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## **To the Members of the Academy of Finance and Readers of the Journal of Finance Issues,**

It is with great pleasure that I present Volume 23, Issue 3 of the Journal of Finance Issues. This issue features a diverse collection of research that delves into critical areas of finance, offering valuable insights into market dynamics, corporate governance, and financial forecasting. We are proud to bring you these contributions from leading scholars in the field.

This issue includes the following papers:

### **A Reliable Approach to Forecast Prices of Precious and Base Metals**

This manuscript presents a pricing model based on geometric Brownian motion simulation to forecast the prices of various precious and base metals. The study finds that one-period ahead forecasts, especially at monthly frequencies and with a higher number of simulations, are more reliable. It also highlights high correlations between GBM-based expected metal prices and actual prices.

### **Earnings Growth Forecast for ETFs**

This study forecasts earnings growth for stock-market-indexed Exchange-Traded Funds (ETFs) over the next five years. The authors utilize two methods: P/E and P/B cross-sectional regression-implied (RI) estimates and earnings growth random-walk (RW) estimates. The findings indicate that both RI and RW forecasts are unbiased for U.S. ETFs but biased for foreign ETFs when compared to actual earnings growth. Furthermore, the RI method generates smaller forecast errors than the RW method for U.S. ETFs, though it offers no advantage for foreign ETFs. The paper concludes that the RI forecast can be a useful method for U.S. ETFs during the 2000-2023 sample period but may not be suitable for foreign ETFs.

### **CEO Power, CEO Compensation, and Firm Performance**

This study examines the impact of CEO power (measured by CEO pay slice or CPS) on the relationship between CEO compensation (measured by pay-performance sensitivity or PPS) and firm performance. The results indicate that PPS increases firm performance at high levels of CPS, but its impact decreases at low CPS levels, suggesting an important interaction effect between CEO power and incentive compensation.

### **Corporate Social Responsibility (CSR) and Firm Features in the Immediate Aftermath of the COVID-19 Pandemic.**

This paper investigates the relationships between key firm features and Corporate Social Responsibility (CSR) scores (energy and water, ethics, customer service, labor, and governance) in the context of the COVID-19 pandemic and its aftermath. It finds that large market-cap and profitable firms are significantly associated with higher CSR scores, but these investments do not appear to lead to significant short-term changes in financial features like market capitalization, profitability, or growth.

### **The Relationship between Firm Growth, Ownership Structure and Performance: An analysis of U.S. property-liability insurers**

This study explores the relationship between firm growth and profitability in the U.S. insurance industry, focusing on differences between stock and mutual insurers. It examines how financial

performance and strategic priorities vary based on ownership structure, contributing insights into the determinants of profitability and growth in the insurance sector.

We extend our sincere gratitude to all the anonymous reviewers for their invaluable time, expertise, and thoughtful feedback, which significantly contributed to the quality of this issue. We also thank our dedicated associate editors, Larry, David, and Won, for their unwavering commitment to serving the Journal and the members of the Academy of Finance.

Sincerely,

Olgun Fuat Sahin  
Editor, Journal of Finance Issues  
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## A Reliable Approach to Forecast Prices of Precious and Base Metals

Amit Sinha\*

### Abstract

Getting reliable and trustworthy estimates of future metal prices is important. This manuscript applies a pricing model based on geometric Brownian motion simulation to test the reliability of expected price forecasts of silver, aluminum, copper, iridium, nickel, lead, palladium, platinum, rhodium, ruthenium, tin and zinc. Expected prices were estimated by totaling up the product of simulated prices and associated probabilities at the monthly, quarterly and annual frequencies, with historic mean and standard deviation based on a rolling twenty-year window as proxies for drift and diffusion. Results indicate that one-period ahead forecasts based on higher number of simulations are more reliable than those based on only one simulation. Besides monthly forecasts and quarterly forecasts may be more trustworthy than those at the annual frequency.

**Keywords:** geometric Brownian motion, forecasting, commodities, metals, probabilities, asset-pricing, normal distribution, expected value, mathematical methods, mathematical and simulation modeling, ensemble effect

**JEL Classifications:** C0, C6, G0, G12, G170

### I. Introduction

Metals are an essential ingredient in the worldwide economic engine, and accordingly accurate forecasts of future metal prices is of interest to economic agents (Liu et al. 2017). Forecasting metal prices is critically important for a country's production activities and industrial policies, as metals are indispensable raw materials (Kahraman and Akay 2023; Oikonomou and Damigos 2024; Zhao et al. 2023). Numerous manuscripts have been published studying factors that influence prices, developing and testing models (Chen et al. 2014; Dooley and Lenihan 2005; Du et al. 2021; Gil-Alana and Poza 2024) that can be used to obtain accurate forecasts for investment or trading decisions. This manuscript contributes to the academic literature on modeling metal prices by applying a commonly used Econophysics (Sinha 2024e) technique called geometric Brownian motion (GBM), to obtain reliable one-period ahead forecasts of prices of six base, and six precious metals.

An exhaustive literature exists that have applied GBM to forecast asset prices. (Parungrojratt and Kidsom 2019; Reddy and Clinton 2016; Samuelson 1965; Sinha 2021, 2024a; Urama and Ezepue 2018) have applied GBM to forecast stock prices or stock indexes. (Alhagyan 2024; Lynch et al. 2021) priced energy assets using GBM. Similarly, a few of the many have applied GBM forecasting to agricultural products (Ibrahim, Misiran, and Laham 2021; Zelingher and Makowski 2024), and exchange rates (Abbas and Alhagyan 2023; Alhagyan 2022; Farzin, Moghaddam, and Shahbalaee 2024). Although only a few (Black and Scholes 1973; Germansah, Tjahjana, and Herdiana 2023; Stojkoski et al. 2020; Veestraeten 2013) are being mentioned here, some aspect of GBM modeling has been applied to derivatives securities. GBM modeling has been

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applied to metal pricing by (Hamdan, Ibrahim, and Mustafa 2020; Huang et al. 2022; Ramos et al. 2019; Roslan and Halim 2024; Soysal 2023; Wilmot 2019), as well although the approach used in this manuscript differs from these.

Typically, GBM modeling requires the application of a solution to a differential equation represented in equations (2) and (6) of this manuscript. These equations have an observed price that is used to make one period ahead forecast using a drift and diffusion terms, and a standard normal random variable that has a mean of zero and a standard deviation of one. Although, the random variable can take on numerous values, allowing for multiple one period ahead forecasts, usually an insufficient number of simulations are obtained (Kumar et al. 2024; Sinha 2021). If numerous forecasts are simulated, the issue of importance becomes as to which forecast to use. (Sinha 2021) addresses this issue by estimating an expected value from the all the simulated forecasts. As simulated values have associated probabilities, expected values from the simulated forecasts can be obtained, and compared to actual prices. (Sinha 2021, 2024f, 2024a) applied a similar methodology to forecast stock index values, and gold price forecasts (Sinha 2024d). In this manuscript, the methodology is applied to six precious metals, silver, iridium, palladium, platinum and rhodium, and six base metals, aluminum, copper, nickel, lead, tin and zinc at the monthly, quarterly, and annual frequencies, and simulation that vary from one, to one hundred thousand.

Modeling prices or return behavior of these metals is prevalent and continued interest the literature. The motivation to apply the methodology to precious metals (Askari and Askari 2011; Barczak 2014; Jalali and Heidari 2018; Oral and Unal 2019; Pierdzioch and Risse 2020; Tapia Cortez et al. 2018; Wen et al. 2017) and base metals (Dooley and Lenihan 2005; Kahraman and Akay 2023; Kriechbaumer et al. 2014; Oikonomou and Damigos 2024; Pincheira Brown and Hardy 2019; Tapia Cortez et al. 2018; Wang et al. 2021; Zhao et al. 2023) arise from the fact that these have been extensively modeled using different methodologies. Although a few manuscripts have applied GBM to obtain forecast of metal prices, this manuscript contributes to the literature by applying the methodology developed by (Sinha 2021) where expected index value of the S&P 500 were estimated using numerous GBM simulations. A similar methodology was also applied to model gold prices by (Sinha 2024d), and in this manuscript, the approach is applied model the prices of an additional six base and precious metals.

Results indicate that reliable and accurate forecasts for metal prices can be obtained especially for the monthly frequencies, at higher number of simulations. The GBM based expected metal prices estimated using one hundred thousand simulations, and the actual metal price for all twelve metals show very high correlations at the monthly frequency. Although the correlations decrease in the quarterly and annual frequencies, they are still very high at the quarterly level, and greater than 0.5 for eight of the twelve metals at the annual frequency. Regression results show good forecasting ability of the expected GBM metal price, especially at the monthly and quarterly frequencies, and for some of the metals at the annual frequency. Tests for differences in means and standard deviations between actual and expected metal prices show a similar pattern for the monthly, quarterly and annual frequencies. Results also indicate that if the expected values are estimated using only one simulation, then those values have very little if any predictable power. GBM based expected values estimated using at least one hundred simulations are found to have usefulness that may mirror those estimated using even higher number of simulations.

In this manuscript, literature review is section II. Section III is the data, while section IV discusses the methodology. The results are in section V, and section VI is the conclusion.

## II. Literature Review

Developing accurate models to forecast metal prices is of prime importance as even minor enhancements in prediction accuracy can result in enormous benefits (Alameer et al. 2019; Du et al. 2021). Accordingly, a wide variety of models of different degrees of complexity have been developed and applied to forecast the price of a number of different types of metals. For example, (Schwartz 1997) applied three different stochastic differential models along with a Kalman-filter methodology to understand the behavior of crude oil, copper and gold. (Achireko and Ansong 2000) used a three-step process that included multivariate, normally distributed random variable generator, neural networks, and multiple regression analysis to model gold prices. (Chen et al. 2014) applied the PANIC method, namely the Panel Analysis of Non-stationarity in Idiosyncratic and Common Components, a procedure developed by (Bai and Ng 2004) to identify factors that influenced prices of fifty-one commodities that included metals like aluminum, copper, zinc, iron ore, lead, tin, nickel and uranium.

(Dooley and Lenihan 2005) applied an Autoregressive Integrated Moving Average (ARIMA) model to forecasts of lead and zinc prices. (Askari and Askari 2011) use the Grey model for gold price forecasting, while (Barczak 2014) compared the Grey, Holt's exponential smoothing and ARIMA model while forecasting gold prices. (Alipour, Khodaiari, and Jafari 2019) compare forecasts of ARIMA and threshold generalized autoregressive conditional heteroskedastic model (TGARCH) time-series models with models based on Stochastic Differential Equations (SDE) while predicting monthly copper prices. (García and Kristjanpoller 2019) developed a hybrid and non-hybrid approach to copper price volatility that included ARIMA, GARCH and artificial neural networks and fuzzy inference systems (FIS). (Hu, Ni, and Wen 2020) integrate Generalized Autoregressive Conditional Heteroskedasticity (GARCH) modeling and artificial neural networks to model copper price volatility predictions. (Sánchez Lasheras et al. 2015) compared the forecasts of copper prices using neural networks and ARIMA models, and found the neural networks approach to provide superior results. (Wen et al. 2017) applied different algorithms in a support vector machine (SVM) and artificial neural networks to forecast gold prices. (Wang et al. 2019) applied a price volatility network and artificial neural networks to predict copper spot prices.

(Maranon 2018) applied band-pass filters and Elliott Wave Principle to analyze and model metal price cycles. (Tapia Cortez et al. 2018) combined Chaos Theory and machine learning techniques to forecast prices of metals like, aluminum, copper, lead, nickel, silver, zinc, gold, tin, palladium, and platinum among other minerals. (Oral and Unal 2019) applied a vector autoregressive fractionally integrated moving average (VARFIMA) model to estimated local Hurst exponents, along with wavelet coherence analysis to daily prices of gold, silver and platinum. (Risse 2019) forecasts gold returns by combining wavelet decomposition and machine learning. (Kriechbaumer et al. 2014) applied wavelet-based multiresolution analysis preceding an ARIMA fitting to increase forecasting accuracy in the monthly forecasts of aluminum, copper, lead and zinc. (He et al. 2015) used a curvelet based multiscale methodology to forecast metal prices.

(Chen, He, and Zhang 2016) use a novel Grey wave forecasting approach that requires unequal-interval contour lines, and contour time filtrating to predict prices of aluminum and nickel at the monthly frequency. (Gligorić et al. 2020) applied a hybrid model based on Grey System Theory, and a stochastic differential model to monthly lead prices. (Ozdemir, Buluş, and Zor 2022) applied deep learning algorithms that required recurrent neural networks-based on long short-term memory (LSTM) and gated recurrent unit (GRU) networks to forecasted medium- and long-term horizon nickel prices. (Li et al. 2020) develop the group method of data handling (GMDH)

intelligence system to predict the price of iron ore, and compared the results of the technique to ARIMA, support vector regression (SVR), artificial neural network (ANN), and classification and regression tree (CART) approaches.

(Jalali and Heidari 2018) applies the Grey GM(1,1) model to forecast palladium prices. (Pierdzioch and Risse 2020) use a multivariate random forests approach to compute out of sample forecasts of four precious metals namely gold, silver, platinum and palladium. (Du et al. 2021) apply an optimized extreme machine learning and nature inspired meta-heuristic algorithms to predict copper and gold prices. Based on a hybrid prediction framework approach, (Wang et al. 2021) looked at point and interval predictions of daily future prices of zinc, copper and lead. (Gil-Alana and Poza 2024) applied fractional integration techniques to analysis volatility persistence in the prices of gold, silver, platinum, aluminum, palladium, lead, zinc and tin. (Jermann 2024) no-arbitrage model based on minimal structural assumptions was able to capture key patterns in the gold futures prices.

Using monthly price data from the World Bank database on commodity prices, (Oikonomou and Damigos 2024) model the prices of aluminum, copper, lead, tin, nickel and zinc by applying an autoregressive Light Gradient Boosting Machine trained to only use lags of the time series to produce forecasts both as a standalone and as part of an ensemble. (Pincheira Brown and Hardy 2019) attempted to predict the returns of six base metals, namely aluminum, copper, lead, nickel, tin and zinc using the Chilean exchange rate and found evidence of predictability. (Kahraman and Akay 2023) also study the prices of base metals, specifically aluminum, copper, lead, iron, nickel, tin, and zinc, by applying numerous exponential smoothing methods. (Zhao et al. 2023) model copper and aluminum prices using a novel hybrid approach that used both interval and point forecasting. While the literature on different models used to forecast accurate metal prices, is vast and exhaustive, another approach is the application of GBM.

$$\frac{dP}{P} = \mu dt + \sigma dW_t \quad (1)$$

The biologist Robert Brown discovered Brownian motion in 1827 (Maruddani and Trimono 2018) while observing pollen grains under a microscope, and (Bachelier 1900) is widely credited for being the first for applying it to understand the price movement of stocks. (Einstein 1905) provide a stochastic differential solution, and equation (1), is widely used to represent GBM (Benninga 2014; Musiela and Rutkowski 2005; Navin 2007; Sengupta 2004). In this equation,  $dW_t$  is the standard Weiner variable with a mean of zero, and a standard deviation of 1, while  $\mu$  and  $\sigma$ , are the drift and diffusion terms respectively. The change in time is represented by  $dt$ , while  $P$  is the price of the asset, and  $dP$  the change in price.

An implementable close form solution of equation (1), and widely used to price assets is represented by equation (2) (Benninga 2014; Benninga and Mofkadi 2021; Hull 2018; Reddy and Clinton 2016).  $P_t$  and  $P_0$  are prices at time  $t$  and 0 respectively. The exponential term is represented by  $e$ , while  $\Delta t$  is the change in time. The drift and diffusion terms usually measured by historical mean and standard deviation is represented by  $\mu$  and  $\sigma$  respectively, while  $\varepsilon$  a normally distributed random variable that has an average of zero, and a standard deviation of one.

$$P_t = P_0 e^{\left( \left( \mu_t - \frac{1}{2}\sigma_t^2 \right) \Delta t + (\sigma_t \varepsilon \sqrt{\Delta t}) \right)} \quad (2)$$

An exhaustive literature exists in the application of GBM to understand price movements of stocks and stock indexes (Feng et al. 2021; Kayal and Maheswaran 2018; Paluszek and Thomas 2020; Parungrojrat and Kidsom 2019; Reddy and Clinton 2016; Samuelson 1965, 1973; Sinha 2021, 2024a; Urama and Ezepeue 2018). The GBM framework has also been applied to price energy assets (Al-Harthy 2007; Alhagyan 2024; Croghan, Jackman, and Min 2017; Lynch et al. 2021; Postali and Picchetti 2006), agricultural products (Ibrahim et al. 2021; Salaudeen et al. 2023; Zelingher and Makowski 2024), exchange rates (Abbas and Alhagyan 2023; Alhagyan 2022), and derivatives instruments (Black and Scholes 1973; Germansah et al. 2023), and metals and minerals prices (Hamdan et al. 2020; Ramos et al. 2019; Roslan and Halim 2024).

(Hamdan et al. 2020) modelled Malaysian gold prices using GBM. Using daily prices for the *Kijang Emas* prices obtained from the website of Bank Negara Malaysia for the period between January 4, 2016 and December 30, 2016 they model daily observations using historical means and standard deviations as proxies for the drift and diffusion terms respectively. Using data obtained from the Index Mundi database ([www.indexmundi.com](http://www.indexmundi.com)), (Ramos et al. 2019) modeled iron-ore prices by applying GBM and Monte Carlo simulation. (Roslan and Halim 2024) forecast world gold prices for the year 2022 using GBM. (Huang et al. 2022) compared the gold price forecasts using a mean reversion process, GBM, and a time series modeling. (Soysal 2023) applied fractional GBM while investigating super cycles in silver prices between 2005 and 2023. Applying GBM to the prices of gold, silver, copper, zinc and platinum, (Wilmot 2019) suggests that prices should follow a more complex process than GBM.

Summarizing, a few manuscripts have used GBM to study and forecast metals and minerals prices, and this manuscript contributes to the literature by applying the GBM simulations to prices of six base metals, aluminum, copper, nickel lead, tin and zinc, and six precious metal prices, silver, iridium, palladium, platinum, rhodium and ruthenium. Another contribution made in this manuscript is that the price forecasts are not based on a single GBM forecast, but rather simulates an universe of possible forecasts. Using the probabilities of each forecast, expected values are estimated by totaling up the product of the simulated price and its associated probability. This process is similar to ensemble forecasting (Feng et al. 2021), or ensemble averaging (Cherstvy et al. 2021; Semmlow 2018; Vinod et al. 2022), whereby multiple observations are averaged to obtain reliable solution. A similar process is used in (Sinha 2024d) to gold prices.

