYSE-AMEX and NASDAQ Stocks | | | | |
| Size | 0.3367 | 0.2343 | 0.2243 | 0.1017 |
| | [34.25] | [28.40] | [28.86] | [15.34] |
| Value | 0.5037 | 0.4794 | 0.1655 | 0.0923 |
| | [34.92] | [49.91] | [17.47] | [13.05] |
| Liquidity | 0.0563 | 0.0971 | 0.0979 | 0.0052 |
| | [-3.21] | [10.01] | [-7.65] | [0.81] |
| Momentum | 0.3268 | 0.3290 | 0.5047 | 0.5084 |
| | [15.68] | [27.46] | [44.57] | [70.80] |
| Short-term Reversal | 1.7288 | 1.7701 | 1.4887 | 1.4198 |
| | [-61.87] | [-60.54] | [-57.18] | [-62.39] |
| Panel B: NASDAQ stocks | | | | |
| Size | 1.0005 | 0.8628 | 0.5937 | 0.2959 |
| | [17.30] | [18.17] | [11.46] | [6.87] |
| Value | 1.0545 | 0.9910 | 0.5797 | 0.4417 |
| | [24.31] | [27.32] | [13.38] | [10.80] |
| Liquidity | 0.1260 | 0.1099 | 0.1775 | 0.0619 |
| | [-2.50] | [3.16] | [-5.12] | [-2.28] |
| Momentum | 0.0533 | 0.0548 | 0.2909 | 0.3318 |
| | [0.55] | [0.72] | [6.63] | [9.11] |
| Short-term Reversal | 1.9273 | 1.9076 | 1.6690 | 1.4518 |
| | [-28.79] | [-32.03] | [-26.39] | [-34.34] |

This table reports the average monthly absolute alphas from well-documented equity anomaly strategies derived from the Fama and French 5 factor model. Panel A includes all stocks from July 1963 to June 2019, while Panel B focuses on NASDAQ stocks from July 1982 to June 2019. The second column ("Baseline") reports time-series absolute alphas using the Fama and French 5 factor model; the third column adds a volatility factor; the fourth column allows betas in the Fama and French 5 factor model to vary with individual stock return standard deviation; and the final column augments the Fama and French 5 factor model with a volatility factor while also allowing the beta to be conditional on volatility. The Newey-West (1987) corrected t-statistics are reported in the bracket.

## V. Robustness Check

To ensure the robustness of the two volatility-related model modifications, we perform tests on two sub-samples from distinct time periods. First, we evaluate the application of these methods over the most recent 20 years to assess their effectiveness in a market where the significance of anomaly effects has arguably weakened. Second, we analyze how these volatility measures perform during recession periods, when the market experiences heightened stress and volatility.

### The case of recent years (latest 20 years)

McLean and Pontiff (2016) investigated the post-publication return predictability of variables known to forecast cross-sectional stock returns, finding that returns from publication-informed trading are, on average, 32% lower. To assess whether individual stock return volatility continues to mitigate equity anomalies, even as the magnitude of anomaly effects has declined, we analyze data from July 1999 to June 2019.

**Table 9: Robustness Tests: The Case of Recent Years (Latest 20 Years)**

| | Fama and French 5 Factor model | | | |
|---|---|---|---|---|
| | Baseline | Volatility Factor | Conditional Betas | Volatility Factor & Conditional Betas |
| **Panel A: NYSE-AMEX and NASDAQ Stocks** | | | | |
| Size | 0.5068 | 0.3717 | 0.3026 | 0.1708 |
| | [15.04] | [15.63] | [15.33] | [10.84] |
| Value | 0.7261 | 0.7521 | 0.2681 | 0.2643 |
| | [12.38] | [21.72] | [8.60] | [11.12] |
| Liquidity | 0.2454 | 0.3654 | 0.1796 | 0.2105 |
| | [4.03] | [13.23] | [4.59] | [12.15] |
| Momentum | 0.1068 | 0.0811 | 0.2578 | 0.1907 |
| | [-1.28] | [3.19] | [6.79] | [13.51] |
| Short-term Reversal | 0.8568 | 0.9347 | 0.6258 | 0.5703 |
| | [-17.07] | [-16.85] | [-14.68] | [-19.17] |
| **Panel B: NASDAQ stocks** | | | | |
| Size | 0.9127 | 0.8369 | 0.4949 | 0.3845 |
| | [8.66] | [10.85] | [7.40] | [7.32] |
| Value | 0.9643 | 1.0302 | 0.4639 | 0.4525 |
| | [11.41] | [17.18] | [8.66] | [9.89] |
| Liquidity | 0.0235 | 0.2570 | 0.1063 | 0.0196 |
| | [0.25] | [4.61] | [1.96] | [0.60] |
| Momentum | 0.1769 | 0.1510 | 0.2749 | 0.1921 |
| | [-1.93] | [-3.05] | [5.20] | [8.91] |
| Short-term Reversal | 1.1952 | 1.2414 | 0.8884 | 0.7744 |
| | [-13.63] | [-15.09] | [-12.21] | [-15.58] |

This table reports the recent 20 years of average monthly absolute alphas from well-documented equity anomaly strategies derived from the CAPM and the Fama and French 5 factor model. Panel A includes all stocks from July 1999 to June 2019, while Panel B focuses on NASDAQ stocks from July 1999 to June 2019. From the second to the fifth columns, time-series monthly absolute alphas are reported, starting with no modifications (baseline) and then with the addition of a volatility factor, where factor loadings are conditioned for both the CAPM and the Fama and French 5-factor model, respectively. The Newey-West (1987) corrected t-statistics are reported in the bracket.

Table 9 compares the average monthly absolute alphas from zero-investment strategies of anomaly constituents based on the Fama and French five-factor model over the past 20 years.[9] The results align closely with the general patterns observed in earlier sections. Momentum and size anomalies are effectively reduced when the volatility factor is added alone. When beta loadings are conditioned, size, value, and short-term reversal anomalies decrease, while momentum effects are exacerbated. Regarding the liquidity anomaly, we observe mixed results:

[9] We find consistent results using the other asset pricing models tested.

positive impacts on all stocks but negative impacts for NASDAQ stocks. When both volatility modifications are incorporated into the baseline model, we see more pronounced improvements in size, value, and short-term reversal anomalies, with noticeable positive effects on the liquidity anomaly. However, the modified model fails to enhance its explanatory power for the momentum anomaly.

Another point of interest is the comparison between the entire sample (Table 8) and the most recent 20 years. In recent years, anomaly return expectations derived from firm characteristics such as size, value, and liquidity have moderately increased, while those based on past return variables have significantly decreased. For NASDAQ stocks, return expectations have declined across all anomaly strategies except for momentum effects. This finding is particularly interesting as it suggests that market participants remain hesitant to fully exploit certain equity anomaly constituents, despite numerous studies highlighting strong relationships between stock portfolio returns and these anomalies. This result contrasts with the general conclusions of McLean and Pontiff (2016). Moreover, the difference in anomaly returns between all stocks and NASDAQ-listed stocks have narrowed considerably in the recent 20-year subsample.

**The case of recessions**

Several researchers have proposed that fluctuations in stock return volatility during recession periods can significantly influence portfolio returns. For instance, Daniel and Moskowitz (2016) suggest that momentum strategies can experience infrequent but persistent strings of negative returns during periods of market stress or panic. They recommend an optimal dynamic momentum strategy that hedges against these momentum crashes by adjusting portfolio weights based on both return premia and risk premia (variance). Similarly, Moreira and Muir (2017) advocate for volatility-managed portfolios during market crises, which reduce risk by scaling down exposure when volatility is high. These portfolios, they report, yield high alphas and Sharpe ratios. Against this backdrop, we further examine how stock return volatility impacts equity anomalies during recession periods.

Table 10 presents the average monthly absolute alphas for all stocks from zero-investment strategies of anomaly constituents based on the Fama and French five-factor model during recession periods.[10] Notably, abnormal return expectations from liquidity and short-term reversal constituents are significantly higher than those estimated for the entire sample, both in the baseline figures and with the two volatility modifications. This disparity likely reflects the distinctive behavior of stock returns during recessions.

The impacts of volatility adjustments largely align with previous findings, though with somewhat enhanced performance during recessions. When the volatility factor is added alone, we observe substantial improvements in the size, value, liquidity, and momentum anomalies, though the short-term reversal anomaly slightly increases. When beta loadings are conditioned, size, value, liquidity, and short-term reversal anomalies decrease, but momentum effects worsen. When both volatility modifications are applied to the baseline model, we find consistent improvements in size, value, and short-term reversal anomalies, with significant positive effects on the liquidity anomaly. However, the modified model continues to fall short in improving its explanatory power for the momentum anomaly.

[10] We identify 6 recession periods (83 months) during our sample period, using NBER database.