### III. Data

The data in this manuscript was extracted from DataStream through Refinitiv, and comprises of prices of metals at the yearly, quarterly and monthly frequencies. The prices of the following twelve metals: silver, aluminum, copper, iridium, nickel, lead, palladium, platinum, rhodium, ruthenium, tin and zine were extracted. *Table 1* provides basic information about these metals, including their chemical symbols. The chemical symbols are not necessary for the analysis carried out in this manuscript but were used to sort the data, while running the programing codes, and accordingly have been presented in the table. The mnemonic used to extract metal prices are also presented as well as the first date of availability of the metal in DataStream. The classification of metals into precious metals and metals is based on that Refinitiv's classification. Six of theses, silver, iridium, palladium, platinum, rhodium, and ruthenium, are classified as precious metals while aluminum, copper, nickel, lead, tin and zinc are classified just as metals, or base metals. This classification into metals and precious metals is rather generic with precious metals being those that are of high economic value and comparatively rare. The prices of precious metals are reported

**Table 1: Data Sources and Basic Information**

<b>Metal</b>	<b>Symbol</b>	<b>Mnemonic</b>	<b>Type</b>	<b>Availability</b>	<b>Source</b>	<b>Unit</b>	<b>Price Quote</b>
Silver	Ag	SILVERH	Precious Metals	1/2/1979	Handy & Harman	Troy Ounce	USD per oz
Aluminum	Al	LAHCASH	Metals	1/31/1957	London Metal Exchange	Metric Ton	USD per ton
Copper	Cu	LCPCASH	Metals	1/31/1957	London Metal Exchange	Metric Ton	USD per ton
Iridium	Ir	JMIRIAS	Precious Metals	7/1/1992	Johnson Matthey	Troy Ounce	USD per oz
Nickel	Ni	LNI3MTH	Metals	4/23/1979	London Metal Exchange	Metric Ton	USD per ton
Lead	Pb	LEDCASH	Metals	7/5/1993	London Metal Exchange	Metric Ton	USD per ton
Palladium	Pd	PALLADM	Precious Metals	1/5/1987	London Metal Exchange	Troy Ounce	USD per oz
Platinum	Pt	PLATFRE	Precious Metals	1/2/1976	London Metal Exchange	Troy Ounce	USD per oz
Rhodium	Rh	JMRHOUS	Precious Metals	1/31/1980	Johnson Matthey	Troy Ounce	USD per oz
Ruthenium	Ru	JMRUTEU	Precious Metals	7/1/1992	Johnson Matthey	Troy Ounce	USD per oz
Tin	Sn	LTICASH	Metals	1/31/1957	London Metal Exchange	Metric Ton	USD per ton
Zinc	Zn	LZZCASH	Metals	1/31/1957	London Metal Exchange	Metric Ton	USD per ton

The availability column shows the first date from which pricing data is available in Refinitiv DataStream. The type column provides the classification as Metals or Precious Metals. The symbol is the chemical notation typically used for the corresponding metals. The DataStream mnemonic used for data extraction, the unit of weight, and price quote, as well as DataStream's source for price data are also provided.

**Table 2: Select Days and Prices**

<b>Metal</b>	<b>First Day Values</b>		<b>Maximum Values</b>		<b>Minimum Values</b>		<b>Last Day Values</b>	
	<b>Date</b>	<b>Value</b>	<b>Date</b>	<b>Value</b>	<b>Date</b>	<b>Value</b>	<b>Date</b>	<b>Value (\$)</b>
Silver	12/30/1983	8.95	4/29/2011	48.55	2/25/1993	3.54	12/29/2023	24.25
Aluminum	12/30/1983	1,549.86	3/4/2022	3,877.50	11/29/1985	950.20	12/29/2023	2,345.50
Copper	12/30/1983	1,415.40	10/18/2021	11,299.50	10/31/1984	1,272.10	12/29/2023	8,463.92
Iridium	7/1/1992	200.00	4/28/2021	6,300.00	6/1/1995	60.00	12/29/2023	5,000.00
Nickel	12/30/1983	4,842.00	5/4/2007	51,600.00	1/12/1987	3,423.11	12/29/2023	16,603.00
Lead	7/5/1993	385.50	10/10/2007	3,989.00	10/5/1993	357.00	12/29/2023	2,034.50
Palladium	1/5/1987	118.25	3/7/2022	3,015.00	8/16/1991	78.25	12/29/2023	1,119.00
Platinum	12/30/1983	391.82	3/4/2008	2,273.00	3/11/1985	244.25	12/29/2023	1,006.00
Rhodium	12/30/1983	350.00	3/22/2021	29,800.00	1/30/1997	200.00	12/29/2023	4,425.00
Ruthenium	7/1/1992	32.00	2/9/2007	870.00	8/18/1993	18.00	12/29/2023	450.00
Tin	12/30/1983	12,341.50	3/8/2022	48,865.00	2/19/2002	3,601.00	12/29/2023	25,175.00
Zinc	12/30/1983	857.60	11/24/2006	4,603.00	11/29/1985	597.46	12/29/2023	2,640.00

The first and last day of the metal and the prices on those days are presented in this table. The last day for all the metals is the same, but the first day is the same for only eight of the metals. The maximum and minimum prices along with the dates are also presented.

**Table 3: Descriptive Statistics**

Metal	Annual				Quarterly				Monthly			
	Mean	Stdev	Skew	Kurt	Mean	Stdev	Skew	Kurt	Mean	Stdev	Skew	Kurt
Silver	2.5%	21.8%	0.42	0.15	0.6%	11.6%	-0.07	0.77	0.2%	7.8%	0.02	1.31
Aluminum	1.0%	24.2%	0.37	0.12	0.3%	11.3%	-0.41	2.04	0.1%	6.6%	0.05	2.53
Copper	4.5%	31.5%	0.54	1.69	1.1%	14.0%	-0.59	5.75	0.4%	7.3%	-0.34	5.21
Iridium	11.9%	44.8%	-0.95	1.65	2.7%	20.5%	1.43	5.94	0.9%	9.1%	2.38	14.80
Lead	4.9%	31.1%	-0.22	3.39	1.4%	14.0%	-0.63	3.00	0.5%	7.8%	-0.32	1.65
Nickel	3.1%	41.5%	0.38	0.03	0.8%	17.9%	0.60	0.53	0.3%	9.7%	0.35	2.53
Palladium	6.1%	38.8%	-0.27	-0.21	1.5%	16.4%	-1.21	4.84	0.5%	9.4%	-0.18	2.50
Platinum	2.4%	20.6%	-0.31	0.41	0.6%	11.2%	-1.70	10.52	0.2%	6.4%	-0.41	4.07
Rhodium	6.3%	65.9%	-0.44	-0.10	1.6%	27.0%	-0.31	3.26	0.5%	13.7%	0.17	9.84
Ruthenium	9.3%	65.3%	0.81	2.23	2.1%	26.8%	0.84	5.55	0.7%	11.8%	1.41	10.55
Tin	1.8%	26.7%	0.55	-0.10	0.4%	13.2%	-0.54	1.64	0.1%	6.8%	-0.21	2.47
Zinc	2.8%	31.5%	0.43	0.86	0.7%	13.0%	-0.05	-0.01	0.2%	7.5%	-0.37	1.77

The mean, standard deviation (stdev), skewness (skew), and kurtosis (kurt) of the log-returns of the twelve metals are presented. These are calculated for the sample period between when the pricing information becomes available to the last day of the sample period. The descriptive statistics for all three frequencies: annual, quarterly and monthly, are presented.

on troy ounces, where an ounce is approximately 31.10 grams, while those of base metals are on metric tons with a metric ton equal to one thousand kilograms. The source used by DataStream to obtain the metal prices are also presented in *Table 1*.

The data extracted for the manuscript was between December 30, 1983 and December 29, 2023 giving a forty-year sample period once log-returns are calculated. As can be seen in *Table 2*, the prices of eight metals, namely silver, aluminum, copper, nickel, platinum, rhodium, tin and zinc were available for the whole sample period. The first availability of iridium and ruthenium is from July 1, 1992, while those for lead and palladium were from July, 5, 1993 and January 5, 1987 respectively. The table also presents the observed prices of the metals on the first and last day of the sample period. The maximum and minimum prices along with the associated dates are also provided in the table. While presenting the maximum and minimum prices, only the dates for the first occurrence has been presented if those prices were observed on multiple days.

## IV. Methodology

### Preliminary Estimations

The returns are calculated by taking the natural log of the division of a metal price by the immediately preceding price, as shown in equation (3). In this equation,  $P_{t,i}$  represents the price of metal  $i$  at time  $t$ , while  $P_{t-1,i}$  is the preceding price of metal  $i$ . The returns were calculated for all twelve metals and for all three frequencies, and were used to estimate the drift and diffusion terms used in the GBM simulation as represented in equations (2) and (6).

$$r_{t,i} = \ln\left(\frac{P_{t,i}}{P_{t-1,i}}\right) \quad (3)$$

The log-returns estimated using equation (3) were used to estimate the mean, standard deviation, skewness and kurtosis for the whole sample period and are presented as descriptive statistics in Table 3. The formulae used to estimate are standard statistical formulae, and are estimated at the annual, quarterly, and monthly frequencies. The highest average return at the annual frequency is observed for iridium at 11.9%, while the lowest is aluminum at 1.0%. The range for standard deviation is from a low of 20.6% for platinum to a high of 65.9% for rhodium, although ruthenium also has a very high standard deviation at 65.3%. Five of the twelve metals show negative skewness. For kurtosis, both positive and negative numbers are observed making some of the distribution platykurtic, and others leptokurtic. The kurtosis values for some of the metals are observed to be close to zero. Iridium is also observed to have the highest average returns, at the quarterly and monthly frequency, while aluminum has the lowest average returns at the quarterly frequency. Aluminum and tin have the lowest returns at 0.1% at the monthly frequency. The skewness numbers at the quarterly frequency show that the returns of more metals are negatively skewed than positively skewed. Besides, eleven of the twelve metals are leptokurtic, with tin being platykurtic, although the number is very close to zero at 0.1. At the monthly frequency, some metals show negative skewness, and others show positive skewness, and all metals show leptokurtosis. Numerically, the monthly frequency numbers for kurtosis of some metals are larger than those at the annual and quarterly frequencies. As the GBM modeling process is mostly dependent on the first two statistical moments, namely drift and diffusion, the impact of

skewness and kurtosis on the efficacy of the simulation has not being explored in this manuscript.

### GBM Simulations and Expected Value Estimation

The drift ( $\mu_{t,i}$ ) and diffusion ( $\sigma_{t,i}$ ) term are proxied by the estimating average returns and standard deviation within a rolling window period, that are based on a twenty-year period as shown in equations (4) and (5) respectively.  $n$  has a value of twenty, eighty, and two hundred and forty at the yearly, quarterly and monthly frequencies. These formulae are standard formulae for estimating averages and standard deviations.

$$\mu_{t,i} = \frac{\sum_1^n r_i}{n} \quad (4)$$

$$\sigma_{t,i} = \sqrt{\frac{\sum_1^n (r_i - \bar{r}_i)^2}{n-1}} \quad (5)$$

One thing to note here is that although the initial data sample is a forty-years, the first twenty years of the data constitutes the first estimation window for the drift and diffusion terms as shown in equations (4) and (5). However, as mentioned in section III and shown in *Table 2*, not all metals have the same starting date in the sample. Due to data availability, eight of the twelve metals, silver, aluminum, copper, nickel, platinum, rhodium, tin and zinc, have their first window based on twenty-years of data. For palladium, the first window drift and diffusion terms were estimated using sixteen years of data, while for ruthenium and iridium the first sample was based on eleven years of data. For lead, it was ten. In the rolling window, the one-periods data was dropped at the back-end, and one added at the front end. For the four metals, with less than twenty-years data for the first window, the sample principal was applied, although data was added at the front end, and the back-end data were excluded only after the window reached twenty-years of data. The twenty-year rolling window ensures twenty-years of data at the annual frequency, eighty-quarters at the quarterly frequency, and two hundred and forty months at the monthly frequency. A similar approach was used in (Sinha 2021, 2024a, 2024b, 2024d, 2024f).

The GBM formula used in this manuscript, and shown in equation (6), has been used previously in a number of published articles and books (Benninga and Mofkadi 2021; Hull 2018; Maruddani and Trimono 2018; Musiela and Rutkowski 2005; Navin 2007; Reddy and Clinton 2016; Sinha 2024f, 2021, 2024a, 2024b, 2024d). In equation (6),  $P_{t,i}$  is the price of metal  $i$  at time  $t$ , while  $P_{sim,t+1,i}$  is the simulated price at time  $t+1$ , with  $\Delta t$  taking a value of one, for one-period ahead forecasts.  $\mu_{t,i}$  and  $\sigma_{t,i}$  are the drift and diffusion terms for metal  $i$  estimated using equations (4) and (5) respectively, at time  $t$ . The letter  $e$  represents the exponential, and  $\varepsilon$  is a standard normal Weiner variable with a mean of zero and a standard deviation of 1.

$$P_{sim,t+1,i} = P_{t,i} e^{\left( (\mu_{t,i} - \frac{1}{2}\sigma_{t,i}^2)\Delta t + (\sigma_{t,i}\varepsilon\sqrt{\Delta t}) \right)} \quad (6)$$

As the Weiner variable,  $\varepsilon$ , is a random variable, numerous values of  $P_{sim,t+1,i}$  can be simulated by change its value with each simulation. By allowing  $\varepsilon$  to change randomly but still maintaining the restriction of a mean of zero, and a standard deviation of 1, one hundred thousand simulated values were obtained for each metal, for each data frequency and each window. Using equation (7), log-returns,  $r_{sim,t+1,i}$ , for each of the simulated value was calculated. After sorting the

simulated returns from low to high, the probabilities of the returns were estimated by taking the difference between the values of the cumulative distribution function of adjacent return values. Using the probabilities and simulated metal prices, the GBM expected metal price was estimated by multiplying them, and totaling them up, as shown in equation (8). As part of the robustness check, the GBM-based expected metal price was also calculated using various numbers of simulations, specifically 1, 100, 1,000, 10,000, and 50,000 simulations.

$$r_{sim,t+1,i} = \ln\left(\frac{P_{sim,t+1,i}}{P_{t,i}}\right) \quad (7)$$

$$Exp\_P_{t,i} = \sum_1^{100,000} P_{sim,t,i} * prob_{P_{sim,t,i}} \quad (8)$$

## Testing

The expected metal price estimated using equation (8), and compared to the actually values and then tested for reliability using a simple regression equation (9), and also for differences in standard deviations and means using equations (10) and (11) respectively. The actual and expected metal prices were plotted against time at the annual (*Figure 1*), quarterly (*Figure 2*) and monthly (*Figure 3*) frequencies for silver. The plots of actual prices against the GBM based expected prices for silver along with a trendline were also generate. These plots are in *Figure 4*, *Figure 5* and *Figure 6* at the annual quarterly and monthly frequencies respectively. While producing the figures 1 to 3, the dollar prices were used, and the natural log of the actual and expected prices were used for producing figures 4 to 6. For these figures, the GBM based expected prices were based on one hundred thousand simulations. In the manuscript though, the plots for only silver is presented.

$$\ln(P_{t,i}) = \alpha_i + \beta_i \ln(Exp\_P_{t,i}) + \varepsilon_i \quad (9)$$

In the simple regression equation, the actual metal price,  $P_{t,i}$ , is the dependent variable while,  $Exp\_P_{t,i}$ , the GBM based expected metal price as obtained by equation (8) is the independent variable. If the GBM based expected metal price is an accurate and reliable predictor of the actual metal price, the coefficient  $\beta_i$ , will have a value of one, and the intercept,  $\alpha_i$ , will have a value equal to zero. A similar logic was used in (Sinha 2021, 2024a, 2024f) while testing the accuracy of expected stock index values and (Sinha 2024d) while testing the accuracy of gold prices. (Grinblatt and Titman 1992) also used a similar approach while testing for persistence in mutual fund performance. As equation (9) is a simple regression, the correlation coefficients between the actual metal prices and the GBM based expected price, are obtained by computing the square root of R-square of the regression equation. These results are presented at the annual frequency are presented in *Table 4*, while those that at quarterly frequency are in *Table 5*, and the monthly frequency in *Table 6*.

The differences in standard deviation and means are tested using a standard statistical procedure discussed in many textbooks on statistics like (Berenson, Levine, and Krehbiel 2015). The hypothesis being tested here is whether the time series of the GBM based expected prices of metals have standard deviation and means similar to those of the actual metal prices. (Sinha 2021, 2024a, 2024d, 2024f) also used a similar logic to test the differences between the actual values and the GBM based expected values. The F-stat, as estimated using equation (10) is used to test for differences in standard deviation, with the hypothesis being that the difference between the standard deviation between the expected prices and actual prices are equal to zero.

**Table 4: Annual Frequency Simple Regression Results**

Metal	Intercept		GBM Expected Price			R <sup>2</sup>	Adj R <sup>2</sup>	$\rho$	F-stat
	Coefficient	t-stat (H <sub>0</sub> = 0)	Coefficient	t-stat (H <sub>0</sub> = 0)	t-stat (H <sub>0</sub> = 1)				
Silver	0.92	<b>3.00</b>	0.68	<b>6.41</b>	<b>3.08</b>	69.5%	67.8%	0.83	<b>41.03</b>
Aluminum	5.51	<b>3.59</b>	0.28	1.38	<b>3.60</b>	9.5%	4.5%	0.31	1.90
Copper	4.90	<b>3.65</b>	0.44	<b>2.89</b>	<b>3.69</b>	31.6%	27.8%	0.56	<b>8.33</b>
Iridium	0.92	1.65	0.87	<b>10.52</b>	1.64	86.0%	85.2%	0.93	<b>110.77</b>
Nickel	6.06	<b>2.99</b>	0.37	1.80 <sup>a</sup>	<b>3.06</b>	15.3%	10.6%	0.39	3.24 <sup>b</sup>
Lead	5.06	<b>4.27</b>	0.33	2.11 <sup>a</sup>	<b>4.33</b>	19.8%	15.4%	0.45	4.45 <sup>a</sup>
Palladium	1.09	1.68	0.82	<b>8.49</b>	1.80 <sup>b</sup>	80.0%	78.9%	0.89	<b>72.03</b>
Platinum	3.62	2.75 <sup>a</sup>	0.48	2.58 <sup>a</sup>	2.80 <sup>a</sup>	27.0%	22.9%	0.52	<b>6.66</b>
Rhodium	2.31	2.05 <sup>b</sup>	0.69	<b>4.97</b>	2.22 <sup>a</sup>	57.8%	55.5%	0.76	<b>24.66</b>
Ruthenium	1.88	2.14 <sup>a</sup>	0.61	<b>3.68</b>	2.39 <sup>a</sup>	43.0%	39.8%	0.66	<b>13.57</b>
Tin	3.84	<b>3.00</b>	0.61	<b>4.66</b>	<b>3.02</b>	54.7%	52.2%	0.74	<b>21.72</b>
Zinc	5.58	<b>4.03</b>	0.28	1.56	<b>4.08</b>	12.0%	7.1%	0.35	2.44

The results of the simple regression equation with natural log of the actual price as the dependent variable, and the natural log of the GBM based expected price are presented in this table. The GBM based expected metal prices are based on one-hundred thousand simulations. The t-stat for the intercept and the slope coefficient being equal to zero are presented. The t-stat for the coefficient equal to one are also provided. Equation statistics like R-square, adjusted R-square and F-stats are also provided.  $\rho$  is the correlation coefficient between the actual and expected values, and were obtained by taking square root of R-square. The highlighted values indicate significance at 1%, while the superscripts *a* and *b* indicate significance at 5% and 10% respectively. The regression was based on twenty-years of data for the period between 2004 and 2023.

**Table 5: Quarterly Frequency Simple Regression Results**

Metal	Intercept		GBM Expected Price			R <sup>2</sup>	Adj R <sup>2</sup>	$\rho$	F-stat
	Coefficient	t-stat (H <sub>0</sub> = 0)	Coefficient	t-stat (H <sub>0</sub> = 0)	t-stat (H <sub>0</sub> = 1)				
Silver	0.30	<b>2.95</b>	0.89	<b>24.98</b>	<b>3.02</b>	88.89%	88.74%	0.94	<b>623.79</b>
Aluminum	1.52	<b>3.11</b>	0.80	<b>12.47</b>	<b>3.13</b>	66.60%	66.17%	0.82	<b>155.54</b>
Copper	1.59	<b>3.63</b>	0.82	<b>16.43</b>	<b>3.66</b>	77.59%	77.30%	0.88	<b>270.09</b>
Iridium	0.30	1.84 <sup>b</sup>	0.96	<b>40.38</b>	1.81 <sup>b</sup>	95.43%	95.38%	0.98	<b>1630.30</b>
Nickel	1.35	2.53 <sup>a</sup>	0.86	<b>15.73</b>	2.58 <sup>a</sup>	76.03%	75.72%	0.87	<b>247.43</b>
Lead	1.40	<b>3.64</b>	0.81	<b>15.90</b>	<b>3.68</b>	76.41%	76.11%	0.87	<b>252.70</b>
Palladium	0.30	1.78 <sup>b</sup>	0.95	<b>37.47</b>	1.86 <sup>b</sup>	94.74%	94.67%	0.97	<b>1,404.18</b>
Platinum	1.02	<b>2.64</b>	0.85	<b>15.48</b>	<b>2.69</b>	75.45%	75.13%	0.87	<b>239.70</b>
Rhodium	0.48	1.95 <sup>a</sup>	0.94	<b>30.25</b>	2.05 <sup>a</sup>	92.15%	92.05%	0.96	<b>915.17</b>
Ruthenium	0.33	1.76 <sup>b</sup>	0.93	<b>25.23</b>	1.95 <sup>a</sup>	89.08%	88.94%	0.94	<b>636.59</b>
Tin	1.16	<b>2.99</b>	0.88	<b>22.17</b>	<b>3.00</b>	86.30%	86.13%	0.93	<b>491.55</b>
Zinc	1.12	<b>3.11</b>	0.85	<b>18.21</b>	<b>3.13</b>	80.96%	80.72%	0.90	<b>331.69</b>

The results of the simple regression equation with natural log of the actual price as the dependent variable, and the natural log of the GBM based expected price are presented in this table. The GBM based expected metal prices are based on one-hundred thousand simulations. The t-stat for the intercept and the slope coefficient being equal to zero are presented. The t-stat for the coefficient equal to one are also provided. Equation statistics like R-square, adjusted R-square and F-stats are also provided.  $\rho$  is the correlation coefficient between the actual and expected values, and were obtained by taking square root of R-square. The highlighted values indicate significance at 1%, while the superscripts *a* and *b* indicate significance at 5% and 10% respectively. The regression was based on eighty-quarters of data for the period between 1<sup>st</sup> quarter of 2004 and the last quarter of 2023.

**Table 6: Monthly Frequency Simple Regression Results**

Metal	Intercept		GBM Expected Price			R <sup>2</sup>	Adj R <sup>2</sup>	$\rho$	F-stat
	Coefficient	t-stat (H <sub>0</sub> = 0)	Coefficient	t-stat (H <sub>0</sub> = 0)	t-stat (H <sub>0</sub> = 1)				
Silver	0.11	<b>2.92</b>	0.96	<b>70.26</b>	<b>3.02</b>	95.40%	95.38%	0.98	<b>4,936.58</b>
Aluminum	0.42	<b>2.79</b>	0.94	<b>47.59</b>	<b>2.80</b>	90.49%	90.45%	0.95	<b>2,265.13</b>
Copper	0.44	<b>3.39</b>	0.95	<b>63.95</b>	<b>3.42</b>	94.50%	94.48%	0.97	<b>4,090.11</b>
Iridium	0.09	2.13 <sup>a</sup>	0.99	<b>167.58</b>	1.98 <sup>a</sup>	99.16%	99.16%	1.00	<b>28,082.30</b>
Nickel	0.42	2.40 <sup>a</sup>	0.96	<b>53.47</b>	2.45 <sup>a</sup>	92.32%	92.28%	0.96	<b>2,859.14</b>
Lead	0.47	<b>3.52</b>	0.94	<b>53.52</b>	<b>3.56</b>	92.33%	92.30%	0.96	<b>2,864.73</b>
Palladium	0.09	1.77 <sup>b</sup>	0.99	<b>121.26</b>	1.85 <sup>b</sup>	98.41%	98.40%	0.99	<b>14,704.90</b>
Platinum	0.30	<b>2.44</b>	0.96	<b>55.49</b>	<b>2.48</b>	92.83%	92.80%	0.96	<b>3,079.46</b>
Rhodium	0.14	1.93 <sup>a</sup>	0.98	<b>108.27</b>	2.00 <sup>a</sup>	98.01%	98.00%	0.99	<b>11,723.20</b>
Ruthenium	0.08	1.60	0.98	<b>102.39</b>	1.69 <sup>b</sup>	97.78%	97.77%	0.99	<b>10,484.20</b>
Tin	0.34	<b>2.91</b>	0.97	<b>80.61</b>	<b>2.92</b>	96.47%	96.45%	0.98	<b>6,497.50</b>
Zinc	0.38	<b>3.04</b>	0.95	<b>58.54</b>	<b>3.06</b>	93.51%	93.48%	0.97	<b>3,426.68</b>

The results of the simple regression equation with natural log of the actual price as the dependent variable, and the natural log of the GBM based expected price are presented in this table. The GBM based expected metal prices are based on one-hundred thousand simulations. The t-stat for the intercept and the slope coefficient being equal to zero are presented. The t-stat for the coefficient equal to one are also provided. Equation statistics like R-square, adjusted R-square and F-stats are also provided.  $\rho$  is the correlation coefficient between the actual and expected values, and were obtained by taking square root of R-square. The highlighted values indicate significance at 1%, while the superscripts *a* and *b* indicate significance at 5% and 10% respectively. The regression was based on two-hundred and forty months of data for the period between January 2004 and December 2023.

$$F - stat_i = \frac{(\ln(Exp_{P_{t,i}}))^2}{(\ln(P_{t,i}))^2} \quad (10)$$

The differences between the means of the GBM based expected metal prices, and the actual metal prices is tested using a t-stat calculated using equation (12), where the pooled variance was calculated equation (11). The hypothesis tested here is  $(\bar{\mu}_{Exp_{P_{t,i}}} - \bar{\mu}_{P_{t,i}}) = 0$ , with  $n_1 = n_2$ . The results for the tests for differences in standard deviation and means are presented in *Table 7*, *Table 8* and *Table 9* for the annual, quarterly and monthly frequencies respectively when the expected metal prices are based on one hundred thousand simulations.

$$\sigma_{pooled,i}^2 = \frac{(n_1-1)\sigma_{P_{t,i}}^2 + (n_2-1)\sigma_{Exp_{P_{t,i}}}^2}{(n_{1,i}-1) + (n_{2,i}-1)} \quad (11)$$

$$t_{stat,i} = \frac{(\bar{\mu}_{Exp_{P_{t,i}}} - \bar{\mu}_{P_{t,i}}) - (\mu_{1,i} - \mu_{2,i})}{\sqrt{\sigma_{pooled,i}^2 \left( \frac{1}{n_{1,i}} + \frac{1}{n_{2,i}} \right)}} \quad (12)$$

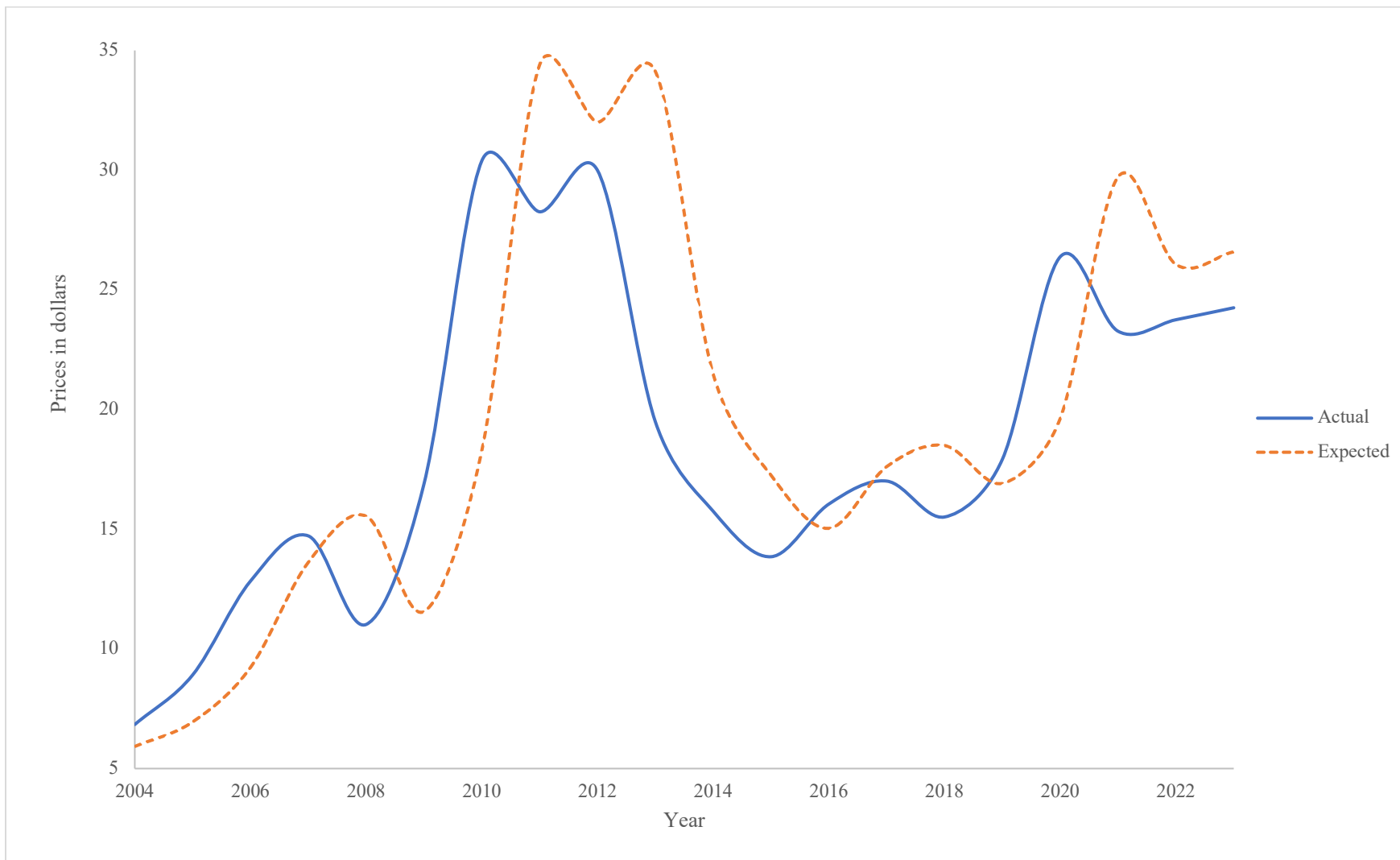
As mentioned earlier and as part of robustness check, the expected prices were estimated using different number of simulations, specifically one, one hundred, one thousand, ten thousand, and fifty thousand. These expected prices were also tested using the regression, and difference in standard deviation and means. The regression results are presented in *Table 10*, *Table 11*, and *Table 12* at the annual, quarterly and monthly frequencies. The results of the differences in standard deviation and means at the annual, quarterly and monthly frequencies are presented *Table 13*, *Table 14* and *Table 16* respectively.

## V. Results

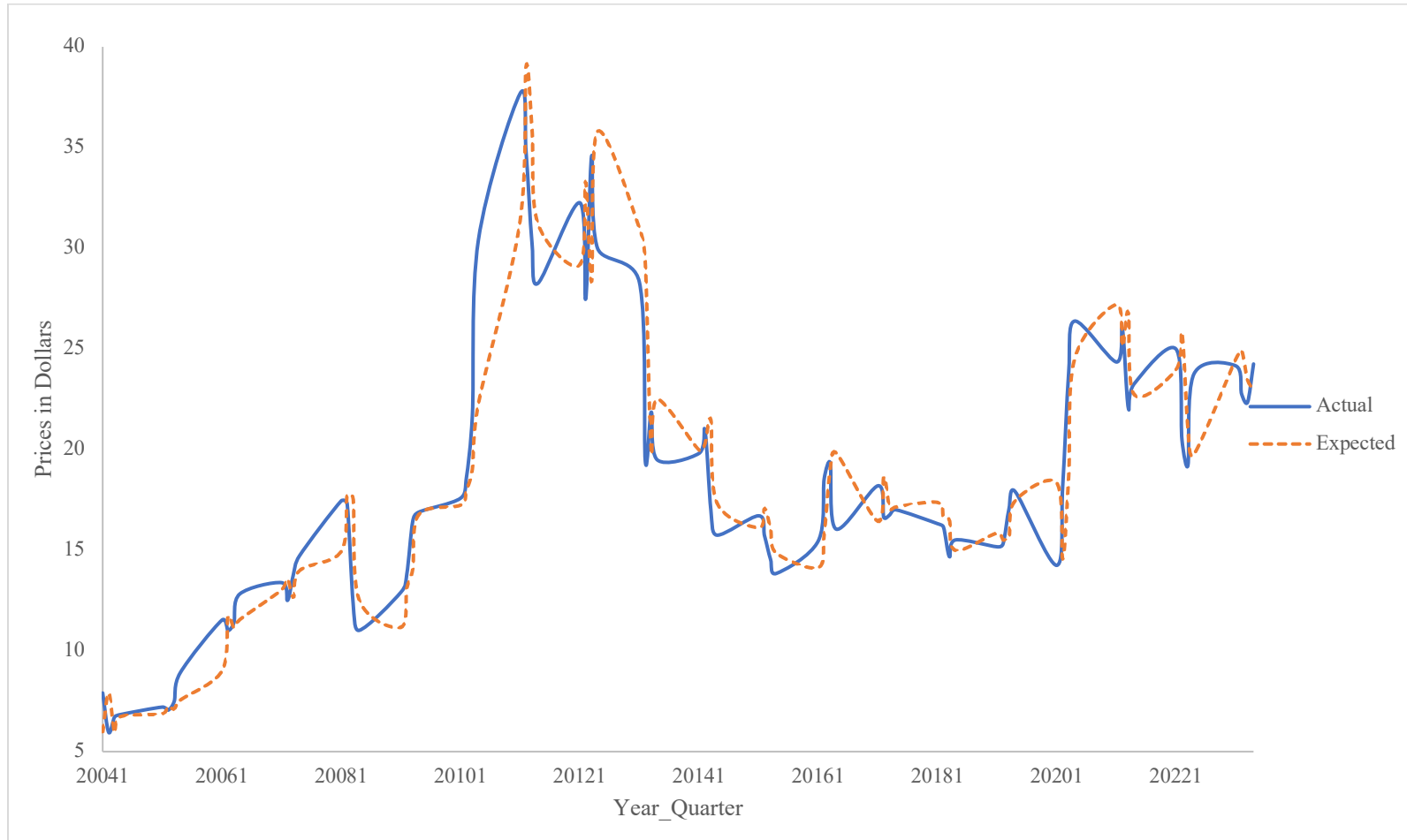
The plots of the actual metal price and the GBM based expected prices for silver are presented in *Figure 1* for the annual frequency, *Figure 2* for the quarterly frequency, and *Figure 3* for the monthly frequency. In these plots the actual year-end dollar price, and the GBM based expected metal price in dollars are plotted. In these plots, we can observe that the expected and actual prices follow the similar pattern, but the gap between the actual and expected decreases from the annual to the monthly frequency.

The plots of natural log of the year-end actual price for silver against the natural log of the GBM based expected silver price are in *Figure 4*, *Figure 5* and *Figure 6* at the annual, quarterly and monthly frequencies respectively. The plot for the annual frequency seems to be too scattered for a trend to not be very pronounced to the naked eye, although a linear trendline can be fitted to the plot. A trend in the quarterly and monthly frequency seem to be more pronounced to the naked eye, and the trendline in both these plots seems to be starting from the intersection of the x-axis and y-axis, and rising upwards to perhaps suggest a linear relationship. The trend seems to be more

**Figure 1: Annual Actual and Expected Silver Prices**

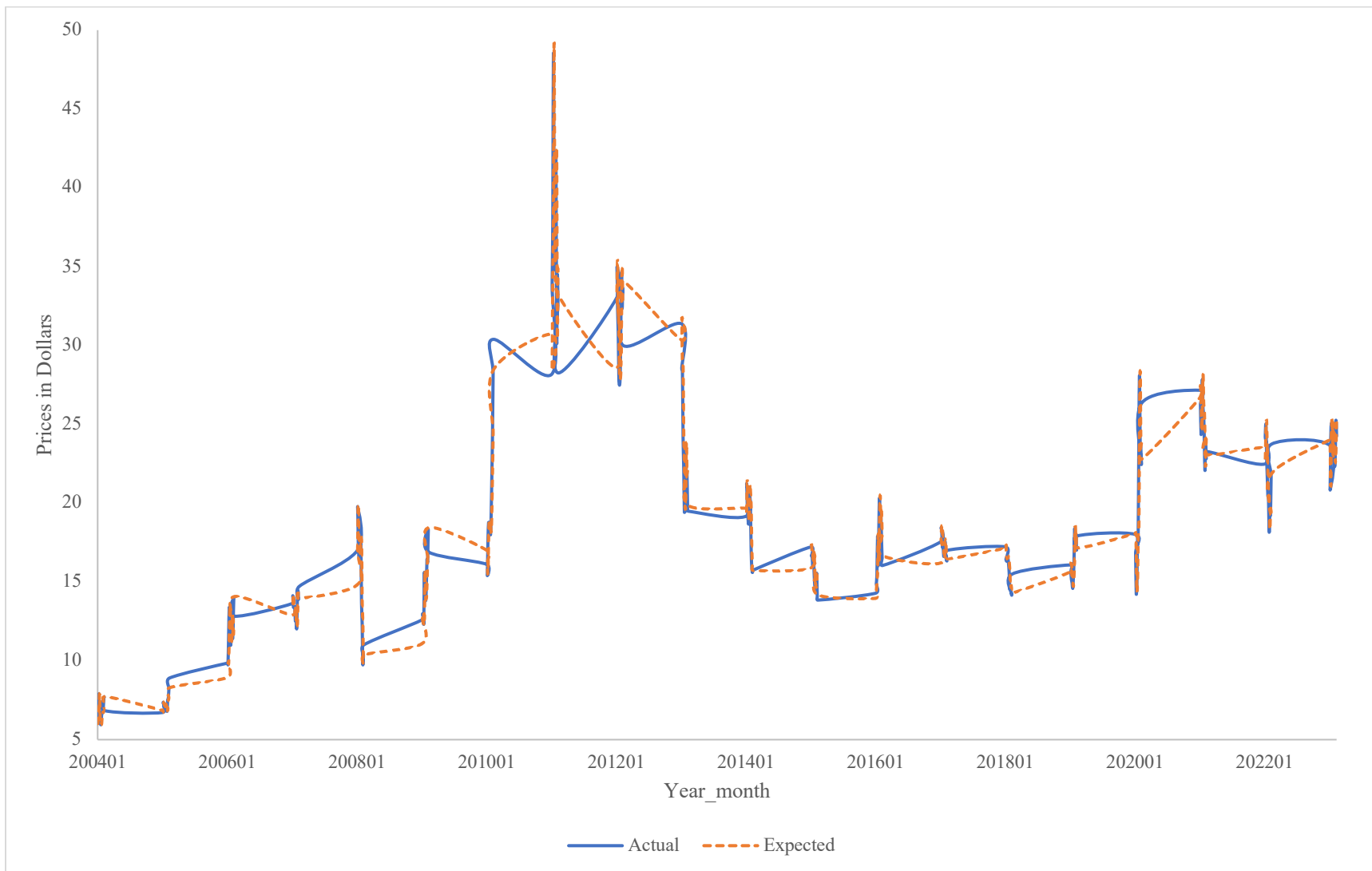


The actual and GBM based expected silver prices are plotted in this figure. The prices are actual year end dollar prices for the period between 2004 and 2023. The expected prices are based on one hundred thousand simulations.