**Table 10: Robustness Tests: The Case of Recessions**

| | Fama and French 5 Factor model | | | |
|---|---|---|---|---|
| | Baseline | Volatility Factor | Conditional Betas | Volatility Factor & Conditional Betas |
| NYSE-AMEX and NASDAQ Stocks | | | | |
| Size | 0.4314<br>[4.75] | 0.1792<br>[3.70] | 0.0522<br>[1.26] | 0.1832<br>[-7.17] |
| Value | 0.4057<br>[2.27] | 0.2526<br>[3.22] | 0.2534<br>[4.12] | 0.3500<br>[-9.99] |
| Liquidity | 1.0488<br>[-16.75] | 0.6777<br>[-10.66] | 0.9632<br>[-20.93] | 0.6606<br>[-14.97] |
| Momentum | 0.2123<br>[-0.71] | 0.1198<br>[1.26] | 0.2402<br>[2.52] | 0.4295<br>[8.15] |
| Short-term Reversal | 2.9810<br>[-22.30] | 3.1451<br>[-27.49] | 2.4723<br>[-18.84] | 2.3277<br>[-25.89] |

This table reports the average monthly absolute alphas from well-documented equity anomaly strategies derived from the CAPM and the Fama and French 5 factor model during the 6 recession periods identified by NBER (83 months). From the second to the fifth columns, time-series monthly absolute alphas are reported, starting with no modifications (baseline) and then with the addition of a volatility factor, where factor loadings are conditioned for both the CAPM and the Fama and French 5-factor model, respectively. The Newey-West (1987) corrected t-statistics are reported in the bracket.

## VI. Conclusions

We address well-documented equity anomalies by enhancing existing asset pricing models with a direct risk measure: the volatility of individual stock returns. In an efficient market, fluctuations in stock returns should reflect all available information, including firm characteristics and various investor behavior biases, which are known contributors to equity anomalies. Our findings indicate that individual stock return volatility exhibits offsetting relationships across all tested anomaly constituents, suggesting that volatility has significant potential to mitigate equity anomalies and enhance the parsimony of asset pricing models.

Examining the effects of two volatility modifications on various asset pricing models, we find that adding the volatility factor alone effectively reduces momentum effects and further improves the size anomaly. Beta conditioning strengthens the models' explanatory power for size, value, and short-term reversal anomalies. When both volatility adjustments are applied together, we observe the most significant improvements in mitigating size, value, and short-term reversal anomalies; however, the inclusion of beta conditioning worsens momentum effects in the baseline models. Regarding the liquidity anomaly, both volatility adjustments prove effective in mitigating this anomaly within more sophisticated models. NASDAQ-listed stocks generally exhibit higher levels of anomalies, except for the momentum effect. Nonetheless, the effects of adding the volatility factor and conditioning on beta for NASDAQ stocks closely resemble those observed for all stocks.

While the estimated volatility of individual stock returns is an important consideration, it is crucial to recognize that the effectiveness of capturing equity anomalies varies based on how the volatility measure is integrated into asset pricing models. Despite consistent offsetting relationships between volatility and all anomaly constituents, the volatility factor mitigates momentum effects across most tested models when applied alone; its impact, however, diminishes when combined with the beta conditioning process. This does not imply that the volatility factor is unimportant; rather, it suggests that certain model frameworks could better harness the potential of individual stock return volatility to alleviate equity anomalies. These findings open avenues for future research, such as exploring how the incorporation of other factors alongside the volatility factor can more effectively explain variations in stock returns. Additionally, investigating alternative frameworks or methods for utilizing stock return volatility—beyond merely constructing a volatility factor or employing beta conditioning—could provide deeper insights into addressing equity anomalies.

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# An Update on Sector Rotation in the "Sell in May and Go Away" Strategy

Steven Dolvin and Bryan Foltice*

## Abstract

A common mantra on Wall Street is to "Sell in May and Go Away." This strategy follows documented seasonal patterns that point to higher (lower) market returns during the months of November to April (May to October). We examine whether this pattern continues to exist in more recent periods, and we also explore whether such a strategy can be improved by simultaneously rotating into and out of cyclical (or defensive) sectors. Our results suggest that investors can still generate positive alpha by following the traditional "Sell in May" strategy, albeit at a slightly reduced level in more recent years. We also find that a sector rotation strategy that moves into cyclical (noncyclical) sectors during the November to April (May to October) period can provide significant incremental alpha, but primarily when implemented in concert with a counterbalanced short selling strategy.

**Keywords:** Sector Rotation, Sell in May, Halloween Indicator, Seasonality, Short Selling

**JEL Classification:** G11

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## I Introduction

In general, it is difficult for investors to consistently outperform the market on a risk-adjusted basis, i.e., to "beat the market." This difficulty, however, has not stopped both researchers and investors alike from seeking out ways to try to do so, as identifying such an anomaly would create an opportunity for significant excess return. Given the amount of time and energy spent in this pursuit, it is not surprising that a number of so-called market anomalies have been identified. For example, the January Effect (see Thaler, 1987) denotes that stocks generally have higher returns in the month of January. Similarly, Tinic and West (1984) note that small-cap stocks have higher returns in January, termed the Turn-of-the-year Effect, and Cross (1973) finds that market returns tend to be lower on Mondays, i.e., the Day-of-the-Week Effect.

Obviously, these anomalies are identified using historical information, so, as Sullivan, Timmermann and White (2001) note, there is a risk of data mining, as well as with whether such anomalies even continue to exist over time or are randomly confined to a particular period. In fact, controlling for factors related to size, value, profitability, and conservatism (i.e., the Fama and French (2014) five-factor model), often eliminates many of these previously identified anomalies, essentially producing alphas (i.e., abnormal returns) that are insignificantly different from zero. Moreover, a number of these anomalies have effectively shrunk over time, either because they did

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not truly exist or because trading of a related strategy has effectively arbitraged the impact (what Dimson and Marsh (1999) refer to as Murphy's Law).

One anomaly that is still debated, however, is the Halloween Effect, or more commonly known as the "Sell in May and Go Away" strategy. While there is uncertainty as to the origin of this approach, Bouman and Jacobsen (2002), building on practitioner strategies developed by O'Higgins and Downes (1990), provide a formal analysis, confirming that average market returns tend to be higher in the months of November through April as compared to the other months of the year (i.e., May through October). Subsequent studies examine this issue, attempting to provide some potential explanations for the phenomenon, which include seasonal affective disorder (e.g., Kamstra, Kramer, and Levi, 2003) or the timing of vacations (e.g., Bouman and Jacobsen, 2002); however, these reasons generally lack support as a complete explanation for the phenomenon.

We add to the existing research by further exploring the Sell in May and Go Away (SMGA) strategy. In particular, we examine a more recent time period to determine if monthly returns still differ across the November to April (Nov-Apr) period as compared to the May to October (May-Oct) period, finding that the average monthly return in the Nov-Apr time period remains larger than the May-Oct period. Our primary contribution, however, is to examine whether the base strategy of being invested in the market during Nov-Apr and out of the market in May-Oct can be improved upon. For example, Doeswijk (2008) suggests that cyclical sectors outperform in strong market periods and defensive sectors outperform in weaker periods, so going long in cyclicals in the Nov-Apr period and long in noncyclicals (or defensives) in the May-Oct period could produce improved performance. We find no support for this strategy; however, we also explore the impact of overlaying a short-sale strategy in the respective monthly periods, finding that this approach not only reduces net market exposure (i.e., beta risk), but also improves alpha. In total, we find that the SMGA strategy is still feasible for producing positive risk-adjusted performance and that this performance can be improved through sector rotation that is implemented in tandem with a combined short selling strategy.

## II Background

As noted, Bouman and Jacobsen (2002) explore the SMGA approach, confirming the benefits of its use both in the United States and across almost all (36 of 37) developed markets, as well as across a broad sample of emerging markets. Further, they note that the positive results do not appear to be driven by other anomalies such as the January Effect, nor do they find any compelling reason for its continued existence. While the SMGA approach received a great deal of attention in earlier years, additional exploration has been relatively muted. Nonetheless, the few studies that have been added to the literature generally confirm the benefit of the SMGA strategy. For example, Degenhardt and Auer (2018) find that the SMGA strategy is also profitable when applied within futures contracts, and Pavlova, Whitworth, and de Boyrie (2022) note that it can also be applied within a subset of ESG (i.e., environmental, social and governance) focused investments.

On a broader basis, Andrade, Chhaochharia, and Fuerst (2013) find that the SMGA strategy continued to generate positive risk-adjusted returns through their extended time period, as do Zhang and Jacobson (2021) across the globe. However, some studies question (the extent of) the continued efficacy of the SMGA approach. For example, Jones and Lundstrum (2009) find that the size and significance of favorable results is heavily based on the time period examined. In addition, Dichtl and Drobetz (2015) find that the level of excess return associated with the SMGA approach has weakened in more recent years.

While the SMGA approach continues to be explored, the level of activity in this area has slowed as compared to early periods. This lack of attention is particularly noteworthy given the introduction of market and sector ETFs that effectively allow investors to undertake the SMGA strategy in a relatively easy, low-cost way – something that was more difficult at the time these anomalies were first noted. Moreover, these strategies have made their way into funds that are available to the broad investing public. For example, the Pacer CFRA-Stovall Equal Weight Seasonal Rotation ETF (i.e., exchange traded fund) generally follows the strategy developed by Doeswijk (2008), but with slightly different sectors defined as either cyclical or defensive. Similarly, private managers such as Peloton Capital Management have integrated the SMGA strategy into managed funds that are available to a more narrow set of accredited investors.

While each fund is similar in its broad application of the SMGA (and related sector rotation) strategy, each manager has the discretion to define which sectors they deem cyclical or not. For example, Table 1 provides a list of sectors employed by each of the three respective sources: (1) the sector rotation strategy defined by Doeswijk (2008), i.e., *Doeswijk*, (2) the managed fund offered by Peloton Capital Management (i.e., *Peloton*), and (3) the Pacer CFRA-Stovall Equal Weight Seasonal Rotation ETF (i.e., *Pacer*). Given the continued use of the SMGA strategy in financial markets, it is important to explore whether the strategy continues to be valid, and, if so, which sectors are optimal for implementing a related sector rotation approach. Thus, we believe an update in this area is warranted.

**Table 1: Defining Cyclical and Noncyclical Sectors**

| | **Doeswijk** | **Peloton** | **Pacer** |
|---|---|---|---|
| Cyclical: | | | |
| Energy | Y | Y | |
| Discretionary | Y | Y | Y |
| Industrials | Y | Y | Y |
| Materials | Y | Y | Y |
| Technology | | | Y |
| Noncyclical (Defensive) | | | |
| Staples | Y | Y | Y |
| Utilities | Y | Y | |
| Healthcare | Y | Y | Y |
| Technology | | Y | |

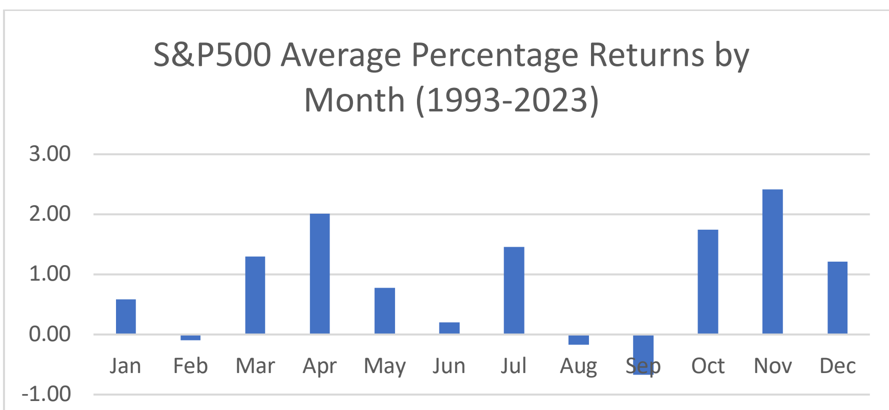
This table identifies which sectors are classified as either cyclical or noncyclical by three sources (Y = yes). *Doeswijk* is based on Doeswijk (2008); *Peloton* is from the strategy employed by Peloton Capital Management; and *Pacer* is based on the Pacer CFRA-Stovall Seasonal Rotation ETF (ticker: SZNE). The sectors of communications, financials, and real estate are not classified as either defensive or cyclical by any source, so they are excluded from the analysis.

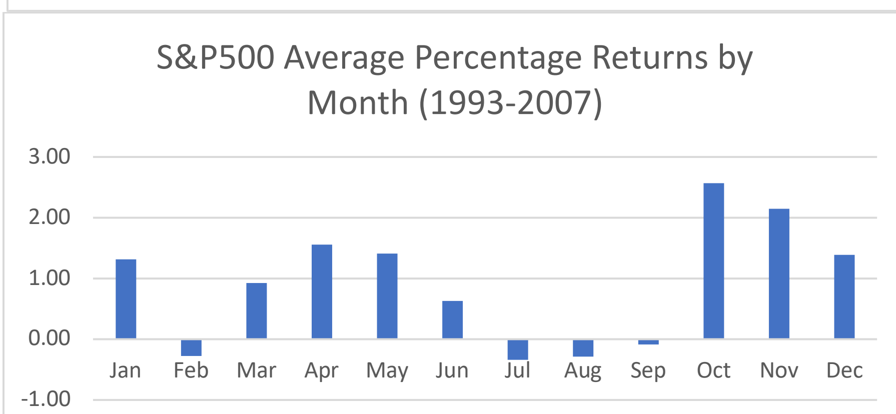
## III Data and Summary Statistics

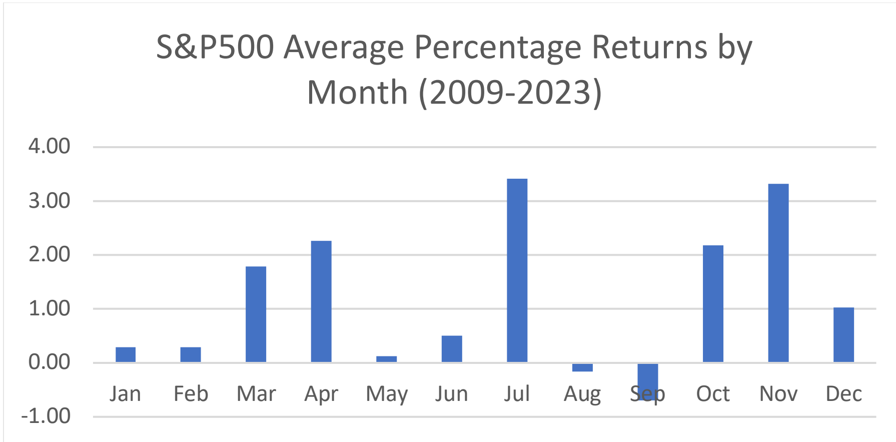
We begin by collecting monthly (index) returns data from YCharts for both the S&P500 and each underlying sector within the S&P500 over the 1993 to 2023 time period. We are particularly interested in whether individual investors are able to capture potential returns associated with the SMGA strategy, which would likely be accomplished via a trading approach employing liquid ETFs. Given this focus, we begin our period of analysis in 1993 when the S&P500 SPDR ETF (ticker SPY) began trading. This provides a 31-year period, which we also split into two subperiods: 1993-2007 and 2009-2023. This segmentation is primarily designed to examine if the potential efficacy of the SMGA strategy has declined over time. Note that we exclude 2008 from our subperiod analysis given the unusual nature of that particular year. Fortunately, this provides us with a natural breakpoint and two equal 15-year subperiods pre- and post-2008. Further, keeping 2008 would significantly skew the results depending on which of the two subperiods it is placed. Nonetheless, for robustness, we explore the impact of excluding 2008 in a forthcoming section.

As a high-level overview, we report monthly average returns for the S&P500 in Figure 1. The first chart provides average monthly returns across the entire 1993 to 2023 time period, while the remaining two charts provide average monthly returns for the respective subperiods (i.e., 1993-2007 and 2009-2023). While the charts illustrate that average monthly returns vary through time, there is a consistently noticeable pattern of higher returns over the months of Nov-Apr. Specifically, over the entire period, the average monthly return is 0.90 percent. For the respective months, the average monthly return is 1.23 percent over the Nov-Apr period and 0.57 over the May-Oct period, representing a difference of 0.66 percent (consistent with the SMGA philosophy). For the 1993-2007 subperiod, the average monthly return is 0.91 percent. The average monthly return is 1.18 percent over the months of Nov-Apr and 0.65 over the months of May-Oct, representing a difference of 0.53 percent. For 2009-2023 subperiod, the average monthly return is 1.19 percent. During these years, the average monthly return is 1.50 percent over the months of Nov-Apr and 0.89 over the months of May-Oct, representing a difference of 0.66 percent. Thus, the potential of the SMGA strategy appears to remain intact even in more recent years.

**Figure 1: S&P500 Average Returns by Month**






This figure illustrates average S&P returns by month for the 1993-2023 time period, as well as for two subperiods (1993-2007 and 2009-2023).

To explore the impact of isolating 2008, in unreported results we examine the returns from the SMGA strategy in that single year. Monthly returns, as one would expect given the global financial crisis, are negative across the entire year; however, the SMGA strategy is still favorable, providing a net monthly average return difference of 3.55 percent, which is larger than the average difference in either subperiod. Therefore, excluding 2008 in the subperiod analysis is a conservative approach as it likely biases us against finding any excess return through use of the basic SMGA strategy.

## IV Primary Analysis

### Base SMGA strategy

The primary SMGA strategy involves going long in the market during Nov-Apr and sitting out May-Oct, during which time an investor would earn the risk-free rate of return. To provide a baseline for comparison, we begin by calculating some univariate statistics for the S&P500 and the base SMGA strategy for the 1993-2023 period, as well as for our two subperiods (i.e., 1993-2007 and 2009-2023). We report these values in Table 2. Specifically, we provide monthly average (and median) percentage returns, as well as the *p*-value and standard deviation of the monthly returns. We also include the Sharpe and Sortino ratios (e.g., the ratio of average monthly returns to deviation for the Sharpe and to downside deviation for the Sortino).

Given that the SMGA strategy effectively sits out of the market half the year, we might expect to earn a significantly lower return. However, this is not necessarily the case. In fact, over the early part of the period studied, the average monthly return is actually higher for the SMGA strategy. Combined with a lower deviation, the SMGA strategy, as would be expected, creates a significantly higher Sharpe (and Sortino) ratio. Given the strong market post-2008, the SMGA strategy fails to deliver the same relative performance. Yet, even in this period, the average return is in line with the overall average, and the deviation is slightly below the overall S&P500.

To provide a broader level of control, we also conduct a Fama and French (2014) analysis that captures five factors known to impact portfolio returns. These include overall market returns (*Market*), as well as factors that control for size (*SMB*, or small minus big), value versus growth (*HML*, or high book-to-market minus low book-to-market), profitability (*RMW*, or robust minus weak), and conservativeness (*CMA*, or conservative minus aggressive). All factor data, including the risk-free rate, are collected from Ken French's website. In this analysis, the coefficient on each factor identifies the level of impact the respective factor has on the underlying portfolio return. The most important metric in this performance analysis is the intercept, which represents alpha (α), the standard industry measure of risk-adjusted, or abnormal, return.