**Figure 2: Quarterly Actual and Expected Silver Prices**

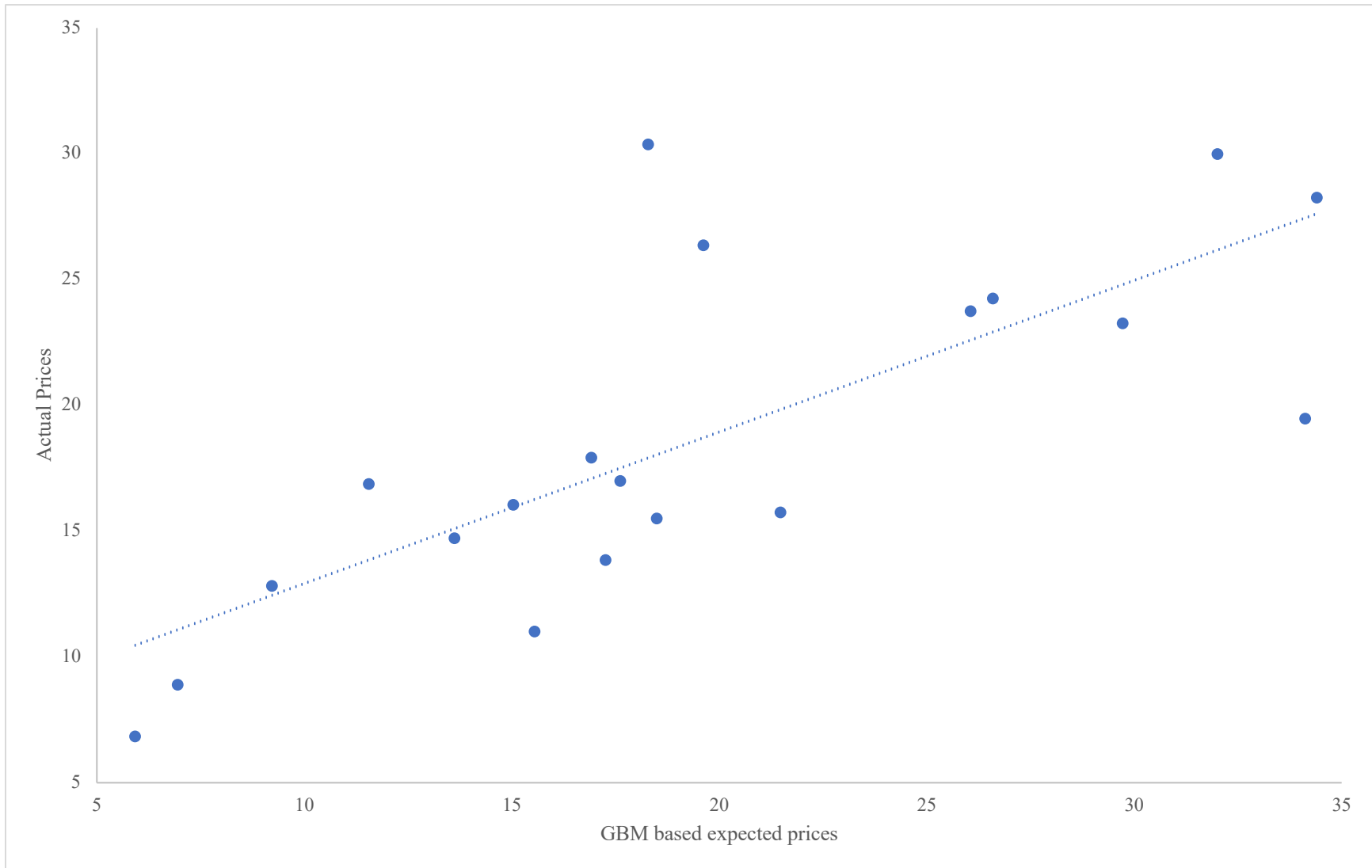
The actual and GBM based expected silver prices are plotted in this figure. The prices are natural logs of the quarter-end prices for the period between 2004 and 2023. The expected prices are based on one hundred thousand simulations.

**Figure 3: Monthly Actual and Expected Silver Prices**



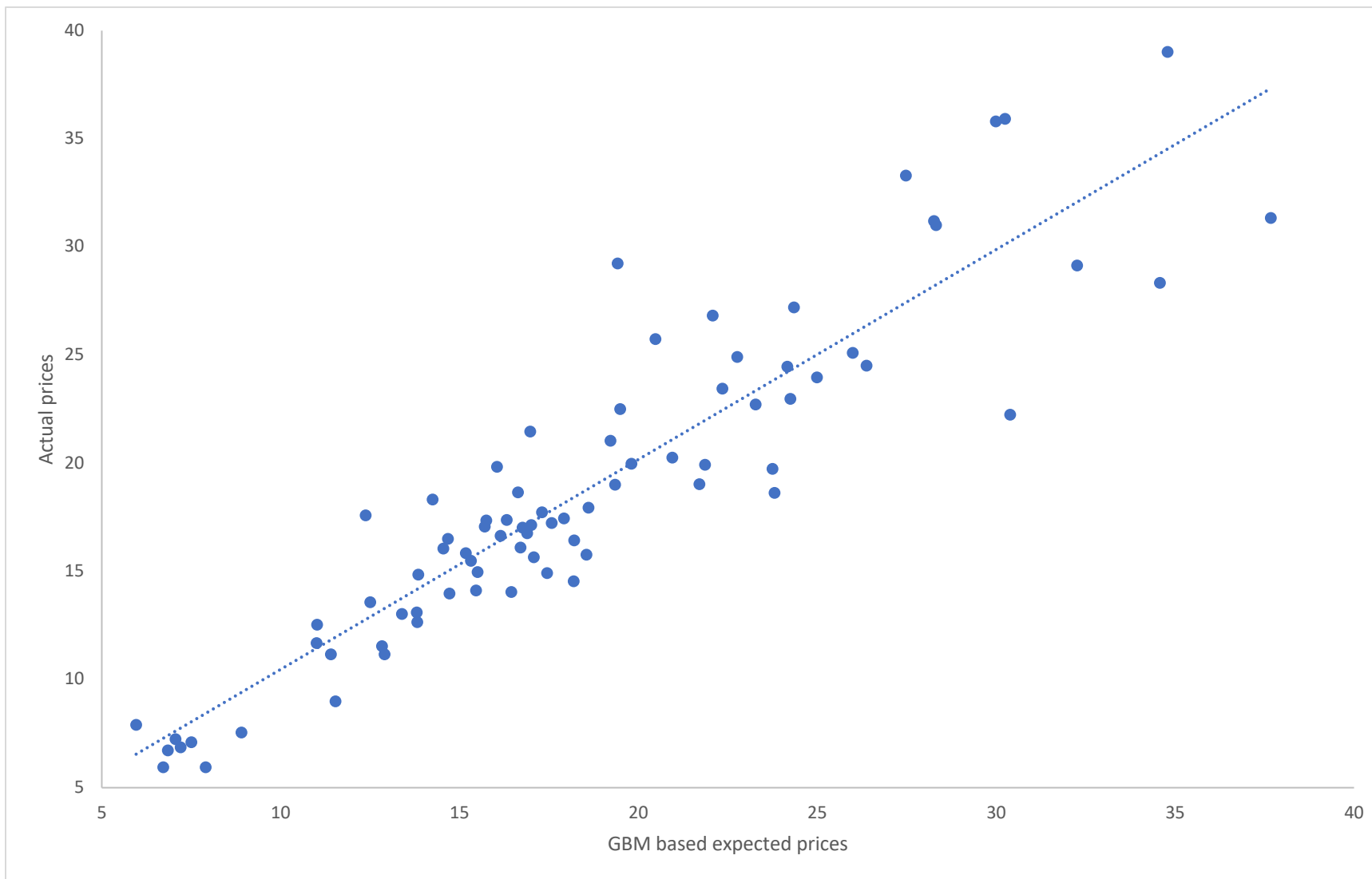
The actual and GBM based expected silver prices are plotted in this figure. The prices are natural logs of the month-end prices for the period between 2004 and 2023. The expected prices are based on one hundred thousand simulations.

**Figure 4: Annual Actual Vs Expected Silver Prices**



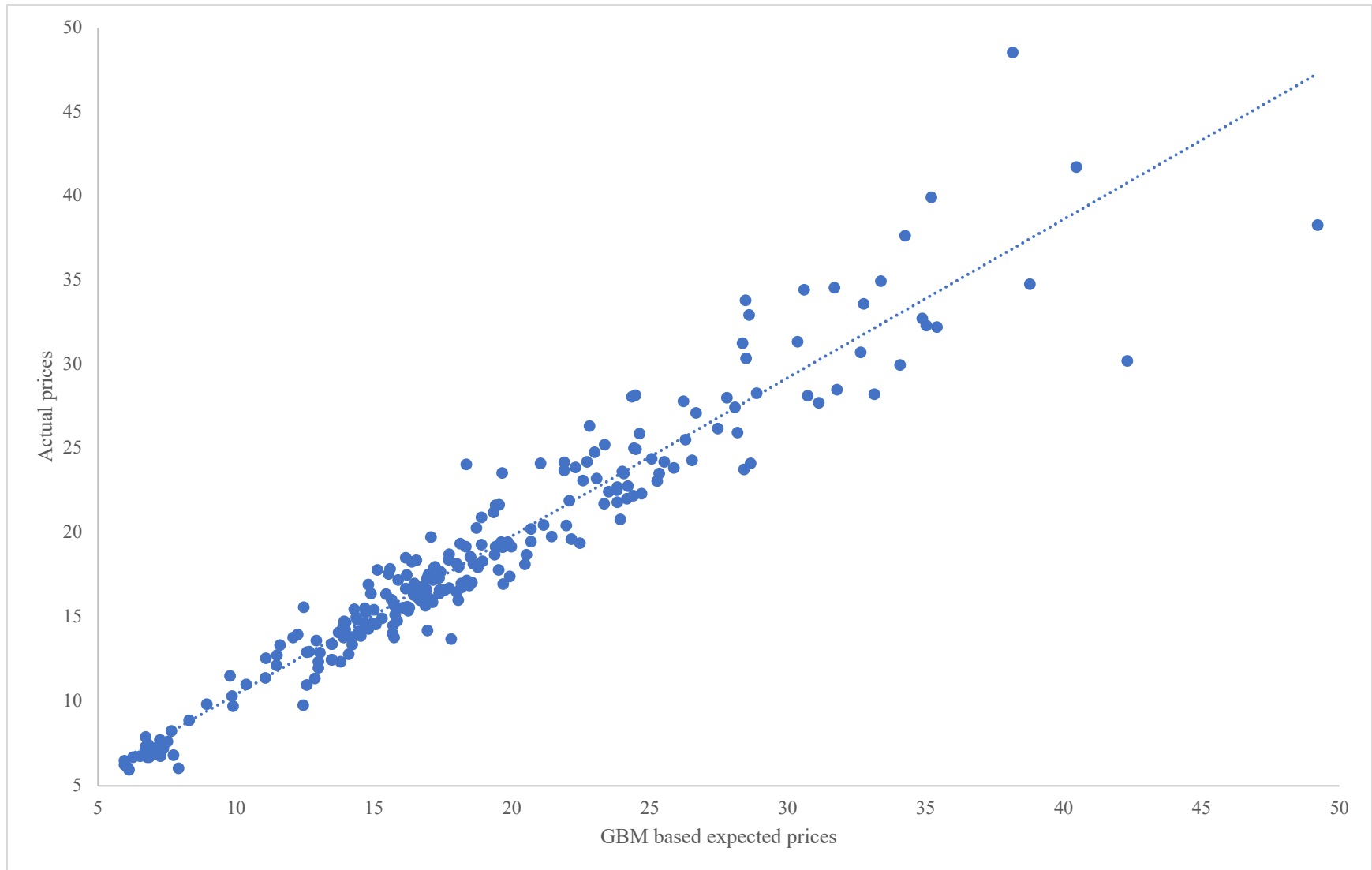
In this figure the actual prices are plotted against the GBM based expected Silver year-end prices for the period between 2003 and 2004. The expected prices are based on one hundred thousand simulations.

**Figure 5: Quarterly Actual Vs Expected Silver Prices**



In this figure the actual prices are plotted against the GBM based expected Silver quarter-end prices for the period between 2003 and 2004. The expected prices are based on one hundred thousand simulations.

**Figure 6: Monthly Actual and Expected Silver Prices**



In this figure the actual prices are plotted against the GBM based expected Silver monthly prices for the period between 2003 and 2004. The expected prices are based on one hundred thousand simulations.

**Table 7: Annual Frequency Differences in Means and Standard Deviation Results**

Metal	Standard Deviation					Mean				
	Actual	Expected	Pooled	F-stat	p-value	Actual	Expected	Difference	t-stat	p-value
Silver	0.40	0.49	0.45	1.52	0.37	2.86	2.87	-0.01	-0.08	0.94
Aluminum	0.18	0.20	0.19	1.25	0.64	7.63	7.65	-0.02	-0.40	0.69
Copper	0.32	0.41	0.36	1.64	0.29	8.77	8.81	-0.05	-0.42	0.68
Iridium	0.98	1.05	1.02	1.15	0.76	6.75	6.73	0.02	0.05	0.96
Nickel	0.36	0.38	0.37	1.11	0.82	9.70	9.84	-0.14	-1.22	0.23
Lead	0.29	0.39	0.34	1.85	0.19	7.56	7.62	-0.07	-0.62	0.54
Palladium	0.77	0.83	0.80	1.18	0.73	6.55	6.62	-0.07	-0.28	0.78
Platinum	0.23	0.25	0.24	1.18	0.73	7.00	7.06	-0.06	-0.82	0.42
Rhodium	0.99	1.09	1.04	1.21	0.68	7.88	8.05	-0.17	-0.52	0.61
Ruthenium	0.88	0.96	0.92	1.17	0.74	5.06	5.25	-0.18	-0.63	0.53
Tin	0.43	0.52	0.47	1.49	0.40	9.78	9.80	-0.02	-0.12	0.91
Zinc	0.32	0.40	0.36	1.56	0.34	7.74	7.80	-0.05	-0.48	0.64

The table provides the results for the differences in standard deviation and means of the natural log of the actual and GBM based expected metal prices. In this table, 'Actual' is the natural log the actual metal price, while 'Expected' is the natural log of the GBM based expected metal price estimated using one hundred thousand simulations, while 'Pooled' is the pooled standard deviation. F-stat is used to test for the differences in standard deviations, while the pooled standard deviation is used to estimate the t-stat for differences in means. The table reports the p-values for the F-stats and t-stats. This table is for the annual frequency and was based on twenty-years of data for the period between 2004 and 2023.

**Table 8: Quarterly Frequency Differences in Means and Standard Deviation Results**

Metal	Standard Deviation					Mean				
	Actual	Expected	Pooled	F-stat	p-value	Actual	Expected	Difference	t-stat	p-value
Silver	0.40	0.43	0.42	1.12	0.62	2.84	2.84	0.00	-0.06	0.95
Aluminum	0.19	0.20	0.20	1.04	0.86	7.62	7.63	0.00	-0.16	0.87
Copper	0.30	0.33	0.32	1.16	0.51	8.76	8.77	-0.01	-0.20	0.84
Iridium	0.95	0.97	0.96	1.04	0.86	6.75	6.74	0.01	0.05	0.96
Nickel	0.35	0.36	0.36	1.03	0.89	9.72	9.74	-0.03	-0.48	0.63
Lead	0.30	0.32	0.31	1.16	0.51	7.53	7.55	-0.02	-0.31	0.76
Palladium	0.75	0.77	0.76	1.04	0.85	6.53	6.54	-0.01	-0.10	0.92
Platinum	0.25	0.26	0.26	1.04	0.86	7.02	7.03	-0.02	-0.40	0.69
Rhodium	1.00	1.03	1.01	1.05	0.83	7.87	7.89	-0.02	-0.13	0.90
Ruthenium	0.87	0.89	0.88	1.03	0.88	5.06	5.09	-0.03	-0.22	0.82
Tin	0.41	0.43	0.42	1.11	0.64	9.78	9.78	0.00	-0.07	0.95
Zinc	0.31	0.33	0.32	1.11	0.64	7.71	7.72	-0.01	-0.12	0.90

The table provides the results for the differences in standard deviation and means of the natural log of the actual and GBM based expected metal prices. In this table, 'Actual' is the natural log the actual metal price, while 'Expected' is the natural log of the GBM based expected metal price estimated using one hundred thousand simulations, while 'Pooled' is the pooled standard deviation. F-stat is used to test for the differences in standard deviations, while the pooled standard deviation is used to estimate the t-stat for differences in means. The table reports the p-values for the F-stats and t-stats. This table is for the quarterly frequency and was based on eighty-quarters of data for the period between 1<sup>st</sup> quarter of 2004 and the last quarter of 2023.