**Table 2: Base SMGA Strategy Returns**

| | S&P500 1993-2023 | S&P500 1993-2007 | S&P500 2009-2023 | SMGA 1993-2023 | SMGA 1993-2007 | SMGA 2009-2023 |
|---|---|---|---|---|---|---|
| Mean | 0.90 | 0.91 | 1.19 | 0.90 | 0.99 | 0.90 |
| (p-value) | (<.0001) | (.0021) | (.0004) | (<.0001) | (<.0001) | (.0034) |
| Median | 1.38 | 1.31 | 1.88 | 0.31 | 0.40 | 0.09 |
| Deviation | 4.32 | 3.92 | 4.44 | 3.52 | 2.91 | 4.06 |
| Sharpe | 0.21 | 0.23 | 0.27 | 0.26 | 0.34 | 0.22 |
| Sortino | 0.28 | 0.31 | 0.37 | 0.39 | 0.61 | 0.32 |

This table provides monthly average (and median) percentage returns for the S&P500, as well as for the base SMGA strategy, which entails going long in the market during the months of Nov-Apr and into a risk-free asset for the other months. In addition, we report the *p*-value and standard deviation of the monthly returns, as well as the Sharpe and Sortino ratios (e.g., the ratio of average monthly returns to deviation for the Sharpe and to downside deviation for the Sortino). We provide values for the entire 1993-2023 time period, as well as for our two subperiods (i.e., 1993-2007 and 2009-2023)

We report the results of this analysis in Table 3 for our base SMGA strategy over the full 1992-2023 period, as well as across our two subperiods. Given that the S&P500 is a broad market index, we would expect to find alphas for an index strategy that are effectively zero and betas that are essentially equal to one. We confirm these baseline expectations in unreported results of SPY. Given that the S&P500 is a large-cap index, it is also not surprising that we find the coefficient on SMB is negative, nor that there is a slight value, profitability, or conservative tilt, since the market factor used in the regression is a broader index that would capture mid- and small-cap stocks as well. Again, these unreported results are all as would be expected for an index, particularly with regard to alpha.

We find, as would be expected, that the market beta for the base SMGA strategy is effectively cut in half (i.e., 0.5 vs. 1.0) relative to a full-year index strategy, which is consistent with the fact that we are essentially in the market only half the year, with the remaining time being in a risk-free asset. Of most interest, we find that once we control for the market and other factors, the SMGA strategy provides a positive alpha. For example, an alpha of 0.31 indicates a gross annualized excess return of 3.72 percent. Consistent with the results in Table 2, and the findings of Dichtl and Drobetz (2015), we find that the alpha value has declined slightly in recent years (both in size and significance), yet it continues to remain economically meaningful. Taken as a whole, we find that the SMGA strategy continues to be capable of increasing portfolio efficiency, but possibly not to the same degree as in earlier periods.

**Table 3: Fama and French (2014) Alphas for the Base SMGA Strategy**

| | (1)<br>SMGA<br>'93-'23 | (2)<br>SMGA<br>'93-'07 | (3)<br>SMGA<br>'09-'23 |
|---|---|---|---|
| α | 0.31 | 0.32 | 0.25 |
| | (.0320) | (.0910) | (.2625) |
| $B_{Market}$ | 0.50 | 0.47 | 0.53 |
| | (<.0001) | (<.0001) | (<.0001) |
| SMB | 0.01 | -0.10 | 0.11 |
| | (.9167) | (.1205) | (.2420) |
| HML | 0.31 | 0.21 | 0.34 |
| | (<.0001) | (.0286) | (<.0001) |
| RMW | 0.10 | 0.05 | 0.13 |
| | (.1297) | (.5251) | (.2652) |
| CMA | -0.17 | -0.06 | -0.25 |
| | (.0484) | (.6062) | (.0683) |
| n | 372 | 372 | 372 |
| Adj. $R^2$ | (.4464) | (.3573) | (.5209) |

This table provides regression results from a standard Fama and French (2014) analysis of the base SMGA strategy defined in Table 2. Alpha (α) is the risk-adjusted monthly excess return from the respective strategy, and $B_{Market}$ is the beta versus the market index. The remaining values represent coefficient estimates for the other market factors: small size (SMB), value (HML), profitability (RMW), and conservative (CMA). P-values for the estimated coefficients are also provided (in parenthesis). All factor data is collected from Ken French's website.

## Sector rotation

Following Doeswijk (2008), we next explore whether concentrating on specific sectors in the SMGA strategy can increase alpha. In particular, Doeswijk (2008) suggests that cyclical stocks should do relatively better during the higher return months of Nov-Apr, while defensive (or non-cyclical) sectors should do relatively better during May-Oct. So, we examine a set of strategies built around this sector rotation approach. More specifically, we explore performance for the three strategies described in Table 1 (i.e., *Doeswijk*, *Peloton*, and *Pacer*). For each of these, the sector rotation strategy is the essentially the same; the only difference is which sectors are identified as cyclical or defensive. For sector definitions, we use the segmentation employed by State Street in their market leading sector ETFs. Again, this approach allows us to examine the performance as employed by a typical retail investor.

We report the results of this analysis in Table 4. For each strategy we report the average monthly (equal-weighted) return (i.e., *Gross*), as well as the average monthly return net of the S&P500 return (i.e., *Premium*). We report these values for the entire period in Panel A, as well as for the subperiods in Panels B (1993-2007) and C (2009-2023). We find that the return *Premium* is positive and significantly different from zero for all strategies across the entire period, as well as for the first subperiod (193-2007). While still positive, on average, *Premium* loses much of its significance within the more recent subperiod (i.e., Panel C), suggesting that the sector rotation approach to the SMGA strategy may have lost efficacy. To get a more complete picture, however, we return to the Fama-French (2014) analysis to explore the risk-adjusted return for each strategy, and we report these results in Table 5.

## Table 4: Returns to Sector Rotation Strategy

**Panel A: Average Monthly Returns 1993-2023**

| | Doeswijk | Peloton | Pacer |
|---|---|---|---|
| Gross | 1.16 | 1.22 | 1.22 |
| | (<.0001) | (<.0001) | (<.0001) |
| Premium | 0.26 | 0.32 | 0.32 |
| | (.0428) | (.0018) | (.0061) |

**Panel B: Average Monthly Returns 1993-2007**

| | Doeswijk | Peloton | Pacer |
|---|---|---|---|
| Gross | 1.27 | 1.31 | 1.33 |
| | (<.0001) | (<.0001) | (<.0001) |
| Premium | 0.35 | 0.40 | 0.42 |
| | (.0642) | (.0079) | (.0234) |

**Panel C: Average Monthly Returns 2009-2023**

| | Doeswijk | Peloton | Pacer |
|---|---|---|---|
| Gross | 1.27 | 1.36 | 1.31 |
| | (<.0001) | (<.0001) | (<.0001) |
| Premium | 0.07 | 0.17 | 0.12 |
| | (.6885) | (.2425) | (.4260) |

This table provides monthly (equal-weighted) average percentage returns for each strategy as defined in Table 1 (i.e., *Doeswijk*, *Peloton*, and *Pacer*). For each we report the gross average monthly return (*Gross*), as well as the average monthly return net of the return of the S&P500 (*Premium*). For each average, we also report the p-value from a difference test examining whether the average return is different from zero. Panel A provides values for the entire 1993-2023 time period, while Panel B (Panel C) provides averages for the 1993-2007 (2009-2023) time periods.

## Table 5: Fama and French (2014) Alphas for Sector Rotation Strategy

| | (1) Doeswijk | (2) Peloton | (3) Pacer |
|---|---|---|---|
| α | 0.24 | 0.26 | 0.32 |
| | (.0441) | (.0112) | (.0080) |
| $B_{Market}$ | 0.86 | 0.94 | 0.90 |
| | (<.0001) | (<.0001) | (<.0001) |
| SMB | -0.08 | -0.06 | -0.11 |
| | (.0498) | (.0882) | (.0094) |
| HML | 0.19 | 0.12 | 0.08 |
| | (<.0001) | (.0024) | (.1044) |
| RMW | 0.19 | 0.19 | 0.15 |
| | (.0005) | (<.0001) | (.0040) |
| CMA | 0.11 | 0.08 | 0.07 |
| | (.1381) | (.1941) | (.3419) |
| n | 372 | 372 | 372 |
| Adj. $R^2$ | (.7324) | (.8232) | (.7547) |

This table repeats the analysis provided in Table 3, but it focuses on various sector rotation strategies. Column 1 examines the strategy proposed by Doeswijk (2008). Columns 2 and 3 examine the high-level sector strategies employed by Peloton and Pacer, respectively.

Examining Table 5, we find that each of the sector rotation strategies has a beta below one, suggesting a smaller amount of market risk. Again, since the sectors are from the S&P500, we find a negative coefficient on SMB and a positive coefficient on the other factors. Once all factors are controlled, each strategy, most notably, creates a positive alpha across the full time period. Thus, one might be tempted to conclude that the sector rotation strategy is value enhancing, and this might be true relative to a straight investment in the market. However, we need to compare these alphas to those provided by the base SMGA strategy (without sector rotation), as well as across the subperiods.

Regarding the first comparison, recall that the (monthly) alpha generated from the base SMGA strategy across the entire period was 0.31 (see column 4 in Table 3). Comparing this to the alphas generated by sector rotation (i.e., Table 5), we note that the alphas are at the same level or lower when sector rotation is employed, begging the question of whether the added complexity is worth it. We also note that the sector rotation carries a higher market beta, which is consistent with longer times invested in the market. Our results, therefore, do not necessarily imply that sector rotation produces a lower average gross return, but they do indicate that on a risk-adjusted basis sector rotation does not appear to improve the efficiency of the base SMGA strategy. In fact, adding leverage to the base SMGA strategy to achieve the same level of beta as the sector rotation strategy would generate a higher alpha.

Regarding the second comparison, we repeat the sector rotation analysis for the subperiods, and we report these results in Table 6. For brevity, however, we exclude the factor coefficients and only include the estimated alphas. Note that the alphas for the entire period match those reported in Table 5. For the subperiods, we find that the alphas decline over time, albeit slightly, which is consistent with our earlier findings from Table 4. Comparing the subperiod alphas to the base SMGA strategy (i.e., Table 3), we continue to find that the sector rotation strategy fails to increase efficiency. While we follow the sector definitions provided by Pacer and Peloton, our results are a high-level view of the strategies, as Peloton, in particular, uses proprietary timing and rotation strategies that are intended to boost alpha.

**Table 6: Sector Rotation Fama and French (2014) Alphas by Period**

| | **Doeswijk** | | | **Peloton** | | | **Pacer** | | |
|---|---|---|---|---|---|---|---|---|---|
| | All | 93-07 | 09-23 | All | 93-07 | 09-23 | All | 93-07 | 09-23 |
| α | 0.24 | 0.24 | 0.20 | 0.26 | 0.25 | 0.22 | 0.32 | 0.40 | 0.19 |
| | (.0441) | (.1825) | (.2695) | (.0112) | (.1053) | (.1323) | (.0080) | (.0421) | (.2051) |

This table repeats the analysis provided in Table 5, but also looks at subperiod estimates (i.e., 1993-2007 and 2009-2023) for each strategy. For brevity, we only report the alpha estimates, as the remaining factor coefficients remain relatively stable.

## Impact of short selling

While the long-only sector rotation strategy does not appear to add incremental alpha relative to the base SMGA approach, it does offer the opportunity to incorporate short selling as part of the broader investment plan, which could potentially increase its efficacy. With this in mind, we next explore a long-short strategy that goes long (short) in cyclicals (noncyclicals) in Nov-Apr and long (short) in defensives (cyclicals) in May-Oct. We report the results of this analysis in Table 7. Panel

A provides average monthly returns, and Panel B reports the alphas and market betas from the standard Fama and French (2014) regression (other factors excluded for brevity).

**Table 7: Long-Short Strategy Returns**

**Panel A: Average Monthly Returns**

| | Doeswijk | Peloton | Pacer |
|---|---|---|---|
| Gross | 0.55 | 0.53 | 0.52 |
| | (.0063) | (.0014) | (.0128) |
| Premium | -0.35 | -0.37 | -0.39 |
| | (.2287) | (.1760) | (.1951) |

**Panel B: Fama and French (2014) Alphas**

| | Doeswijk | | | Peloton | | | Pacer | | |
|---|---|---|---|---|---|---|---|---|---|
| | All | 93-07 | 09-23 | All | 93-07 | 09-23 | All | 93-07 | 09-23 |
| α | 0.49 | 0.41 | 0.46 | 0.37 | 0.19 | 0.42 | 0.50 | 0.55 | 0.34 |
| | (.0174) | (.1541) | (.1504) | (.0314) | (.4219) | (.1227) | (.0200) | (.1112) | (.2249) |
| $B_{Market}$ | -0.03 | 0.02 | -0.07 | -0.00 | 0.10 | -0.05 | -0.05 | -0.04 | -0.06 |
| | (.5043) | (.8481) | (.3731) | (.9608) | (.1626) | (.4523) | (.3755) | (.6915) | (.3384) |

This table examines the results of a long-short strategy that goes long (short) in cyclicals (noncyclicals) in November through April and long (short) in noncyclicals (cyclicals) in May through October for the 1993-2023 period. Panel A replicates the results from Table 4 (Panel A), while Panel B replicates the Fama and French (2014) analysis as performed in Tables 5 and 6.

Examining Panel A, the long-short sector rotation strategy produces an average monthly return of between 0.52 and 0.55 percent, depending on the specific sectors defined as cyclical or defensive (i.e., the varying definitions suggested by *Doeswijk*, *Peloton*, and *Pacer*). The average monthly S&P500 return is 0.90 (from Table 2), which produces a net *Premium* of between -0.35 and -0.39 percent for the long-short strategies. While the return is lower, the long-short strategy should also significantly reduce net market exposure. In fact, Panel B illustrates that a long-short approach results in a beta that is effectively 0. Moreover, the alpha is positive and significant across most time periods and across the varying definitions of cyclical/defensive sectors. Comparing this to the base SMGA approach, our results suggest that sector rotation can be efficiency enhancing when a broader long-short strategy is employed. As such, implementation of a sector rotation strategy among investment managers is likely best positioned within a market-neutral hedge fund structure that has the capacity to implement this more robust trading philosophy and that is in search of positive alpha with zero net market exposure.

### Robustness tests

To examine the veracity of our results, we conduct a series of robustness tests. We begin by examining a shift in the start date at which we shift back into the market. In particular, the charts in Figure 1 illustrate that monthly returns in October may be on the higher side. So, we explore a strategy that continues to sell in May, but that re-enters the market in October instead of November. We report the results of this analysis within the sector rotation strategy in Table 8. We find that changing the monthly window does not improve performance over the standard November re-entry date. In particular, gross average monthly returns remain effectively unchanged, while the alphas decline, suggesting the October date may have a larger impact in increasing risk than on any change in return. Thus, we recommend retaining the standard monthly periods within the SMGA strategy and related approaches.

**Table 8: October Trigger**

**Panel A: Returns 1993-2023**

| | Doeswijk | Peloton | Pacer |
|---|---|---|---|
| Gross | 1.13 | 1.16 | 1.20 |
| | (<.0001) | (<.0001) | (<.0001) |
| Premium | 0.23 | 0.26 | 0.30 |
| | (.0643) | (.0117) | (.0077) |

**Panel B: Alphas**

| | Doeswijk | | | Peloton | | | Pacer | | |
|---|---|---|---|---|---|---|---|---|---|
| | All | 93-07 | 09-23 | All | 93-07 | 09-23 | All | 93-07 | 09-23 |
| α | 0.14 | 0.17 | 0.26 | 0.13 | 0.12 | 0.14 | 0.26 | 0.35 | 0.17 |
| | (.2557) | (.3486) | (.0276) | (.1922) | (.4334) | (.3642) | (.0276) | (.0649) | (.2541) |

This table examines the results of a cyclical strategy that begins in October instead of November. Thus, investors would invest in cyclicals in October through April and in noncyclicals in May through September. Panel A replicates the results from Table 4 (Panel A), while Panel B replicates the Fama and French (2014) analysis as performed in Tables 5 and 6.

In unreported results, we also explore a combination of the SMGA strategy and sector investing. Specifically, we consider separate strategies that either invest only in cyclicals in May-Nov or only in defensives in May-Oct. These approaches do not improve performance; in fact, we find generally negative alphas associated with these varying methods. We also repeat our standard Fama and French (2014) analyses but with the inclusion of the momentum factor developed by Carhart (1997). This inclusion has little impact on our alphas and conclusions. Lastly, we also repeat our analysis using alternative S&P500 index funds (e.g., IVV and VOO). IVV began in May 2000 and VOO in September 2010. So, for comparison to our base results, we created samples based on similar time periods. The only difference in results (primarily alpha) is driven by slightly different expense ratios among the respective ETFs; however, these differences are very small and therefore result in no meaningful difference. Thus, our results remain robust.

## V Discussion and Conclusion

Given the competitiveness within financial markets, investment managers are continually searching for ways to outperform the market. This research has led to the identification of a number of market anomalies. While most of these are found to be transient and disappear over time or after controlling for other factors, one that has appeared to remain robust is the "Sell in May and Go Away" (SMGA) strategy, which invests in the market during the months of November to April, but exits the market (to a risk-free) asset in May to October.

We examine the SMGA strategy, finding that it continues to be effective at generating positive alpha, albeit at a slightly reduced level in more recent years. We also explore the use of a sector rotation strategy within SMGA that goes long in cyclicals during the Nov-Apr period and long in defensives in the May-Oct period. We find that this approach does not improve upon the more basic SMGA philosophy. However, we do find that adding short selling does improve risk-adjusted performance. Thus, investors who have the knowledge and ability to add this layer may be able to craft strategies that are capable of positive, risk-adjusted performance.

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# A Stochastic Analysis of Buy and Hold Versus Annual Rebalancing Portfolio Strategies

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## Abstract

Numerous articles in the literature develop theoretical mathematical models that prove rebalancing investment portfolios will outperform a buy and hold strategy on a risk adjusted basis. However, a number of authors using varying asset classes and time periods have found instances where buy and hold outperforms a rebalancing strategy on a risk adjusted basis. This research shows that both can be correct. Most of the papers found in the literature use historical prices and price changes to validate their findings. The research effort detailed in this paper simulates a wide range of possible future asset pricing scenarios that do not rely on historical price patterns. These findings suggest that differences between these two strategies, at a more detailed level, favor the rebalancing strategy, but not exclusively.

**Keywords:** Investment Strategies, Investment Portfolio Performance, Portfolio Management

**JEL Classification: C6, G1**

## I. Introduction

Which is better, buy and hold (BH) or annual rebalancing (RB), is a question that seems to linger in the investment management literature. Generally, but not always the research looks back 5, 10 or more years and uses monthly or annual returns to determine which approach results in higher risk adjusted portfolio returns. Using historical returns are essentially a sample that may not be representative of future return trends and patterns. In addition, authors select different types of assets classes for analysis making it difficult to compare the results of one research endeavor with others.