**Table 9: Monthly Frequency Differences in Means and Standard Deviation Results**

<b>Metal</b>	<b>Standard Deviation</b>					<b>Mean</b>				
	<b>Actual</b>	<b>Expected</b>	<b>Pooled</b>	<b>F-stat</b>	<b>p-value</b>	<b>Actual</b>	<b>Expected</b>	<b>Difference</b>	<b>t-stat</b>	<b>p-value</b>
Silver	0.42	0.43	0.42	1.04	0.78	2.84	2.84	0.00	-0.06	0.95
Aluminum	0.19	0.20	0.20	1.01	0.91	7.63	7.63	0.00	-0.09	0.93
Copper	0.31	0.32	0.32	1.05	0.71	8.76	8.77	0.00	-0.08	0.93
Iridium	0.95	0.96	0.95	1.02	0.91	6.72	6.72	0.01	0.08	0.94
Nickel	0.36	0.36	0.36	1.01	0.94	9.72	9.73	-0.01	-0.27	0.79
Lead	0.31	0.31	0.31	1.05	0.71	7.54	7.54	0.00	-0.16	0.87
Palladium	0.75	0.76	0.75	1.01	0.91	6.53	6.53	0.00	-0.05	0.96
Platinum	0.25	0.26	0.25	1.01	0.92	7.03	7.03	0.00	-0.22	0.83
Rhodium	1.02	1.02	1.02	1.02	0.90	7.87	7.87	0.00	-0.05	0.96
Ruthenium	0.87	0.87	0.87	1.01	0.94	5.05	5.06	0.00	-0.04	0.97
Tin	0.41	0.42	0.42	1.04	0.79	9.78	9.78	0.00	-0.02	0.98
Zinc	0.33	0.33	0.33	1.04	0.79	7.71	7.71	0.00	-0.07	0.94

The table provides the results for the differences in standard deviation and means of the natural log of the actual and GBM based expected metal prices. In this table, 'Actual' is the natural log the actual metal price, while 'Expected' is the natural log of the GBM based expected metal price estimated using one hundred thousand simulations, while 'Pooled' is the pooled standard deviation. F-stat is used to test for the differences in standard deviations, while the pooled standard deviation is used to estimate the t-stat for differences in means. The table reports the p-values for the F-stats and t-stats. This table is for the monthly frequency and was based on two-hundred and forty months of data for the period between January 2004 and December 2023.

pronounced in the monthly than in the quarterly figure. Perhaps a stronger relationship between expected price and the actual price are to be expected at the monthly and quarterly frequency, than at the annual frequency.

$$\ln(P_{t,i}) = \hat{\alpha}_i + \hat{\beta}_i \ln(\text{Exp\_}P_{t,i}) \quad (13)$$

Equation (13) represents the output of the simple regression equation represented in equation (9), with  $\hat{\alpha}_i$  being the intercept, and  $\hat{\beta}_i$  the coefficient of the dependent variable  $\text{Exp\_}P_{t,i}$ . As mentioned earlier, if the GBM based expected metal price is an absolute reliable representation of the actual price, the intercept would have a value of zero, and the slope coefficient would have a value of one. Nevertheless, the regression model with a non-zero intercept, and slope different from one, could still be useful in making predictions about the future prices.

Table 4 presents the results of the regression for all twelve metals at the annual frequency, with GBM based expected metal price estimated from one hundred thousand simulations. The value of the intercept ranges from 0.92 for silver to 6.06 for nickel. The hypothesis that the intercept is statistically different from zero is rejected for silver, aluminum, copper, nickel, lead, tin, and zinc at the 1% confidence level, and the 5% for platinum and ruthenium. The hypothesis is rejected at the 10% level for rhodium, with iridium being the only metal for which the hypothesis is not rejected. The value of the slope coefficient ranges from a minimum of 0.28 for aluminum and silver to a maximum of 0.87 for iridium. These coefficients are tested for being different from zero, and one. For aluminum and silver, the hypothesis that the slope coefficient is equal to zero is not rejected, but rejected for silver, copper, iridium, palladium, rhodium, ruthenium, and tin at the 1% confidence level. For nickel, lead, platinum the rejection is at the 5% level. The statistics show the slope to be being statistically different from one, for eleven of the twelve metals at either the one, five or ten percent confidence levels. Only for one metal, iridium, the hypothesis cannot be rejected. Except for aluminum and zinc, the regressions have double digit R-squares and adjusted R-squares, with maximum value observed for iridium. The regression F-stat is significant for ten of the twelve metals, indicating that except for aluminum and zinc the regression equations may be useful to forecast one-period ahead forecasts. Only for iridium, the GBM based expected price could be used as a proxy for a forecast. This may also be surmised from the correlation coefficient between the actual and expected price, which is also the highest for iridium at 0.93. Palladium has a high correlation coefficient at 0.89, while the lowest is found to be for aluminum at 0.31.

The regression results at the quarterly frequency is in Table 5. The hypothesis that the intercept is equal to zero is rejected for all the metals at the one, five or ten percent confidence levels. For silver, aluminum, copper, lead, platinum, tin and zinc, it is at the one percent level, while it is at the five percent level for nickel and rhodium. For iridium and palladium, the rejection is at the ten percent level. The value of the intercept ranges from 0.30 for silver to 1.59 for copper. The value of the slope coefficients, although not equal to one, are numerically close to one ranging from a low of 0.81 for lead, to a high of 0.96 for iridium. The t-stat for the hypothesis that the slope coefficient is equal to zero is rejected at the one percent level for all the metals. The hypothesis that they are statistically equal to one is also rejected for all metals as well, at one confidence level or another. The rejection is at the one percent level for silver, aluminum, copper, lead, platinum, tin and zinc. It is rejected at the five percent level for nickel, rhodium, ruthenium, and at the ten percent level for iridium. The equations have very high and significant F-stat, and all R-squares and adjusted R-squares are greater than 66%. The actual metal price and GBM based expected

prices show very high correlation coefficients, with the lowest being 0.82 for aluminum. The highest is for iridium at 0.98. The results at the quarterly frequency may be interpreted as that although the GBM based expected metal price is not an exact proxy of the actual price, it may be used to obtain reliable one period ahead forecasts of the actual metal prices.

*Table 6* presents the results of equation (13) at the monthly frequency. The regressions for all metals have very high R-squares and adjusted R-squares, with the minimum being 90.49% for aluminum. All metals have very high regression F-stats that are significant at one percent, indicating appropriateness of the simple regression model. The correlation coefficients between the expected and actual prices are also very high at this frequency, with the lowest being 0.95 for aluminum and the highest being 1.00 for iridium. The numerical values of the intercepts are low, with the lowest being for ruthenium at 0.08, and the highest being 0.47 for lead. The hypothesis that the intercept is statistical equal to zero, are rejected for all metals, except ruthenium. This rejection is at the one percent confidence level for silver, aluminum, copper, lead, platinum, tin and zinc. It is at the five percent level for iridium, nickel and rhodium, while it is at the ten percent level for palladium. The numerical value of the slope coefficient,  $\hat{\beta}_i$ , is close to, but not quite one for all metals with the lowest being 0.94 for aluminum and lead, and the highest being 0.99 for iridium and palladium. The hypotheses that these coefficients are statistically equal to zero are rejected for all metals at the one percent significance level. Although, the values of the slope coefficients approach one, they are not equal to one, at least statistically. The hypothesis that they are equal to one are rejected at the one percent level for silver, aluminum, copper, lead, platinum, tin and zinc, and at the five percent level for iridium, nickel and rhodium. The rejection is at the ten percent level for palladium and ruthenium. Given the regression statistics at the monthly frequency, one may conclude that while the GBM based expected metal price may not be an exact proxy for the actual metal price, the simple regression model may be used to produce reliable forecast of the actual while using it as the independent variable.

The results for the test for differences in standard deviation and means are presented in *Table 7*, *Table 8* and *Table 9* at the annual, quarterly and monthly frequency respectively. In these tables, the GBM based expected metal prices are estimated using one hundred thousand simulations. The hypotheses tested is that the differences in standard deviation and means between the natural logs of the actual and GBM based expected prices, are equal to zero. F-stats as estimated using equation (10) were used to test the hypotheses related to standard deviation, while the t-stat estimated using equation (12) was for the differences in means. The results from all three tables indicate that the hypotheses related to standard deviation cannot be rejected at all three frequencies, implying that the standard deviation of the GBM based expected prices are not statistically different from those of the actual prices. Similarly, the hypotheses related to differences in means for all the metals cannot be rejected at all three frequencies, indicating that the means of the GBM based expected prices are not statistically different from those of the actual prices.

As part of robustness check, the GBM based expected metal prices were estimate at different simulation runs, specifically one, one hundred, one thousand, ten thousand and fifty thousand. The prices were then compared to actual prices using simple regressions, and also tested for differences in standard deviation and means. The simple regression results are presented in *Table 10*, *Table 11* and *Table 12* at the annual, quarterly and monthly frequencies respectively. The results of the tests for the differences in standard deviation and means at the annual, quarterly and monthly frequencies are provided in *Table 13*, *Table 14*, and *Table 15* respectively. These tables are organized into panels, with one panel for each metal.

**Table 10: Annual Simple Regression Tests for Different Simulation Runs**

No. of Simulations	Intercept		GBM Expected Price			R <sup>2</sup>	Adj R <sup>2</sup>	$\rho$	F-stat
	Coef.	t-stat (H <sub>0</sub> = 0)	Coef.	t-stat (H <sub>0</sub> = 0)	t-stat (H <sub>0</sub> = 1)				
Panel A: Silver									
1	2.53	<b>8.78</b>	0.13	1.20	<b>7.74</b>	7.41%	2.26%	0.27	1.44
100	0.94	<b>3.22</b>	0.67	<b>6.67</b>	<b>3.25</b>	71.20%	69.60%	0.84	<b>44.50</b>
1,000	0.92	<b>3.00</b>	0.68	<b>6.40</b>	<b>3.08</b>	69.45%	67.75%	0.83	<b>40.92</b>
10,000	0.92	<b>3.00</b>	0.68	<b>6.40</b>	<b>3.08</b>	69.50%	67.80%	0.83	<b>41.01</b>
50,000	0.92	<b>3.00</b>	0.68	<b>6.41</b>	<b>3.08</b>	69.51%	67.82%	0.83	<b>41.04</b>
Panel B: Aluminum									
1	7.47	<b>46.94</b>	0.03	1.05	<b>40.04</b>	5.74%	0.51%	0.24	1.10
100	5.54	<b>3.58</b>	0.27	1.35	<b>3.58</b>	9.23%	4.19%	0.30	1.83
1,000	5.52	<b>3.58</b>	0.28	1.37	<b>3.60</b>	9.45%	4.42%	0.31	1.88
10,000	5.51	<b>3.59</b>	0.28	1.38	<b>3.61</b>	9.55%	4.53%	0.31	1.90
50,000	5.51	<b>3.58</b>	0.28	1.38	<b>3.60</b>	9.54%	4.52%	0.31	1.90
Panel C: Copper									
1	8.24	<b>22.59</b>	0.07	1.46	<b>19.62</b>	10.53%	5.56%	0.32	2.12
100	5.04	<b>3.71</b>	0.42	<b>2.75</b>	<b>3.74</b>	29.58%	25.67%	0.54	<b>7.56</b>
1,000	4.88	<b>3.63</b>	0.44	<b>2.89</b>	<b>3.67</b>	31.65%	27.85%	0.56	<b>8.33</b>
10,000	4.90	<b>3.65</b>	0.44	<b>2.89</b>	<b>3.69</b>	31.63%	27.83%	0.56	<b>8.33</b>
50,000	4.90	<b>3.65</b>	0.44	<b>2.89</b>	<b>3.69</b>	31.64%	27.84%	0.56	<b>8.33</b>
Panel D: Iridium									
1	5.61	<b>11.58</b>	0.25	2.55 <sup>a</sup>	<b>7.74</b>	26.53%	22.45%	0.52	6.50 <sup>a</sup>
100	0.96	1.70	0.86	<b>10.39</b>	1.66	85.70%	84.91%	0.93	<b>107.88</b>
1,000	0.92	1.65	0.87	<b>10.57</b>	1.63	86.12%	85.35%	0.93	<b>111.66</b>
10,000	0.92	1.65	0.87	<b>10.53</b>	1.64	86.02%	85.25%	0.93	<b>110.79</b>
50,000	0.92	1.65	0.87	<b>10.52</b>	1.64	86.02%	85.24%	0.93	<b>110.76</b>
Panel E: Nickel									
1	9.31	<b>21.68</b>	0.04	0.93	<b>20.22</b>	4.63%	-0.67%	0.22	0.87
100	6.23	<b>3.20</b>	0.35	1.79 <sup>b</sup>	<b>3.27</b>	15.08%	10.36%	0.39	3.20 <sup>b</sup>
1,000	5.97	<b>2.92</b>	0.38	1.83 <sup>b</sup>	<b>2.99</b>	15.66%	10.98%	0.40	3.34 <sup>b</sup>
10,000	6.06	<b>2.99</b>	0.37	1.80 <sup>b</sup>	<b>3.06</b>	15.21%	10.50%	0.39	3.23 <sup>b</sup>
50,000	6.06	<b>2.99</b>	0.37	1.80 <sup>b</sup>	<b>3.06</b>	15.25%	10.54%	0.39	3.24 <sup>b</sup>
Panel F: Lead									
1	7.16	<b>18.07</b>	0.06	1.02	<b>16.20</b>	5.50%	0.25%	0.23	1.05
100	5.12	<b>4.35</b>	0.32	2.08 <sup>a</sup>	<b>4.41</b>	19.33%	14.85%	0.44	4.31 <sup>a</sup>
1,000	5.07	<b>4.28</b>	0.33	2.09 <sup>a</sup>	<b>4.34</b>	19.59%	15.12%	0.44	4.39 <sup>a</sup>
10,000	5.06	<b>4.27</b>	0.33	2.11 <sup>a</sup>	<b>4.33</b>	19.79%	15.33%	0.44	4.44 <sup>a</sup>
50,000	5.06	<b>4.27</b>	0.33	2.11 <sup>a</sup>	<b>4.33</b>	19.81%	15.35%	0.45	4.45 <sup>a</sup>
Panel G: Palladium									
1	6.13	<b>16.41</b>	0.09	1.26	<b>13.02</b>	8.09%	2.98%	0.28	1.58
100	1.23	1.86 <sup>b</sup>	0.81	<b>8.11</b>	1.94 <sup>b</sup>	78.52%	77.33%	0.89	<b>65.79</b>
1,000	1.10	1.69	0.82	<b>8.41</b>	1.80 <sup>b</sup>	79.72%	78.59%	0.89	<b>70.75</b>
10,000	1.09	1.68	0.82	<b>8.49</b>	1.80 <sup>b</sup>	80.02%	78.91%	0.89	<b>72.09</b>
50,000	1.09	1.68	0.82	<b>8.49</b>	1.80 <sup>b</sup>	80.01%	78.90%	0.89	<b>72.06</b>
Panel H: Platinum									
1	6.89	<b>25.91</b>	0.02	0.44	<b>21.54</b>	1.05%	-4.45%	0.10	0.19

100	3.73	<b>2.89</b>	0.46	2.53 <sup>a</sup>	<b>2.93</b>	26.30%	22.20%	0.51	6.42 <sup>a</sup>
1,000	3.61	2.75 <sup>a</sup>	0.48	2.59 <sup>a</sup>	2.80 <sup>a</sup>	27.09%	23.04%	0.52	6.69 <sup>a</sup>
10,000	3.62	2.75 <sup>a</sup>	0.48	2.58 <sup>a</sup>	2.80 <sup>a</sup>	27.00%	22.95%	0.52	6.66 <sup>a</sup>
50,000	3.62	2.75 <sup>a</sup>	0.48	2.58 <sup>a</sup>	2.80 <sup>a</sup>	26.99%	22.94%	0.52	6.66 <sup>a</sup>
Panel I: Rhodium									
1	5.55	<b>6.54</b>	0.37	<b>2.81</b>	<b>4.81</b>	30.52%	26.66%	0.55	<b>7.91</b>
100	2.24	2.01 <sup>b</sup>	0.70	<b>5.10</b>	2.15 <sup>a</sup>	59.10%	56.82%	0.77	<b>26.01</b>
1,000	2.31	2.05 <sup>b</sup>	0.69	<b>4.98</b>	2.21 <sup>a</sup>	57.99%	55.66%	0.76	<b>24.85</b>
10,000	2.31	2.05 <sup>b</sup>	0.69	<b>4.96</b>	2.21 <sup>a</sup>	57.76%	55.41%	0.76	<b>24.61</b>
50,000	2.32	2.05 <sup>b</sup>	0.69	<b>4.96</b>	2.22 <sup>a</sup>	57.77%	55.43%	0.76	<b>24.63</b>
Panel J: Ruthenium									
1	4.33	<b>7.97</b>	0.21	1.44	<b>5.54</b>	10.32%	5.34%	0.32	2.07
100	1.81	2.12 <sup>a</sup>	0.63	<b>3.86</b>	2.31 <sup>a</sup>	45.26%	42.22%	0.67	<b>14.89</b>
1,000	1.89	2.15 <sup>a</sup>	0.61	<b>3.67</b>	2.38 <sup>a</sup>	42.76%	39.58%	0.65	<b>13.45</b>
10,000	1.88	2.14 <sup>a</sup>	0.61	<b>3.68</b>	2.39 <sup>a</sup>	43.00%	39.83%	0.66	<b>13.58</b>
50,000	1.88	2.14 <sup>a</sup>	0.61	<b>3.68</b>	2.38 <sup>a</sup>	42.98%	39.81%	0.66	<b>13.57</b>
Panel K: Tin									
1	9.36	<b>20.41</b>	0.05	0.96	<b>17.75</b>	4.82%	-0.46%	0.22	0.91
100	3.78	<b>2.95</b>	0.61	<b>4.70</b>	<b>2.96</b>	55.12%	52.63%	0.74	<b>22.11</b>
1,000	3.83	<b>3.00</b>	0.61	<b>4.67</b>	<b>3.02</b>	54.76%	52.25%	0.74	<b>21.79</b>
10,000	3.83	<b>3.00</b>	0.61	<b>4.66</b>	<b>3.02</b>	54.71%	52.20%	0.74	<b>21.75</b>
50,000	3.84	<b>3.00</b>	0.61	<b>4.66</b>	<b>3.02</b>	54.69%	52.17%	0.74	<b>21.72</b>
Panel L: Zinc									
1	7.58	<b>21.26</b>	0.03	0.47	<b>18.09</b>	1.21%	-4.28%	0.11	0.22
100	5.75	<b>4.21</b>	0.26	1.47	<b>4.25</b>	10.66%	5.70%	0.33	2.15
1,000	5.57	<b>4.02</b>	0.28	1.57	<b>4.06</b>	12.02%	7.13%	0.35	2.46
10,000	5.58	<b>4.03</b>	0.28	1.56	<b>4.08</b>	11.91%	7.02%	0.35	2.43
50,000	5.58	<b>4.03</b>	0.28	1.56	<b>4.08</b>	11.95%	7.05%	0.35	2.44

The results of the simple regression equation with natural log of the actual price as the dependent variable, and the natural log of the GBM based expected price are presented in this table. The GBM based expected metal prices are based on simulations, ranging from one to fifty-thousand. The t-stat for the intercept and the slope coefficient being equal to zero are presented. The t-stat for the coefficient equal to one are also provided. Equation statistics like R-square, adjusted R-square and F-stats are also provided.  $\rho$  is the correlation coefficient between the actual and expected values, and were obtained by taking square root of R-square. The highlighted values indicate significance at 1%, while the superscripts a and b indicate significance at 5% and 10% respectively. The regression was based on twenty-years of data for the period between 2004 and 2023.