This paper presents empirical evidence, derived from extensive simulations conducted over diverse future market scenarios, which reveals RB is superior most of the time, but not always. By analyzing the performance of both buy and hold and annual rebalancing strategies under numerous "futures," the aim is to provide insights that have not been reported in the literature, specifically the details of the risk and returns for each trial, not just the overall results which is typical in the literature. Even Dichtl, H., Drobetz, W., & Wambach, M. (2016) who performed extensive analysis with 1000 simulations including buy & hold and nine different rebalancing strategies using a history-based bootstrap method only reported summary statistics, not specific details.

A large percentage of the literature examining which strategy produces the better returns indicates that annual or some form of periodic portfolio rebalancing leads to lower risk and higher returns. A sample of the papers include Dichtl, Drobetz & Wambach (2014), Maeso & Martellini (2020), Bouchey, Nemtchinov, Paulsen & Stein (2012), Meyer-Bullerdiek (2018) and Farago & Hjalmarsson (2023).

Dichtl, Drobetz & Wambach (2014) conclude that, "Despite cross-country differences, our history-based simulation results show that all rebalancing strategies outperform a buy-and-hold strategy in terms of Sharpe ratios, Sortino ratios, and Omega measures. The differences in risk-adjusted performance are not only statistically significant, but also economically relevant."

Buy and hold has many advocates although there does not appear to be quite as many publications in the literature. A sample of these papers include Hilliard & Hilliard (2018), Spinu (2015), Constable (2021) and Hulbert (2023). Masters (2003) highlights the often repeated conundrum, "While the power of rebalancing to improve returns and reduce risk is generally acknowledged … Who wants to take money from an asset class that has performed extremely well and reinvest it in something that has lagged behind?" The goal of this research is to provide insights at a greater level of detail than those reported in the literature by providing the probabilities of which strategy can provide superior returns, whether risk or non-risk adjusted.

## II. Stochastic Process Methodology

A stochastic process is a "family or ensemble of time functions x(t, ζ) depending on the parameter ζ where the domain of ζ is the set of all experimental outcomes and the domain of t is a set of real numbers (Papoulis, 1991)." The Stochastic Comparison Model (SCM) developed for this research creates three arrays, one for each trial (Trial array), one for the randomly computed precent changes (Pct array) and a summary array for every trial (Outcome array).

The total portfolio value for both the BH and RB analyses is $100,000. The array for each trial is created using the following equations for the eight-asset class model. For the eight-asset class model, the beginning portfolio value for each asset class is set to $12,500 (12.5% of the total portfolio value). The two asset class model formulas are adjusted accordingly. For the 60% / 40% portfolio, the starting equity allocation is $60,000 and the initial bond allocation is $40,000. The value for t=0 is the initial portfolio value for each asset class at the beginning of month 1.

$$\mathrm{bh}(t, n) = \mathrm{bh}(t-1, n) * (1 + \mathrm{pct}(t, n)) \tag{1}$$

$$\mathrm{rb}(t, n) = \mathrm{rb}(t-1, n) * (1 + \mathrm{pct}(t, n)) \tag{2}$$

where t = 1 to 120 (10 years of monthly data), n is the number of asset classes and pct as defined in equation 4.

In the case of the RB portfolio, the year beginning portfolio value for each asset class needs to be reset back to the initial asset class proportions (traditional rebalancing) using the following equation:

$$\mathrm{rb}(\mathrm{t}) = \frac{1}{n}\sum_{i=1}^{n}(\mathrm{rb}(\mathrm{t,i})) \tag{3}$$

where n=8 and when t is 12, 24, 36, … 108 (a multiple of 12) but not reset for t=120.

The values for pct in equations 1 and 2 are created using the Box Muller transform. For each trial, a corresponding array of random asset class percent changes (Pct array) is created with eight columns corresponding to the eight asset classes and 120 rows corresponding to the months for 10 years.

$$\mathrm{pct}(\mathrm{t,n}) = \mu_n + \sigma_n\sqrt{-2\ln(\mathrm{U})} * \cos(2\pi\mathrm{V}) \tag{4}$$

where U is a single random number uniformly distributed in the interval (0,1) and where V is a single random number uniformly distributed in the interval (0,1) and $\mu_n$ = mean for n and $\sigma_n$ = standard deviation for n. The values for the mean and standard deviation are taken from the first eight rows listed in the table in Appendix A. The means are the values in the column labeled Expected Return and the standard deviations are the values in the column by the same name.

Computing the Sharpe ratios requires calculating the mean and standard deviation for the percent change in the total portfolio value for each month (t). The results of equations 6 and 8 are appended to each row of the Trial array.

$$\text{bhtot(t)} = \sum_{i=1}^{n} (\text{bh(t, i)}) \tag{5}$$

$$\text{bhchg(t)} = (\ \text{bhtot(t)} / \text{bhtot(t-1)}\ ) \text{ -1} \tag{6}$$

$$\text{rbtot(t)} = \sum_{i=1}^{n} (\text{rb(t, i)}) \tag{7}$$

$$\text{rbchg(t)} = (\ \text{rbtot(t)} / \text{rbtot(t-1)}\ ) \text{ -1} \tag{8}$$

At the completion of each trial loop, a summary of the Trail array is computed and appended to the Outcome array. A row of the Outcome array includes the final portfolio values for each asset class and both strategies (equations 9 and 10). The final values from equations 5 and 7 (t=120) are also included in the row.

$$\text{bh}(\zeta,\text{n}) = \text{bh(t,n), where t=120 and n = the 8 ending portfolio values} \tag{9}$$

$$\text{rb}(\zeta,\text{n}) = \text{bh(t,n), where t=120 and n = the 8 ending portfolio values} \tag{10}$$

The following equations calculate the variables required for the Sharpe calculations and are appended to each trial row of the Outcome array.

$$\text{bhmean}(\zeta) = \frac{1}{t} \sum_{i=1}^{t} (\text{bhchg(i)}) \tag{11}$$

$$\text{rbmean}(\zeta) = \frac{1}{t} \sum_{i=1}^{t} \left(\text{rbchg(i)}\right) \tag{12}$$

$$\text{bhstd}(\zeta) = \frac{1}{t} \sum_{i=1}^{t} \sqrt{(\text{bhchg(i)} - \text{bhmean(i)})^2} \tag{13}$$

$$\text{rbstd}(\zeta) = \frac{1}{t} \sum_{i=1}^{t} \sqrt{(\text{rbchg(i)} - \text{rbmean(i)})^2} \tag{14}$$

The Outcome array with all 5000 trials is output for post processing. Since the Sharpe ratios from the SCM are monthly, they are annualized using the following formulas:

$$SR_{BH}(\zeta) = 12 * (\text{bhmean}(\zeta) - \text{riskfree}(\zeta))/(\sqrt{12} * \text{bhstd}(\zeta)) \tag{15}$$

$$SR_{RB}(\zeta) = 12 * (\text{rbmean}(\zeta) - \text{riskfree}(\zeta))/(\sqrt{12} * \text{rbstd}(\zeta)) \tag{16}$$

## III. Validating the SCM

Dichtl, H., Drobetz, W., & Wambach, M. (2016) performed extensive analysis of numerous portfolio strategies including BH and nine different RB strategies. They ran 1000 simulations of each strategy using a history-based bootstrap method of Politis and Romano (1994). This bootstrap method randomly selects short segments of actual historical results and is intended to preserve time series properties not present in typical stochastic models. The descriptive statistics of their two asset classes are shown in Table 1 which are the ones applied to the SCM for validation.

**Table 1: Asset Class Financial Statistics Used by Dichtl, et al.**

| | Mean | Standard Deviation |
|---|---|---|
| Stocks | 10.45% | 15.77% |
| Government Bonds | 8.57% | 7.91% |
| Cash | 4.46% | 0.77% |

The first two rows of Table 2 compare the Sharpe ratios from Dichtl's BH and 60% - 40% RB portfolio to the those obtained from the SCM. It indicates that the results are very similar even though one is based on historical patterns and the other on simulated patterns. The third row compares the cross correlation between stocks and bonds for both research efforts.

**Table 2: Comparison of SCM to Dichtl, et al. Results for 60% / 40% Portfolio**

| 60% - 40% Portfolio | Dichtl, et al. | Stochastic Model |
|---|---|---|
| Buy & Hold | .552 | .510 |
| Annual Rebalancing | .579 | .529 |
| Stock/Bonds Correlation | .039 | .0007 |

A test for robustness using a 20% stock and 80% bond portfolio is shown in Table 3. Two numbers are shown for the SCM to test the repeatability using a different set of random numbers. Once again the results are quite similar.

**Table 3: Comparison of SCM to Dichtl, et al. Results for 20% / 80% Portfolio**

| 20% Stocks - 80% Bonds | Dichtl, et al. | Stochastic Model |
|---|---|---|
| Buy & Hold | .668 | .634/.637 |
| Annual Rebalancing | .669 | .638/.642 |
| Stock/Bonds Correlation | .039 | .0007/.0005 |

As reported by Dichtl, et al., many of the results for the 10% stock / 90% bonds and 20% stock / 80% bonds portfolios favored BH over RB strategies, however beyond a 20% equity allocation, RB consistently outperformed BH. It is interesting to note that while they did not give specific statistics for each of the 1000 simulations, Table 4 (joint probability table) provides detailed analysis of the 5000 trials from the SCM and demonstrates why researchers differ on which strategy outperforms. For example, in 2851 of the trials the RB ending portfolio value is greater than the BH portfolio AND the RB Sharpe ratio for the RB portfolio is better than the Sharpe ratio for BH. BH outperforms RB on both dimension for 843 trials of 5000 simulations. There are 926 trials where the BH ending value exceeds the RB portfolio but the RB Sharpe ratio is better and 380 trials where the RB ending portfolio value is greater than BH, but the BH portfolio has a better Sharpe ratio.

**Table 4: Specific Details of the 60%/40% Portfolio Ending Values and Sharpe Ratios**

| 60% Stocks - 40% Bonds | | Sharpe Ratio (Frequency / % of Total) | | |
|---|---|---|---|---|
| | | RB>BH | BH>RB | Total |
| Ending | RB>BH | 2851 / 57.0% | 380 / 7.6% | 3231/ 64.6% |
| Portfolio | BH>RB | 926 / 18.5% | 843 / 16.9% | 1769 / 35.4% |
| Value | Total | 3777 / 75.5% | 1223 / 24.5% | 5000 / 100.0% |

Dichtl et al. (2016) state, "Our results show statistical evidence that rebalancing significantly outperforms buy-and-hold if the portfolio weight of stocks exceeds a certain threshold." The threshold is 20% stocks / 80% bonds. So the 60% stocks / 40% bonds portfolio significantly exceeds the threshold. However, drilling down into the details with SCM paints a more nuanced perspective on performance. Essentially RB outperforms BH 57.0% of the time on a risk adjusted basis, but BH outperforms 16.9% of the time on a risk adjusted basis. In the next section, the SCM is applied to an all-equity portfolio with eight diverse asset classes.

## IV. Comparative Results for an All-Equity Portfolio

Application of the SCM to an all-equity asset class portfolio begins with two portfolios (BH and RB) each with a beginning balance of $100,000 equally distributed among eight equity asset classes with $12,500 initially invested in each asset class. The asset classes, their historical returns and standard deviations are the first eight rows of the table listed in Appendix A, specifically: US large cap growth, US large cap value, US mid-cap growth, US mid-cap value, US small cap growth, US small cap value, non-US developed market stocks and non-US emerging market stocks.

These asset classes were chosen for two reasons. First, they encompass a wide range of investment alternatives of non-overlapping equities. Second, allocating equal dollar amounts in each asset class helps to ensure a high degree of portfolio diversification that would not be obtained using common indexes such as the S&P 500 or the Russell 3000, both of which are market capitalization weighted.

The current iteration of the SCM does not model serial or cross correlation. Examining the random values over short periods, there are numerous times when the values exhibit some serial correlation by chance even though this was not specifically modeled. Cross correlation should have minimal impact on results since the identical random variables are applied to both portfolios at the same point in time.

This research assumes no transaction costs or the tax implication of rebalancing. The transaction cost assumption is now moot since most of the larger brokerage firms no longer charge commissions on stock and bond transactions. If the portfolios being analyzed are in a taxed deferred account then taxes have no impact on the performance of RB relative to BH. For taxable accounts however, the performance of the RB portfolio is impacted by the annual net profits or losses. Mattei (2018), required eight assumptions to calculate a reasonable estimate of the impact of taxes on the RB portfolio. He found that over the twenty-year period from 1997-2016, the annual return on the RB portfolio was 7.57% before taxes and 7.31% after taxes.

The modelling process follows equations 1 through 14 beginning with setting a loop counter to run 5000 separate trials. Table 5 depicts the basic summary statistics comparing the performance of the BH and RB strategies for the simulated futures. The average portfolio value at the end of 10 years for the BH approach is $303,718 and $298,703 for RB. The range for BH is $107,554 to $1,182,630 while for RB the range is $109,422 to $728,328.

The average compound annual growth rate for BH is 11.3% and 11.2% for RB. The CAGR is calculated by dividing the ending portfolio value for each trial and each strategy by $100,000 (the starting value of the portfolio), then taking the 10$^{th}$ root and subtracting 1. The averages shown in Table 5 are the arithmetic averages of the 5000 CAGRs for each strategy. These results indicate that BH generally performs somewhat better than RB on a non-risk adjusted basis, but not by a large amount except in a few cases of BH outliers.

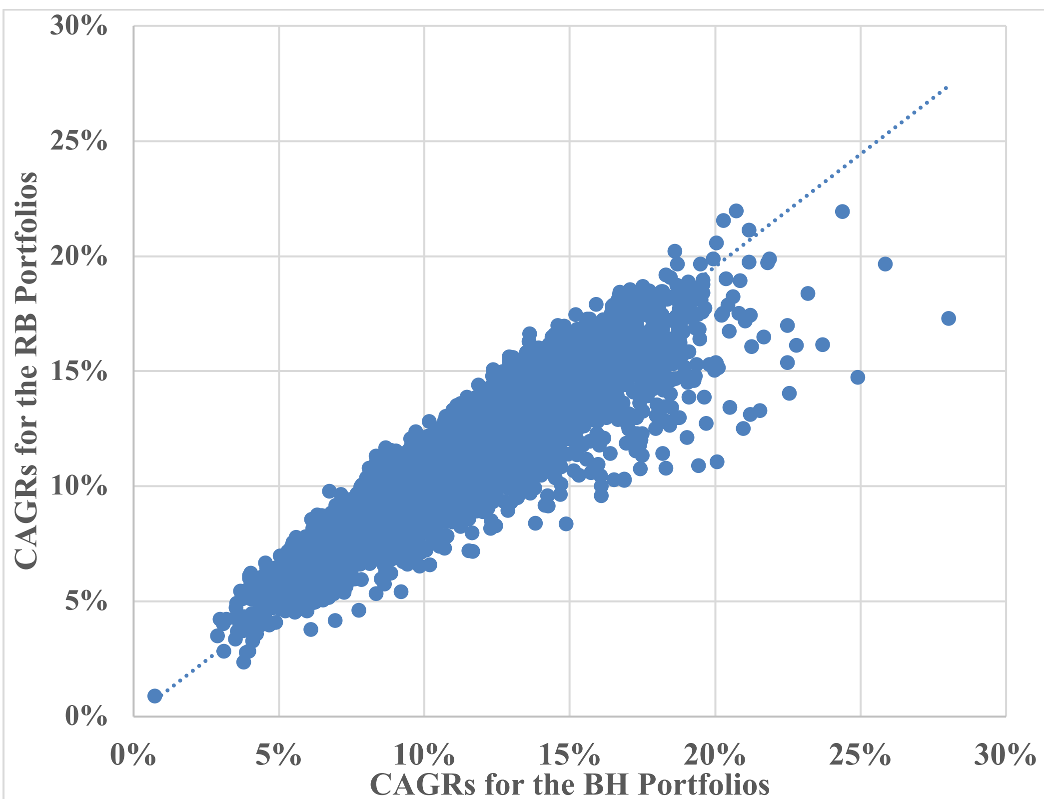
**Table 5: Summary Statistics of Ending Portfolio Values, CAGR and Sharpe Ratios for the Two Strategies**

| Statistic | Buy & Hold | Annual Rebalance |
|---|---|---|
| Average Ending Portfolio Value | $303,718 | $298,703 |
| Min Ending Portfolio Value | $107,554 | $109,422 |
| Max Ending Portfolio Value | $1,182,630 | $728,328 |
| | | |
| Average CAGR | 11.3% | 11.2% |
| Min CAGR | 0.7% | 0.9% |
| Max CAGR | 28.0% | 22.0% |
| | | |
| Average Sharpe Ratio | 1.108 | 1.221 |
| Min Sharpe Ratio | 0.010 | 0.029 |
| Max Sharpe Ratio | 2.323 | 2.452 |

Graphing the individual data points with the CAGR for the BH portfolios on the x-axis and the CAGR for the RB portfolios on the y-axis (similar to a QQ plot) provides a better insight into the performance of each strategy for each of the 5000 trials. Figure 1 is formatted so that the maximum value on the x-axis equals the maximum value on the y-axis to highlight how the values cluster around a trendline at approximately a 45° angle. If a point lands exactly on the trendline then the ending values for that trial are the same. Points above the trendline depict instances where the RB CAGR for a given trial exceeds the BH CAGR. Points below the trendline depict instances where the BH CAGR for a given trial exceeds the RB CAGR.

A linear regression of the data produces the equation: RB CAGR = .977 x BH CAGR with an $R^2 = .985$. The equation indicates that for every 1% increase in the BH CAGR, one can expect an approximately 0.977% increase in the corresponding RB CAGR. Therefore, the BH strategy will provide a slightly higher 10-year CAGR than the RB strategy on a non-risk adjusted basis. The chart also shows how rebalancing reduces variability of returns. The points above the regression line are tightly clustered on or above the regression line while there is a much higher degree of variability below and to the right of the regression line where the BH performance is better than the RB performance.