**Table 11: Quarterly Simple Regression Tests for Different Simulation Runs**

No. of Simulations	Intercept		GBM Expected Price		R <sup>2</sup>	Adj R <sup>2</sup>	$\rho$	F-stat	
	Coef.	H <sub>0</sub> = 0	Coef.	H <sub>0</sub> = 0    H <sub>0</sub> = 1					
Panel A: Silver									
1	2.84	<b>33.31</b>	0.00	-0.05	<b>24.60</b>	0.00%	-1.28%	0.00	0.00
100	0.32	<b>3.17</b>	0.89	<b>24.99</b>	<b>3.13</b>	88.89%	88.75%	0.94	<b>624.35</b>
1,000	0.30	<b>2.96</b>	0.89	<b>24.98</b>	<b>3.03</b>	88.89%	88.74%	0.94	<b>623.79</b>
10,000	0.30	<b>2.95</b>	0.89	<b>24.97</b>	<b>3.02</b>	88.88%	88.74%	0.94	<b>623.55</b>
50,000	0.30	<b>2.95</b>	0.89	<b>24.98</b>	<b>3.03</b>	88.89%	88.74%	0.94	<b>623.77</b>
Panel B: Aluminum									
1	7.24	<b>70.46</b>	0.06	<b>3.82</b>	<b>61.58</b>	15.78%	14.70%	0.40	<b>14.61</b>
100	1.61	<b>3.29</b>	0.79	<b>12.26</b>	<b>3.28</b>	65.82%	65.38%	0.81	<b>150.21</b>
1,000	1.52	<b>3.11</b>	0.80	<b>12.48</b>	<b>3.12</b>	66.63%	66.20%	0.82	<b>155.74</b>
10,000	1.52	<b>3.11</b>	0.80	<b>12.47</b>	<b>3.13</b>	66.60%	66.17%	0.82	<b>155.51</b>
50,000	1.52	<b>3.11</b>	0.80	<b>12.47</b>	<b>3.13</b>	66.60%	66.17%	0.82	<b>155.54</b>
Panel C: Copper									
1	8.29	<b>34.77</b>	0.06	2.01 <sup>a</sup>	<b>30.53</b>	4.92%	3.71%	0.22	4.04 <sup>b</sup>
100	1.62	<b>3.68</b>	0.82	<b>16.30</b>	<b>3.68</b>	77.30%	77.01%	0.88	<b>265.57</b>
1,000	1.59	<b>3.62</b>	0.82	<b>16.42</b>	<b>3.65</b>	77.56%	77.28%	0.88	<b>269.66</b>
10,000	1.59	<b>3.63</b>	0.82	<b>16.43</b>	<b>3.66</b>	77.58%	77.30%	0.88	<b>269.96</b>
50,000	1.59	<b>3.63</b>	0.82	<b>16.43</b>	<b>3.66</b>	77.59%	77.30%	0.88	<b>270.07</b>
Panel D: Iridium									
1	4.83	<b>12.87</b>	0.34	<b>5.27</b>	<b>10.12</b>	26.29%	25.35%	0.51	<b>27.82</b>
100	0.34	2.05 <sup>a</sup>	0.95	<b>38.89</b>	1.95 <sup>b</sup>	95.09%	95.03%	0.98	<b>1,512.15</b>
1,000	0.30	1.85 <sup>b</sup>	0.96	<b>40.39</b>	1.81 <sup>b</sup>	95.44%	95.38%	0.98	<b>1,631.36</b>
10,000	0.30	1.84 <sup>b</sup>	0.96	<b>40.38</b>	1.81 <sup>b</sup>	95.43%	95.38%	0.98	<b>1,630.47</b>
50,000	0.30	1.84 <sup>b</sup>	0.96	<b>40.38</b>	1.81 <sup>b</sup>	95.43%	95.38%	0.98	<b>1,630.54</b>
Panel E: Nickel									
1	9.31	<b>42.09</b>	0.05	1.89 <sup>b</sup>	<b>37.67</b>	4.38%	3.15%	0.21	3.57 <sup>b</sup>
100	1.30	2.46 <sup>a</sup>	0.86	<b>15.95</b>	2.51 <sup>a</sup>	76.52%	76.22%	0.87	<b>254.25</b>
1,000	1.34	2.51 <sup>a</sup>	0.86	<b>15.75</b>	2.56 <sup>a</sup>	76.09%	75.78%	0.87	<b>248.17</b>
10,000	1.35	2.53 <sup>a</sup>	0.86	<b>15.73</b>	2.58 <sup>a</sup>	76.04%	75.73%	0.87	<b>247.52</b>
50,000	1.35	2.53 <sup>a</sup>	0.86	<b>15.73</b>	2.58 <sup>a</sup>	76.03%	75.72%	0.87	<b>247.44</b>
Panel F: Lead									
1	6.84	<b>37.67</b>	0.11	<b>3.90</b>	<b>33.07</b>	16.32%	15.25%	0.40	<b>15.22</b>
100	1.42	<b>3.63</b>	0.81	<b>15.62</b>	<b>3.65</b>	75.77%	75.46%	0.87	<b>243.98</b>
1,000	1.41	<b>3.64</b>	0.81	<b>15.86</b>	<b>3.68</b>	76.34%	76.04%	0.87	<b>251.65</b>
10,000	1.40	<b>3.64</b>	0.81	<b>15.90</b>	<b>3.68</b>	76.41%	76.11%	0.87	<b>252.66</b>
50,000	1.40	<b>3.64</b>	0.81	<b>15.90</b>	<b>3.68</b>	76.42%	76.11%	0.87	<b>252.72</b>
Panel G: Palladium									
1	4.73	<b>14.94</b>	0.33	<b>5.84</b>	<b>11.75</b>	30.43%	29.54%	0.55	<b>34.12</b>
100	0.32	1.89 <sup>b</sup>	0.95	<b>37.39</b>	1.91 <sup>b</sup>	94.72%	94.65%	0.97	<b>1,398.15</b>
1,000	0.30	1.77 <sup>b</sup>	0.95	<b>37.57</b>	1.84 <sup>b</sup>	94.76%	94.70%	0.97	<b>1,411.34</b>
10,000	0.30	1.78 <sup>b</sup>	0.95	<b>37.48</b>	1.86 <sup>b</sup>	94.74%	94.67%	0.97	<b>1,405.01</b>
50,000	0.30	1.78 <sup>b</sup>	0.95	<b>37.48</b>	1.86 <sup>b</sup>	94.74%	94.67%	0.97	<b>1,404.44</b>
Panel H: Platinum									
1	6.69	<b>51.10</b>	0.06	<b>2.53</b>	<b>42.89</b>	7.56%	6.37%	0.27	<b>6.38</b>
100	1.01	2.52 <sup>a</sup>	0.86	<b>15.06</b>	2.54 <sup>a</sup>	74.40%	74.08%	0.86	<b>226.73</b>

1,000	1.02	<b>2.63</b>	0.85	<b>15.52</b>	<b>2.67</b>	75.53%	75.21%	0.87	<b>240.72</b>
10,000	1.02	<b>2.64</b>	0.85	<b>15.48</b>	<b>2.69</b>	75.45%	75.14%	0.87	<b>239.75</b>
50,000	1.02	<b>2.64</b>	0.85	<b>15.48</b>	<b>2.69</b>	75.45%	75.14%	0.87	<b>239.73</b>
Panel I: Rhodium									
1	5.38	<b>16.57</b>	0.38	<b>7.92</b>	<b>13.13</b>	44.57%	43.86%	0.67	<b>62.73</b>
100	0.46	1.88 <sup>b</sup>	0.94	<b>30.50</b>	1.95 <sup>b</sup>	92.26%	92.16%	0.96	<b>930.22</b>
1,000	0.48	1.94 <sup>b</sup>	0.94	<b>30.33</b>	2.03 <sup>a</sup>	92.18%	92.08%	0.96	<b>919.76</b>
10,000	0.48	1.95 <sup>b</sup>	0.94	<b>30.26</b>	2.05 <sup>a</sup>	92.15%	92.05%	0.96	<b>915.59</b>
50,000	0.48	1.95 <sup>b</sup>	0.94	<b>30.25</b>	2.05 <sup>a</sup>	92.15%	92.04%	0.96	<b>915.04</b>
Panel J: Ruthenium									
1	3.27	<b>15.11</b>	0.43	<b>8.74</b>	<b>11.52</b>	49.50%	48.85%	0.70	<b>76.45</b>
100	0.37	1.92 <sup>b</sup>	0.92	<b>25.03</b>	2.03 <sup>a</sup>	88.93%	88.79%	0.94	<b>626.70</b>
1,000	0.33	1.76 <sup>b</sup>	0.93	<b>25.23</b>	1.94 <sup>b</sup>	89.09%	88.95%	0.94	<b>636.64</b>
10,000	0.33	1.76 <sup>b</sup>	0.93	<b>25.23</b>	1.95 <sup>b</sup>	89.09%	88.95%	0.94	<b>636.74</b>
50,000	0.33	1.76 <sup>b</sup>	0.93	<b>25.23</b>	1.95 <sup>b</sup>	89.08%	88.94%	0.94	<b>636.56</b>
Panel K: Tin									
1	9.19	<b>27.35</b>	0.07	1.76 <sup>b</sup>	<b>24.25</b>	3.84%	2.61%	0.20	3.11 <sup>b</sup>
100	1.24	<b>3.16</b>	0.87	<b>21.79</b>	<b>3.15</b>	85.89%	85.71%	0.93	<b>474.74</b>
1,000	1.15	<b>2.96</b>	0.88	<b>22.20</b>	<b>2.97</b>	86.34%	86.16%	0.93	<b>492.81</b>
10,000	1.16	<b>2.99</b>	0.88	<b>22.17</b>	<b>3.00</b>	86.31%	86.13%	0.93	<b>491.65</b>
50,000	1.16	<b>2.99</b>	0.88	<b>22.17</b>	<b>3.00</b>	86.30%	86.13%	0.93	<b>491.52</b>
Panel L: Zinc									
1	7.24	<b>36.04</b>	0.07	2.37 <sup>a</sup>	<b>31.23</b>	6.73%	5.54%	0.26	5.63 <sup>a</sup>
100	1.17	<b>3.26</b>	0.85	<b>18.16</b>	<b>3.26</b>	80.87%	80.62%	0.90	<b>329.73</b>
1,000	1.13	<b>3.11</b>	0.85	<b>18.20</b>	<b>3.12</b>	80.93%	80.69%	0.90	<b>331.13</b>
10,000	1.13	<b>3.11</b>	0.85	<b>18.20</b>	<b>3.13</b>	80.95%	80.70%	0.90	<b>331.37</b>
50,000	1.12	<b>3.11</b>	0.85	<b>18.21</b>	<b>3.13</b>	80.96%	80.72%	0.90	<b>331.68</b>

The results of the simple regression equation with natural log of the actual price as the dependent variable, and the natural log of the GBM based expected price are presented in this table. The GBM based expected metal prices are based on simulations, ranging from one to fifty-thousand. The t-stat for the intercept and the slope coefficient being equal to zero are presented. The t-stat for the coefficient equal to one are also provided. Equation statistics like R-square, adjusted R-square and F-stats are also provided.  $\rho$  is the correlation coefficient between the actual and expected values, and were obtained by taking square root of R-square. The highlighted values indicate significance at 1%, while the superscripts a and b indicate significance at 5% and 10% respectively. The regression was based on equity-quarters of data for the period between 1<sup>st</sup> quarter of 2004 and the last quarter of 2023.

**Table 12: Monthly Simple Regression Tests for Different Simulation Runs**

No. of Simulations	Intercept		GBM Expected Price			R <sup>2</sup>	Adj R <sup>2</sup>	$\rho$	F-stat
	Coef.	H <sub>0</sub> = 0	Coef.	H <sub>0</sub> = 0	H <sub>0</sub> = 1				
Panel A: Silver									
1	2.65	<b>56.16</b>	0.11	<b>4.88</b>	<b>40.93</b>	9.08%	8.70%	0.30	<b>23.78</b>
100	0.14	<b>3.46</b>	0.95	<b>68.96</b>	<b>3.31</b>	95.23%	95.21%	0.98	<b>4,755.13</b>
1,000	0.11	<b>2.93</b>	0.96	<b>70.35</b>	<b>3.00</b>	95.41%	95.39%	0.98	<b>4,948.74</b>
10,000	0.11	<b>2.93</b>	0.96	<b>70.27</b>	<b>3.02</b>	95.40%	95.38%	0.98	<b>4,937.19</b>
50,000	0.11	<b>2.93</b>	0.96	<b>70.26</b>	<b>3.02</b>	95.40%	95.38%	0.98	<b>4,936.31</b>
Panel B: Aluminum									
1	7.43	<b>99.97</b>	0.03	<b>2.72</b>	<b>87.86</b>	3.02%	2.61%	0.17	<b>7.41</b>
100	0.44	<b>2.85</b>	0.94	<b>47.12</b>	<b>2.81</b>	90.32%	90.28%	0.95	<b>2,219.85</b>
1,000	0.42	<b>2.77</b>	0.95	<b>47.77</b>	<b>2.77</b>	90.55%	90.52%	0.95	<b>2,281.84</b>
10,000	0.42	<b>2.79</b>	0.94	<b>47.58</b>	<b>2.80</b>	90.49%	90.45%	0.95	<b>2,264.02</b>
50,000	0.42	<b>2.79</b>	0.94	<b>47.59</b>	<b>2.80</b>	90.49%	90.45%	0.95	<b>2,265.08</b>
Panel C: Copper									
1	8.19	<b>57.88</b>	0.08	<b>4.10</b>	<b>50.57</b>	6.60%	6.21%	0.26	<b>16.82</b>
100	0.49	<b>3.78</b>	0.94	<b>63.15</b>	<b>3.73</b>	94.37%	94.34%	0.97	<b>3,988.12</b>
1,000	0.44	<b>3.40</b>	0.95	<b>63.95</b>	<b>3.42</b>	94.50%	94.48%	0.97	<b>4,090.00</b>
10,000	0.44	<b>3.39</b>	0.95	<b>63.96</b>	<b>3.42</b>	94.50%	94.48%	0.97	<b>4,091.47</b>
50,000	0.44	<b>3.39</b>	0.95	<b>63.96</b>	<b>3.42</b>	94.50%	94.48%	0.97	<b>4,090.60</b>
Panel D: Iridium									
1	4.06	19.48	0.46	<b>13.12</b>	<b>15.14</b>	41.99%	41.74%	0.65	<b>172.24</b>
100	0.10	2.50 <sup>a</sup>	0.99	<b>164.53</b>	2.11 <sup>a</sup>	99.13%	99.12%	1.00	<b>27,070.50</b>
1,000	0.09	2.15 <sup>a</sup>	0.99	<b>167.32</b>	1.98 <sup>a</sup>	99.16%	99.15%	1.00	<b>27,996.80</b>
10,000	0.09	2.14 <sup>a</sup>	0.99	<b>167.56</b>	1.99 <sup>a</sup>	99.16%	99.16%	1.00	<b>28,077.70</b>
50,000	0.09	2.13 <sup>a</sup>	0.99	<b>167.58</b>	1.98 <sup>a</sup>	99.16%	99.16%	1.00	<b>28,082.00</b>
Panel E: Nickel									
1	8.87	<b>54.88</b>	0.10	<b>5.35</b>	<b>49.05</b>	10.74%	10.37%	0.33	<b>28.64</b>
100	0.43	2.42 <sup>a</sup>	0.96	<b>52.59</b>	2.42 <sup>a</sup>	92.08%	92.04%	0.96	<b>2,765.85</b>
1,000	0.42	2.40 <sup>a</sup>	0.96	<b>53.47</b>	2.45 <sup>a</sup>	92.32%	92.28%	0.96	<b>2,859.08</b>
10,000	0.42	2.40 <sup>a</sup>	0.96	<b>53.47</b>	2.45 <sup>a</sup>	92.32%	92.28%	0.96	<b>2,858.99</b>
50,000	0.42	2.40 <sup>a</sup>	0.96	<b>53.47</b>	2.45 <sup>a</sup>	92.32%	92.28%	0.96	<b>2,859.33</b>
Panel F: Lead									
1	7.08	<b>63.36</b>	0.07	<b>4.14</b>	<b>55.00</b>	6.71%	6.32%	0.26	<b>17.13</b>
100	0.47	<b>3.53</b>	0.94	<b>52.73</b>	<b>3.50</b>	92.11%	92.08%	0.96	<b>2,780.27</b>
1,000	0.47	<b>3.55</b>	0.94	<b>53.42</b>	<b>3.58</b>	92.30%	92.27%	0.96	<b>2,853.74</b>
10,000	0.46	<b>3.52</b>	0.94	<b>53.55</b>	<b>3.56</b>	92.34%	92.30%	0.96	<b>2,867.16</b>
50,000	0.47	<b>3.52</b>	0.94	<b>53.52</b>	<b>3.56</b>	92.33%	92.30%	0.96	<b>2,864.86</b>
Panel G: Palladium									
1	4.59	<b>26.69</b>	0.35	<b>11.60</b>	<b>21.28</b>	36.10%	35.83%	0.60	<b>134.47</b>
100	0.10	1.92 <sup>b</sup>	0.99	<b>119.23</b>	1.81 <sup>b</sup>	98.35%	98.35%	0.99	<b>14,215.90</b>
1,000	0.10	1.79 <sup>b</sup>	0.98	<b>121.16</b>	1.85 <sup>b</sup>	98.40%	98.40%	0.99	<b>14,680.20</b>
10,000	0.09	1.77 <sup>b</sup>	0.99	<b>121.28</b>	1.85 <sup>b</sup>	98.41%	98.40%	0.99	<b>14,708.90</b>
50,000	0.09	1.77 <sup>b</sup>	0.99	<b>121.27</b>	1.85 <sup>b</sup>	98.41%	98.40%	0.99	<b>14,705.30</b>
Panel H: Platinum									
1	6.63	<b>77.16</b>	0.07	4.73	<b>65.79</b>	8.60%	8.22%	0.29	<b>22.40</b>
100	0.31	2.47 <sup>a</sup>	0.96	53.89	2.44 <sup>a</sup>	92.42%	92.39%	0.96	<b>2,903.64</b>

1,000	0.30	2.45 <sup>a</sup>	0.96	55.50	2.48 <sup>a</sup>	92.83%	92.80%	0.96	<b>3,080.77</b>
10,000	0.30	2.44 <sup>a</sup>	0.96	55.50	2.48 <sup>a</sup>	92.83%	92.80%	0.96	<b>3,080.38</b>
50,000	0.30	2.44 <sup>a</sup>	0.96	55.49	2.48 <sup>a</sup>	92.83%	92.80%	0.96	<b>3,079.47</b>
Panel I: Rhodium									
1	5.24	<b>24.02</b>	0.39	<b>12.36</b>	<b>19.60</b>	39.10%	38.84%	0.63	<b>152.80</b>
100	0.16	2.22 <sup>a</sup>	0.98	<b>107.27</b>	2.17 <sup>a</sup>	97.97%	97.97%	0.99	<b>11,506.60</b>
1,000	0.14	1.95 <sup>b</sup>	0.98	<b>108.36</b>	2.01 <sup>a</sup>	98.01%	98.01%	0.99	<b>11,742.00</b>
10,000	0.14	1.93 <sup>b</sup>	0.98	<b>108.27</b>	2.00 <sup>a</sup>	98.01%	98.00%	0.99	<b>11,721.90</b>
50,000	0.14	1.93 <sup>b</sup>	0.98	<b>108.27</b>	2.00 <sup>a</sup>	98.01%	98.00%	0.99	<b>11,723.40</b>
Panel J: Ruthenium									
1	3.72	<b>31.24</b>	0.34	<b>12.04</b>	<b>23.60</b>	37.87%	37.61%	0.62	<b>145.04</b>
100	0.10	2.06 <sup>a</sup>	0.98	<b>101.56</b>	1.96 <sup>a</sup>	97.74%	97.74%	0.99	<b>10,314.90</b>
1,000	0.08	1.61	0.98	<b>102.35</b>	1.68 <sup>b</sup>	97.78%	97.77%	0.99	<b>10,476.40</b>
10,000	0.08	1.60	0.98	<b>102.38</b>	1.69 <sup>b</sup>	97.78%	97.77%	0.99	<b>10,482.30</b>
50,000	0.08	1.60	0.98	<b>102.39</b>	1.69 <sup>b</sup>	97.78%	97.77%	0.99	<b>10,483.50</b>
Panel K: Tin									
1	8.79	<b>47.82</b>	0.11	<b>5.46</b>	<b>42.78</b>	11.11%	10.74%	0.33	<b>29.76</b>
100	0.39	<b>3.39</b>	0.96	<b>80.78</b>	<b>3.31</b>	96.48%	96.47%	0.98	<b>6,525.69</b>
1,000	0.34	<b>2.93</b>	0.96	<b>80.62</b>	<b>2.93</b>	96.47%	96.45%	0.98	<b>6,499.20</b>
10,000	0.34	<b>2.91</b>	0.97	<b>80.59</b>	<b>2.92</b>	96.47%	96.45%	0.98	<b>6,495.32</b>
50,000	0.34	<b>2.91</b>	0.97	<b>80.60</b>	<b>2.92</b>	96.47%	96.45%	0.98	<b>6,496.86</b>
Panel L: Zinc									
1	7.04	<b>63.60</b>	0.10	<b>6.12</b>	<b>54.41</b>	13.61%	13.24%	0.37	<b>37.48</b>
100	0.40	<b>3.16</b>	0.95	<b>58.54</b>	<b>3.11</b>	93.51%	93.48%	0.97	<b>3,426.97</b>
1,000	0.38	<b>3.05</b>	0.95	<b>58.65</b>	<b>3.06</b>	93.53%	93.50%	0.97	<b>3,440.06</b>
10,000	0.38	<b>3.04</b>	0.95	<b>58.55</b>	<b>3.06</b>	93.51%	93.48%	0.97	<b>3,427.93</b>
50,000	0.38	<b>3.04</b>	0.95	<b>58.54</b>	<b>3.06</b>	93.51%	93.48%	0.97	<b>3,427.14</b>

The results of the simple regression equation with natural log of the actual price as the dependent variable, and the natural log of the GBM based expected price are presented in this table. The GBM based expected metal prices are based on simulations, ranging from one to fifty-thousand. The t-stat for the intercept and the slope coefficient being equal to zero are presented. The t-stat for the coefficient equal to one are also provided. Equation statistics like R-square, adjusted R-square and F-stats are also provided.  $\rho$  is the correlation coefficient between the actual and expected values, and were obtained by taking square root of R-square. The highlighted values indicate significance at 1%, while the superscripts a and b indicate significance at 5% and 10% respectively. The regression was based on two-hundred and forty months of data for the period between January 2004 and December 2023.

**Table 13: Annual Standard Deviation and Means Tests for Different Simulation**

No. of Simulations	Standard Deviation					Mean				
	Act.	Exp.	P/S	F-stat	p-value	Act.	Exp.	Diff.	t-stat	p-value
Panel A: Silver										
1	0.40	0.81	0.64	4.11	0.00	2.86	2.45	0.41	2.01	0.05
100	0.40	0.50	0.45	1.57	0.33	2.86	2.85	0.00	0.03	0.98
1,000	0.40	0.49	0.45	1.52	0.37	2.86	2.87	-0.01	-0.07	0.94
10,000	0.40	0.49	0.45	1.52	0.37	2.86	2.87	-0.01	-0.08	0.94
50,000	0.40	0.49	0.45	1.52	0.37	2.86	2.87	-0.01	-0.08	0.94
Panel B: Aluminum										
1	0.18	1.69	1.20	88.41	0.00	7.63	6.33	1.30	3.43	0.00
100	0.18	0.20	0.19	1.23	0.66	7.63	7.64	-0.01	-0.10	0.92
1,000	0.18	0.20	0.19	1.24	0.64	7.63	7.65	-0.02	-0.39	0.70
10,000	0.18	0.20	0.19	1.25	0.63	7.63	7.65	-0.02	-0.39	0.70
50,000	0.18	0.20	0.19	1.25	0.64	7.63	7.65	-0.02	-0.40	0.69
Panel C: Copper										
1	0.32	1.49	1.08	22.08	0.00	8.77	7.55	1.21	3.56	0.00
100	0.32	0.41	0.36	1.65	0.28	8.77	8.80	-0.04	-0.32	0.75
1,000	0.32	0.40	0.36	1.63	0.30	8.77	8.81	-0.05	-0.42	0.68
10,000	0.32	0.41	0.36	1.64	0.29	8.77	8.81	-0.05	-0.42	0.68
50,000	0.32	0.41	0.36	1.64	0.29	8.77	8.81	-0.05	-0.42	0.68
Panel D: Iridium										
1	0.98	2.04	1.60	4.32	0.00	6.75	4.58	2.17	4.30	0.00
100	0.98	1.05	1.02	1.15	0.76	6.75	6.71	0.03	0.11	0.92
1,000	0.98	1.05	1.01	1.15	0.77	6.75	6.73	0.02	0.06	0.95
10,000	0.98	1.05	1.01	1.15	0.77	6.75	6.73	0.02	0.05	0.96
50,000	0.98	1.05	1.02	1.15	0.76	6.75	6.73	0.02	0.05	0.96
Panel E: Nickel										
1	0.36	1.75	1.26	23.70	0.00	9.70	8.92	0.78	1.96	0.06
100	0.36	0.39	0.38	1.21	0.69	9.70	9.83	-0.13	-1.10	0.28
1,000	0.36	0.37	0.37	1.09	0.85	9.70	9.84	-0.14	-1.20	0.24
10,000	0.36	0.38	0.37	1.11	0.82	9.70	9.84	-0.14	-1.22	0.23
50,000	0.36	0.38	0.37	1.11	0.82	9.70	9.84	-0.14	-1.22	0.23
Panel F: Lead										
1	0.29	1.13	0.82	15.57	0.00	7.56	6.73	0.82	3.17	0.00
100	0.29	0.39	0.34	1.89	0.18	7.56	7.62	-0.06	-0.56	0.58
1,000	0.29	0.39	0.34	1.85	0.19	7.56	7.62	-0.07	-0.60	0.55
10,000	0.29	0.39	0.34	1.85	0.19	7.56	7.62	-0.07	-0.61	0.54
50,000	0.29	0.39	0.34	1.85	0.19	7.56	7.62	-0.07	-0.62	0.54
Panel G: Palladium										
1	0.77	2.48	1.84	10.41	0.00	6.55	4.75	1.79	3.09	0.00
100	0.77	0.84	0.81	1.21	0.69	6.55	6.59	-0.04	-0.18	0.86
1,000	0.77	0.83	0.80	1.18	0.73	6.55	6.61	-0.07	-0.26	0.79
10,000	0.77	0.83	0.80	1.18	0.73	6.55	6.62	-0.07	-0.28	0.78
50,000	0.77	0.83	0.80	1.18	0.73	6.55	6.62	-0.07	-0.28	0.78
Panel H: Platinum										
1	0.23	1.19	0.86	26.55	0.00	7.00	5.73	1.28	4.71	0.00
100	0.23	0.25	0.24	1.22	0.67	7.00	7.06	-0.05	-0.70	0.49

1,000	0.23	0.25	0.24	1.17	0.73	7.00	7.06	-0.06	-0.80	0.43
10,000	0.23	0.25	0.24	1.18	0.73	7.00	7.06	-0.06	-0.82	0.42
50,000	0.23	0.25	0.24	1.17	0.73	7.00	7.06	-0.06	-0.82	0.42
Panel I: Rhodium										
1	0.99	1.49	1.27	2.24	0.09	7.88	6.30	1.58	3.94	0.00
100	0.99	1.09	1.04	1.20	0.70	7.88	8.02	-0.14	-0.42	0.68
1,000	0.99	1.09	1.04	1.21	0.69	7.88	8.04	-0.16	-0.48	0.63
10,000	0.99	1.09	1.04	1.21	0.68	7.88	8.05	-0.17	-0.51	0.61
50,000	0.99	1.09	1.04	1.21	0.68	7.88	8.05	-0.17	-0.51	0.61
Panel J: Ruthenium										
1	0.88	1.38	1.16	2.42	0.06	5.06	3.55	1.52	4.15	0.00
100	0.88	0.95	0.92	1.16	0.75	5.06	5.20	-0.13	-0.46	0.65
1,000	0.88	0.95	0.92	1.16	0.75	5.06	5.24	-0.17	-0.60	0.55
10,000	0.88	0.96	0.92	1.17	0.74	5.06	5.25	-0.18	-0.63	0.54
50,000	0.88	0.96	0.92	1.17	0.74	5.06	5.25	-0.18	-0.63	0.54
Panel K: Tin										
1	0.43	1.83	1.33	18.50	0.00	9.78	8.39	1.40	3.33	0.00
100	0.43	0.51	0.47	1.46	0.41	9.78	9.79	0.00	-0.02	0.98
1,000	0.43	0.52	0.47	1.48	0.40	9.78	9.80	-0.02	-0.11	0.91
10,000	0.43	0.52	0.47	1.49	0.40	9.78	9.80	-0.02	-0.12	0.91
50,000	0.43	0.52	0.47	1.49	0.40	9.78	9.80	-0.02	-0.12	0.91
Panel L: Zinc										
1	0.32	1.39	1.01	18.91	0.00	7.74	6.48	1.27	3.97	0.00
100	0.32	0.41	0.37	1.62	0.30	7.74	7.78	-0.04	-0.31	0.76
1,000	0.32	0.40	0.36	1.55	0.35	7.74	7.79	-0.05	-0.45	0.66
10,000	0.32	0.40	0.36	1.56	0.34	7.74	7.80	-0.05	-0.48	0.64
50,000	0.32	0.40	0.36	1.56	0.34	7.74	7.80	-0.05	-0.48	0.64

The table provides the results for the differences in standard deviation and means of the natural log of the actual and GBM based expected metal prices. In this table, 'Actual' is the natural log the actual metal price, while 'Expected' is the natural log of the GBM based expected metal price, while 'Pooled' is the pooled standard deviation. F-stat is used to test for the differences in standard deviations, while the pooled standard deviation is used to estimate the t-stat for differences in means. The table reports the p-values for the F-stats and t-stats. This table is for the annual frequency and was based on twenty-years of data for the period between 2004 and 2023. The results in this table are based on different number of simulations.

**Table 14: Quarterly Standard Deviation and Means Test Results for Different Simulation**

No. of Simulations	Standard Deviation					Mean				
	Act.	Exp.	Pooled	F-stat	p-value	Act.	Exp.	Diff.	t-stat	p-value
Panel A: Silver										
1	0.40	1.12	0.84	7.72	0.00	2.84	1.77	1.07	7.99	0.00
100	0.40	0.43	0.42	1.13	0.60	2.84	2.83	0.01	0.11	0.92
1,000	0.40	0.43	0.42	1.12	0.62	2.84	2.84	0.00	-0.05	0.96
10,000	0.40	0.43	0.42	1.12	0.62	2.84	2.84	0.00	-0.06	0.95
50,000	0.40	0.43	0.42	1.12	0.62	2.84	2.84	0.00	-0.06	0.95
Panel B: Aluminum										
1	0.19	1.31	0.94	46.16	0.00	7.62	6.59	1.03	6.98	0.00
100	0.19	0.20	0.20	1.06	0.80	7.62	7.62	0.00	0.11	0.91
1,000	0.19	0.20	0.20	1.04	0.86	7.62	7.63	0.00	-0.13	0.90
10,000	0.19	0.20	0.20	1.04	0.86	7.62	7.63	0.00	-0.16	0.87
50,000	0.19	0.20	0.20	1.04	0.86	7.62	7.63	0.00	-0.16	0.87
Panel C: Copper										
1	0.30	1.09	0.80	12.91	0.00	8.76	7.68	1.08	8.55	0.00
100	0.30	0.33	0.32	1.16	0.51	8.76	8.76	0.00	0.01	0.99
1,000	0.30	0.33	0.31	1.16	0.52	8.76	8.77	-0.01	-0.17	0.86
10,000	0.30	0.33	0.32	1.16	0.51	8.76	8.77	-0.01	-0.20	0.85
50,000	0.30	0.33	0.32	1.16	0.51	8.76	8.77	-0.01	-0.20	0.84
Panel D: Iridium										
1	0.95	1.42	1.21	2.24	0.00	6.75	5.60	1.15	6.01	0.00
100	0.95	0.97	0.96	1.05	0.83	6.75	6.73	0.02	0.13	0.90
1,000	0.95	0.97	0.96	1.04	0.86	6.75	6.74	0.01	0.06	0.95
10,000	0.95	0.97	0.96	1.04	0.86	6.75	6.74	0.01	0.05	0.96
50,000	0.95	0.97	0.96	1.04	0.86	6.75	6.74	0.01	0.05	0.96
Panel E: Nickel										
1	0.35	1.54	1.12	19.18	0.00	9.72	8.61	1.11	6.25	0.00
100	0.35	0.36	0.35	1.02	0.91	9.72	9.74	-0.02	-0.40	0.69
1,000	0.35	0.36	0.36	1.03	0.90	9.72	9.74	-0.03	-0.46	0.64
10,000	0.35	0.36	0.36	1.03	0.89	9.72	9.74	-0.03	-0.48	0.63
50,000	0.35	0.36	0.36	1.03	0.89	9.72	9.74	-0.03	-0.48	0.63
Panel F: Lead										
1	0.30	1.15	0.84	14.66	0.00	7.53	6.61	0.92	6.93	0.00
100	0.30	0.32	0.31	1.15	0.53	7.53	7.54	-0.01	-0.17	0.87
1,000	0.30	0.32	0.31	1.16	0.51	7.53	7.55	-0.01	-0.28	0.78
10,000	0.30	0.32	0.31	1.16	0.51	7.53	7.55	-0.01	-0.31	0.76
50,000	0.30	0.32	0.31	1.16	0.51	7.53	7.55	-0.02	-0.31	0.76
Panel G: Palladium										
1	0.75	1.25	1.03	2.76	0.00	6.53	5.43	1.10	6.76	0.00
100	0.75	0.77	0.76	1.05	0.84	6.53	6.53	0.00	-0.02	0.98
1,000	0.75	0.77	0.76	1.04	0.85	6.53	6.54	-0.01	-0.08	0.93
10,000	0.75	0.77	0.76	1.04	0.85	6.53	6.54	-0.01	-0.09	0.92
50,000	0.75	0.77	0.76	1.04	0.85	6.53	6.54	-0.01	-0.10	0.92
Panel H: Platinum										
1	0.25	1.26	0.91	24.45	0.00	7.02	5.82	1.20	8.37	0.00
100	0.25	0.26	0.25	1.02	0.94	7.02	7.02	-0.01	-0.13	0.89
1,000	0.25	0.26	0.26	1.04	0.87	7.02	7.03	-0.02	-0.37	0.71

10,000	0.25	0.26	0.26	1.04	0.87	7.02	7.03	-0.02	-0.40	0.69
50,000	0.25	0.26	0.26	1.04	0.86	7.02	7.03	-0.02	-0.40	0.69
Panel I: Rhodium										
1	1.00	1.78	1.44	3.15	0.00	7.87	6.61	1.26	5.53	0.00
100	1.00	1.02	1.01	1.04	0.85	7.87	7.88	-0.01	-0.08	0.94
1,000	1.00	1.03	1.01	1.05	0.83	7.87	7.89	-0.02	-0.11	0.91
10,000	1.00	1.03	1.01	1.05	0.83	7.87	7.89	-0.02	-0.13	0.90
50,000	1.00	1.03	1.01	1.05	0.83	7.87	7.89	-0.02	-0.13	0.90
Panel J: Ruthenium										
1	0.87	1.42	1.18	2.66	0.00	5.06	4.15	0.91	4.88	0.00
100	0.87	0.89	0.88	1.04	0.86	5.06	5.07	-0.02	-0.11	0.91
1,000	0.87	0.88	0.88	1.03	0.89	5.06	5.09	-0.03	-0.21	0.83
10,000	0.87	0.89	0.88	1.03	0.88	5.06	5.09	-0.03	-0.22	0.82
50,000	0.87	0.89	0.88	1.03	0.88	5.06	5.09	-0.03	-0.22	0.82
Panel K: Tin										
1	0.41	1.19	0.89	8.34	0.00	9.78	8.67	1.11	7.94	0.00
100	0.41	0.44	0.42	1.12	0.60	9.78	9.77	0.01	0.08	0.93
1,000	0.41	0.43	0.42	1.11	0.64	9.78	9.78	0.00	-0.05	0.96
10,000	0.41	0.43	0.42	1.11	0.64	9.78	9.78	0.00	-0.06	0.95
50,000	0.41	0.43	0.42	1.11	0.64	9.78	9.78	0.00	-0.07	0.95
Panel L: Zinc										
1	0.31	1.16	0.85	13.50	0.00	7.71	6.65	1.06	7.90	0.00
100	0.31	0.33	0.32	1.12	0.60	7.71	7.71	0.00	0.05	0.96
1,000	0.31	0.33	0.32	1.11	0.64	7.71	7.71	-0.01	-0.11	0.92
10,000	0.31	0.33	0.32	1.11	0.64	7.71	7.72	-0.01	-0.12	0.90
50,000	0.31	0.33	0.32	1.11	0.64	7.71	7.72	-0.01	-0.12	0.90

The table provides the results for the differences in standard deviation and means of the natural log of the actual and GBM based expected metal prices. In this table, 'Actual' is the natural log the actual metal price, while 'Expected' is the natural log of the GBM based expected metal price, while 'Pooled' is the pooled standard deviation. F-stat is used to test for the differences in standard deviations, while the pooled standard deviation is used to estimate the t-stat for differences in means. The table reports the p-values for the F-stats and t-stats. This table is for the quarterly frequency and was based on eighty-quarters of data for the period between 1<sup>st</sup> quarter of 2004 and the last quarter of 2023. The results in this table are based on different number of simulations.