**Figure 1: Scatter Chart of the CAGR for BH versus RB Portfolios**




The most extreme outlier provides some noteworthy insights (the isolated point on the far right of the graph). The CAGR for BH is 28.02% and it is 17.31% for RB. The ending portfolio values are $1,182,602 and $493,548 respectively. The Sharpe ratios are quite close, 1.635 for BH and 1.685 for RB. While the BH portfolio has an ending value of more than twice the RB portfolio and the Sharpe ratios are quite close, the final asset class percentages are strikingly different (Table

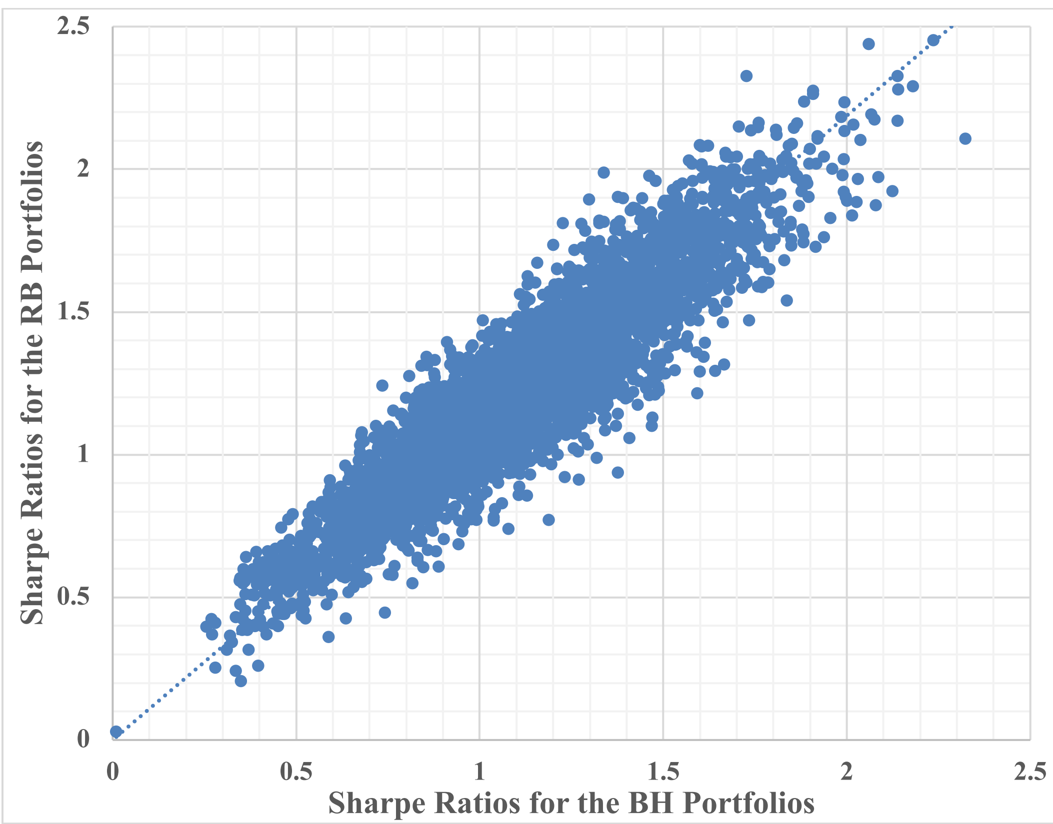
6). It should be noted that the RB portfolio is not rebalanced at the end of the final 12-month period so the values will not be exactly 12.5%. While there is a chance this outcome could occur in "real life," the simulation predicts that is will occur only once in 5000 futures or a .02% probability.

**Table 6: Ending Asset Class Percent of the Total Portfolio Ending Value for the Extreme Outlier Example**

| | Asset Class 1 | Asset Class 2 | Asset Class 3 | Asset Class 4 | Asset Class 5 | Asset Class 6 | Asset Class 7 | Asset Class 8 |
|---|---|---|---|---|---|---|---|---|
| BH | 1.0% | 1.6% | 2.5% | 1.6% | 1.6% | 8.5% | 8.9% | 74.4% |
| RB | 10.0% | 10.1% | 12.5% | 14.4% | 13.6% | 11.1% | 10.4% | 17.8% |

The Sharpe ratio chart (Figure 2) is also formatted so that the maximum value on the x-axis equals the maximum value on the y-axis to highlight how the values cluster around a trendline at approximately a 45° angle. If a point lands exactly on the trendline then the Sharpe ratios for that trial are the same. Points above the trendline depict instances where the RB Sharpe ratios for a given trial exceeds the BH Ratio. Points below the trendline depict instances where the BH Sharpe ratio for a given trial exceeds the RB Ratio.

**Figure 2: Scatter Chart of the Sharpe Ratios for BH versus RB**



A linear regression of the data points in Figure 2 produces the equation where the RB Sharpe ratio = 1.094 x BH Sharpe ratio with an $R^2 = .9886$. The equation indicates that for every 0.1 increase in the BH Sharpe ratio, one can expect an approximately .1094 increase in the corresponding RB Sharpe ratio. Therefore, while the BH strategy provides slightly higher CAGRs and ending portfolio values, the RB strategy provides slightly higher risk adjusted rates of return.

## V. Conclusions

Most authors agree that rebalancing a portfolio reduces risk, but it can result in reduced performance by selling high performing assets too soon. This has led some researchers such as Dai, TS., Chen, BJ., Sun, YJ. et al. (2024), Sandy Rattray, Nick Granger et al. (2020) and others to propose alternate rebalancing strategies to take advantage of "holding winners" longer. Essentially trying to capture some of the benefits of BH while also reducing the risk inherent in a BH strategy.

Table 7 of joint probability distributions (value on the right) provides a detailed breakdown of the performance differences between the two portfolio strategies by ending portfolio value and Sharpe ratios. The ending portfolio value for the RB portfolios is higher than BH 55.3% of the time. In all but 35 trials, the Sharpe ratio is higher for the RB portfolio. While the ending value for the BH portfolios are higher 44.7% of the time the Sharpe rations for the BH strategy is better than RB strategy only 17.6% of the time. These results seem to support the potential for alternate RB strategies that can capture some of the BH performance without the higher risks associated with asset concentration.

**Table 7: Specific Details of the Ending Portfolio Values and Corresponding Sharpe Ratios**

| 8 Equity Asset Classes | | Sharpe Ratio Frequency / % of Total | | |
|---|---|---|---|---|
| | | RB>BH | BH>RB | Total |
| Ending | RB>BH | 2730 / 54.6% | 35 / 0.7% | 2765 / 55.3% |
| Portfolio | BH>RB | 1357 / 27.1% | 878 / 17.6% | 2235 / 44.7% |
| Value | Total | 4087 / 81.7% | 913 / 18.3% | 5000 / 100.0% |

A closer look at the risks induced by asset class concentrations are shown in Table 8. For the BH portfolios, the asset class weights after 10 years are shown in the third column and the ending percentages for the "traditionally" rebalanced portfolio are in the fourth column. There is a very high correlation (.994) between the expected return of the asset class and its weight at the end of 10 years for the BH portfolio.

**Table 8: Ending Asset Class Weights for BH and RB Compared to Expected Returns**

| Asset Class | Expected Return | Average Ending Asset Class Weight in the Portfolio for the BH Strategy | Average Ending Asset Class Weight in the Portfolio for the RB Strategy |
|---|---|---|---|
| U.S. Large-cap Growth Stocks | 8.25% | 9.5% | 12.1% |
| U.S. Large-cap Value Stocks | 9.60% | 11.1% | 12.4% |

| | | | |
|---|---|---|---|
| U.S. Mid-cap Growth Stocks | 10.41% | 11.8% | 12.4% |
| U.S. Mid-cap Value Stocks | 12.48% | 14.2% | 12.7% |
| U.S. Small-cap Growth Stocks | 9.98% | 10.9% | 12.3% |
| U.S. Small-cap Value Stocks | 13.29% | 15.3% | 12.7% |
| Non-U.S. Dev Stocks | 10.02% | 11.5% | 12.4% |
| Non-U.S. Emerging Stocks | 14.35% | 15.8% | 12.9% |

The results of this research demonstrate that one cannot know which strategy will outperform the other even as the market is unfolding, but the traditional RB strategy outperforms BH 81.7% of the time on a risk adjusted basis. Over an 80% probability of outperformance strongly supports the RB strategy. However the results of this research also indicate that BH outperforms RB on a risk adjusted and ending portfolio value basis 17.6% of the time. This result strongly supports the use of non-traditional rebalancing strategies that can capture some of the benefits of a BH strategy without the inherent weaknesses.

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**Appendix A: Historical Returns for Common Asset Classes from Morningstar (2024)**

| Asset Class | Return Data Series (Benchmark) | Expected Return | Standard Deviation |
|---|---|---|---|
| U.S. Large-cap Growth Stocks | Russell Top 200 Growth | 8.25% | 21.82% |
| U.S. Large-cap Value Stocks | Russell Top 200 Value | 9.60% | 17.47% |
| U.S. Mid-cap Growth Stocks | Russell Midcap Growth | 10.41% | 23.22% |
| U.S. Mid-cap Value Stocks | Russell Midcap Value | 12.48% | 19.17% |
| U.S. Small-cap Growth Stocks | Russell 2000 Growth | 9.98% | 27.62% |
| U.S. Small-cap Value Stocks | Russell 2000 Value | 13.29% | 22.46% |
| Non-U.S. Dev Stocks | MSCI EAFE | 10.02% | 20.62% |
| Non-U.S. Emerging Stocks | MMSCI Emerging Mkts | 14.35% | 29.65% |
| U.S. Investment Grade Bonds | Barclays US Agg Bond TR USD | 3.36% | 7.08% |
| U.S. High-Yield Bonds | Barclay US Corp High Yield | 7.37% | 11.33% |
| Non-U.S. Dev Bonds | Citi WGBI Non-USD USD | 3.08% | 11.22% |
| Cash | Citi Treasury Bill 3 Mon USD | 0.97% | 1.67% |
| Commodities | DJ UBS Commodity TR | 4.48% | 17.86% |
| U.S. Real Estate | FTSE NAREIT-Equity | 8.92% | 23.55% |