**Table 15: Monthly Standard Deviation and Means Test Results for Different Simulation**

No. of Simulations	Standard Deviation					Mean				
	Act.	Exp.	POOLED	F-stat	p-value	Act.	Exp.	Diff.	t-stat	p-value
Panel A: Silver										
1	0.42	1.18	0.89	8.02	0.00	2.84	1.81	1.03	12.74	0.00
100	0.42	0.43	0.42	1.05	0.73	2.84	2.83	0.01	0.19	0.85
1,000	0.42	0.43	0.42	1.04	0.78	2.84	2.84	0.00	-0.04	0.97
10,000	0.42	0.43	0.42	1.04	0.77	2.84	2.84	0.00	-0.06	0.95
50,000	0.42	0.43	0.42	1.04	0.78	2.84	2.84	0.00	-0.06	0.95
Panel B: Aluminum										
1	0.19	1.12	0.81	33.41	0.00	7.63	6.63	0.99	13.49	0.00
100	0.19	0.20	0.20	1.01	0.91	7.63	7.62	0.01	0.36	0.72
1,000	0.19	0.20	0.20	1.01	0.92	7.63	7.63	0.00	-0.03	0.97
10,000	0.19	0.20	0.20	1.01	0.91	7.63	7.63	0.00	-0.09	0.93
50,000	0.19	0.20	0.20	1.01	0.91	7.63	7.63	0.00	-0.09	0.93
Panel C: Copper										
1	0.31	1.07	0.79	11.73	0.00	8.76	7.66	1.10	15.31	0.00
100	0.31	0.32	0.32	1.06	0.66	8.76	8.76	0.01	0.23	0.82
1,000	0.31	0.32	0.32	1.05	0.71	8.76	8.76	0.00	-0.05	0.96
10,000	0.31	0.32	0.32	1.05	0.71	8.76	8.76	0.00	-0.08	0.94
50,000	0.31	0.32	0.32	1.05	0.71	8.76	8.76	0.00	-0.08	0.93
Panel D: Iridium										
1	0.95	1.33	1.15	1.95	0.00	6.72	5.74	0.98	9.35	0.00
100	0.95	0.96	0.95	1.02	0.90	6.72	6.71	0.02	0.19	0.85
1,000	0.95	0.96	0.95	1.02	0.91	6.72	6.72	0.01	0.09	0.93
10,000	0.95	0.96	0.95	1.02	0.91	6.72	6.72	0.01	0.08	0.94
50,000	0.95	0.96	0.95	1.02	0.91	6.72	6.72	0.01	0.08	0.94
Panel E: Nickel										
1	0.36	1.19	0.88	11.10	0.00	9.72	8.71	1.02	12.68	0.00
100	0.36	0.36	0.36	1.01	0.95	9.72	9.72	0.00	0.01	0.99
1,000	0.36	0.36	0.36	1.01	0.94	9.72	9.73	-0.01	-0.24	0.81
10,000	0.36	0.36	0.36	1.01	0.94	9.72	9.73	-0.01	-0.26	0.79
50,000	0.36	0.36	0.36	1.01	0.94	9.72	9.73	-0.01	-0.27	0.79
Panel F: Lead										
1	0.31	1.13	0.83	13.70	0.00	7.54	6.51	1.03	13.53	0.00
100	0.31	0.31	0.31	1.05	0.72	7.54	7.53	0.00	0.16	0.87
1,000	0.31	0.31	0.31	1.05	0.70	7.54	7.54	0.00	-0.13	0.90
10,000	0.31	0.31	0.31	1.05	0.71	7.54	7.54	0.00	-0.16	0.87
50,000	0.31	0.31	0.31	1.05	0.71	7.54	7.54	0.00	-0.16	0.87
Panel G: Palladium										
1	0.75	1.28	1.05	2.90	0.00	6.53	5.51	1.03	10.70	0.00
100	0.75	0.76	0.75	1.01	0.92	6.53	6.52	0.01	0.10	0.92
1,000	0.75	0.76	0.75	1.01	0.91	6.53	6.53	0.00	-0.03	0.97
10,000	0.75	0.76	0.75	1.01	0.91	6.53	6.53	0.00	-0.05	0.96
50,000	0.75	0.76	0.75	1.01	0.91	6.53	6.53	0.00	-0.05	0.96
Panel H: Platinum										
1	0.25	1.11	0.81	19.10	0.00	7.03	5.96	1.07	14.59	0.00
100	0.25	0.26	0.25	1.01	0.94	7.03	7.02	0.00	0.13	0.90
1,000	0.25	0.26	0.25	1.01	0.92	7.03	7.03	0.00	-0.18	0.86

10,000	0.25	0.26	0.25	1.01	0.92	7.03	7.03	0.00	-0.21	0.83
50,000	0.25	0.26	0.25	1.01	0.92	7.03	7.03	0.00	-0.22	0.83
Panel I: Rhodium										
1	1.02	1.64	1.36	2.61	0.00	7.87	6.78	1.09	8.71	0.00
100	1.02	1.03	1.02	1.02	0.88	7.87	7.86	0.01	0.06	0.95
1,000	1.02	1.02	1.02	1.02	0.90	7.87	7.87	0.00	-0.04	0.97
10,000	1.02	1.02	1.02	1.02	0.90	7.87	7.87	0.00	-0.05	0.96
50,000	1.02	1.02	1.02	1.02	0.90	7.87	7.87	0.00	-0.05	0.96
Panel J: Ruthenium										
1	0.87	1.58	1.27	3.32	0.00	5.05	3.94	1.11	9.54	0.00
100	0.87	0.87	0.87	1.02	0.90	5.05	5.05	0.01	0.08	0.94
1,000	0.87	0.87	0.87	1.01	0.94	5.05	5.05	0.00	-0.03	0.98
10,000	0.87	0.87	0.87	1.01	0.94	5.05	5.06	0.00	-0.04	0.97
50,000	0.87	0.87	0.87	1.01	0.94	5.05	5.06	0.00	-0.04	0.97
Panel K: Tin										
1	0.41	1.22	0.91	8.69	0.00	9.78	8.78	1.00	12.03	0.00
100	0.41	0.42	0.42	1.05	0.73	9.78	9.77	0.01	0.27	0.79
1,000	0.41	0.42	0.42	1.04	0.78	9.78	9.78	0.00	0.00	1.00
10,000	0.41	0.42	0.42	1.04	0.79	9.78	9.78	0.00	-0.02	0.98
50,000	0.41	0.42	0.42	1.04	0.79	9.78	9.78	0.00	-0.02	0.98
Panel L: Zinc										
1	0.33	1.19	0.87	13.31	0.00	7.71	6.60	1.11	13.97	0.00
100	0.33	0.33	0.33	1.04	0.78	7.71	7.70	0.01	0.23	0.82
1,000	0.33	0.33	0.33	1.04	0.79	7.71	7.71	0.00	-0.04	0.97
10,000	0.33	0.33	0.33	1.04	0.79	7.71	7.71	0.00	-0.07	0.95
50,000	0.33	0.33	0.33	1.04	0.79	7.71	7.71	0.00	-0.07	0.94

The table provides the results for the differences in standard deviation and means of the natural log of the actual and GBM based expected metal prices. F-stat is used to test for the differences in standard deviations, while the pooled standard deviation is used to estimate the t-stat for differences in means. The table reports the p-values for the F-stats and t-stats. This table is for the monthly frequency and was based on two-hundred and forty months of data for the period between January 2004 and December 2023. The results in this table are based on different number of simulations.

The simple regression results for equation (13) at the annual frequency in *Table 10*, show that when the expected value is based on only one simulation, the hypothesis that the intercept of the equation is equal to zero, is rejected for all twelve metals. Numerically, the intercept ranges from a minimum of 2.53 for silver to a maximum of 9.36 for tin. The slope coefficients range from a minimum of 0.02 for platinum, to a maximum of 0.37 for rhodium. Statistically, however, these coefficients are not different from zero for eleven of twelve metals. For iridium, it takes the value of 0.25 and the hypothesis is rejected at the five percent confidence level. The correlation coefficient between the actual and expected metal prices are observed to be rather low, with only iridium and rhodium with a value greater than 0.50 but still less than 0.60. These regressions also have low R-squares and adjusted R-squares, and equation F-stats do not reject the inappropriateness of the model for ten out of twelve metals. Only for iridium and rhodium the hypotheses are rejected at five and one percent confidence level respectively.

It is observed that as the number of simulations increases towards fifty thousand, regression R-squares and adjusted R-squares increases, especially when the number of simulations increases from one to one hundred, although relatively speaking the improvement is small for aluminum, nickel and zinc. The increase in R-squares and adjusted R-squares for simulations higher than one hundred are not as dramatic than the increase from one to one hundred simulations. A similar pattern is observed for equation F-stats. As the number of simulations increases from one to one hundred, the F-stat value increases from insignificance to significance at one percent confidence level for silver, copper, palladium, rhodium, ruthenium and tin. For nickel, the improved is from insignificance to significance at the ten percent level, while for platinum the improvement is to a five percent significance level. For iridium the improvement is from a five to one percent significance level. The F-stat, however, does not improve to have a significance levels for aluminum and zinc.

The value of the regression equation intercepts decreases as the number of simulations increases, although the decreases is not uniform for all the metals. In most of the cases, the value of the intercept does not change much after one hundred or one thousand simulations. For iridium and palladium, the hypothesis that the value is not different from zero can be rejected. The value of the slope coefficient decreases for all metals with increases in simulation numbers, and the hypotheses that they are equal to zero are rejected for all metals except for aluminum and zinc. The rejection is at the one percent level for silver copper, iridium, palladium, rhodium, ruthenium and tin, if expected values are estimate with one hundred or higher simulations. For lead and platinum, the rejection is at one percent, while it is at the five percent level for nickel. The numerical value of the intercept limits towards one, although it is not quite one for all metals. The hypothesis that they are equal to one is rejected for all metals with at least ten percent confidence except iridium.

The results of the equation (13) simple regression at the quarterly frequency in *Table 11* is similar to that of the annual frequency in *Table 10* but with some exceptions. When expected values are based on only one simulation, F-stats are insignificant, and the regressions have poor R-squares and adjusted R-squares. The values of the regression intercepts are not statistically equal to zero for all metals as the hypotheses are reject at the one percent confidence level. The intercept values decrease and stabilizes at either one hundred or one thousand simulations. The hypotheses that the values are equal to zero are also rejected at one hundred or higher simulations at the ten percent, five percent or one percent significance levels. The values of the slope coefficients are low (or equal to zero in case of silver), but increases as the number of simulations increases. The hypothesis that they are equal to zero are also rejected for all metals except for silver when

expected value is based on only one simulation. The slope coefficient values are also not statistically equal to one, but numerically they are greater than 0.8 for simulation runs greater than one hundred for all twelve metals. The correlation coefficient between expected and actual values are also greater than 0.81 for all twelve metals for simulations runs one hundred or greater.

The expected and actual metal prices demonstrate very high correlation at the monthly frequency, as can be inferred from the correlation coefficients in *Table 12*, especially when the expected prices were based on at least one hundred simulations. The lowest correlation coefficient for simulation runs one hundred or greater is 0.95 for aluminum, and for iridium it is 1.00. Even when the expected price was based on only one simulation run, some of the metals have very high correlation coefficient between the actual and expected prices, though not as good as those for greater number of simulations. The F-stats for regression equation (13) are significant for all equations at all simulation runs including one. The R-squares and adjusted R-squares are greater than 90% for all metals at simulation runs one hundred or greater, although they are comparatively lower when expected prices are estimated using only one simulation run. The hypothesis that the intercept value is statistically equal to zero is rejected for all metals at all simulation runs, except for ruthenium for simulations runs one thousand or greater. Numerically, though, the numbers are in decimals for all metals when expected prices were estimated using at least one hundred simulations. The value of the slope coefficient is not statistically equal to zero for all metals for all simulation runs as the hypothesis that they are equal to zero can be rejected at the one percent confidence level. The hypothesis that the slope coefficient is statistically equal to one, can also be rejected for all metals, with the hypotheses rejected at the one percent level for silver, aluminum, copper, lead, ruthenium, tin and zinc at the one percent confidence level. The hypotheses can be rejected at the five percent level for iridium, nickel and rhodium, though for iridium the rejection for expected price based on one simulation run is at the one percent level. The hypotheses are rejected with ten percent confidence for palladium. Numerically, however, the slope coefficients are greater than 0.94 for all metals when the expected prices are estimated using at least one hundred simulations.

The results for the test for differences in standard deviations and means between the actual and expected metal prices when the expected prices were estimated at different simulations runs are in *Table 13*, *Table 14* and *Table 15* at the annual, quarterly and monthly frequencies respectively. Results show that the hypotheses of no difference between the standard deviations of the natural log of the actual and expected metal prices cannot not be rejected for all metals and all simulation runs, except when the expected prices were estimated using only one simulation. The rejection is with one percent confidence. Similarly, the hypothesis that the differences between the means of the natural logs of actual and expected metal prices are equal to zero can be reject for all metals and all frequencies when the expected prices were estimated using at least one hundred simulations. The hypotheses that the differences are equal to zero cannot be rejected for all metals for all frequencies with at least ten percent confidence level, when the expected prices were estimated using only one simulation run.

Summarizing, this study evaluates the reliability of GBM-based forecasts for twelve metal prices, six base and six precious, across annual, quarterly, and monthly frequencies. Graphical analysis shows that the GBM-expected and actual prices align more closely at higher frequencies. Regression analyses based on a log-linear model reveal that while the GBM forecasts are not perfect proxies, they display strong statistical relationships with actual prices, particularly at the monthly level. The strength of the relationship improves with the number of simulations, with at least 100 simulations yielding significantly better predictive power than lower number of

simulation runs. Tests for differences in means and standard deviations of the natural log of actual and expected prices further affirm the statistical similarity, especially at higher simulation counts. Robustness checks confirm that forecasts based on a single simulation are unreliable, while those based on 100 or more, could be useful. The findings underscore that GBM-based models, when properly specified and sufficiently simulated, can produce useful one-period-ahead forecasts, especially for shorter time horizons. These results consistent with earlier studies on gold (Sinha 2024d) and equity index forecasts (Sinha 2021, 2024a, 2024f).

## VI. Conclusion

Empirical research in metal prices has mushroomed (Pierdzioch and Risse 2020), as accurate metal price forecasts are of great importance to economic agents (Liu et al. 2017). Researchers have studied and established links between vital economic variables like exchange rates (Balcilar, Gupta, and Pierdzioch 2017; Pierdzioch, Risse, and Rohloff 2016a, 2016b; Pukthuanthong and Roll 2011; Reboredo 2013b), interest rates (Agbeyegbe 1989; Baffes and Savescu 2014), inflation rates (Beckmann and Czudaj 2013a), oil prices (Beckmann and Czudaj 2013b; Reboredo 2013a), business cycle (Fama and French 1988), and mineral activity (Huang et al. 2022) to mention a few. Accordingly, a number of different methodologies have been developed to study factors influencing and forecast metal prices (Chen et al. 2014; Dooley and Lenihan 2005; Du et al. 2021; Gil-Alana and Poza 2024). Among the various forecasting methodological approach is GBM, a commonly applied Econophysics (Sinha 2024e, 2024c) technique.

GBM is a stochastic differential equation-based modeling approach that requires drift and diffusion terms along with a standard normal random variable that has an average value of zero, and a standard deviation of one. This approach has been applied to forecast stock prices and indexes (Parungrojratt and Kidsom 2019; Reddy and Clinton 2016; Samuelson 1965; Urama and Ezepue 2018), energy assets (Alhagyan 2024; Lynch et al. 2021), agricultural products (Ibrahim et al. 2021; Zelingher and Makowski 2024), and exchange rates are (Abbas and Alhagyan 2023; Alhagyan 2022), and metals (Hamdan et al. 2020; Huang et al. 2022; Ramos et al. 2019; Roslan and Halim 2024; Soysal 2023; Wilmot 2019). Although each iteration of the random term, generates a forecast, most manuscripts use only a limited number of simulations (Kumar et al. 2024; Sinha 2021). If more than one simulation is used, and each simulation producing a forecast, the issue becomes as to which forecast to use. This was resolved in (Sinha 2021) by estimating expected forecasted values from the forecasts and their associate probabilities. (Sinha 2024a, 2024f) applied this approach to forecast stock index values, while (Sinha 2024d) applied this approach to forecast gold prices.

This study estimates expected metal prices using the (Sinha 2021) framework for six base metals—aluminum, copper, nickel, lead, tin, and zinc—and six precious metals—silver, iridium, palladium, platinum, and rhodium. Forecasts are generated at monthly, quarterly, and annual frequencies using an initial set of 100,000 simulations. The results reveal strong correlations between actual market prices and GBM-derived expected prices at the monthly and quarterly levels across all metals, and at the annual level for most. Regression analyses and statistical tests for differences in means and standard deviations further support the reliability of the GBM-based forecasts. Robustness assessments demonstrate that forecasts based on a single simulation are unreliable, whereas forecasts derived from a minimum of 100 simulations provide reliable estimates.

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## Earnings Growth Forecast for ETFs

Yan He and Junbo Wang<sup>1</sup>

### Abstract

We forecast earnings growth in the next 5 years for stock-market-indexed exchange-traded funds (ETFs). Our methods include the P/E and P/B cross-sectional regression-implied (RI) estimates and the earnings growth random-walk (RW) estimates. Our results show that compared with the actual earnings growth, both the RI and RW forecasts of earnings growth are unbiased for the U.S. ETFs but biased for the foreign ETFs. In addition, the RI method generates smaller forecast errors than the RW method for the U.S. ETFs but holds no advantage over the RW method for the foreign ETFs. Therefore, the RI forecast may be a useful method for the U.S. ETFs during our sample period of 2000-2023 but may not be so for the foreign ETFs.

**Keywords:** Earnings growth forecast; Regression-implied forecast; Random-walk forecast

**JEL classification:** G10

### I. Introduction

A firm's earnings forecast is important to both investors and managers. Numerous studies have been conducted on earnings predictions for individual stocks. In addition, as the exchange-traded funds (ETFs) expand quickly, some researchers have paid attention to the earnings perspectives for equity ETFs. Hence, we divide the existing literature into three categories: analyst estimates of expected earnings for stocks, model-implied estimates of expected earnings for stocks, and earnings perspectives for equity ETFs.

#### Analyst estimates of expected earnings for stocks

First, analyst estimates of expected earnings are compared with actual earnings. Zacks (1979) explains that companies whose analyst consensus forecasts underestimate actual earnings growth outperform the market, whereas companies whose analyst consensus forecasts overestimate actual earnings growth underperform the market. Francis and Philbrick (1993), McNichols and O'Brien (1997), Das, Levine, and Sivaramakrishnan (1998), Lin and McNichols (1998), Lim (2001), Hong and Kubik (2003), and Hand and Martin (2017) provide evidence of analyst optimism. That is, analysts tend to underreact to negative but overreact to positive information, inducing an optimistic bias in their earnings forecasts. The direction of analysts' forecast errors reveals systematic and time-persistent upward bias about firm performance. Jung, Keeley, and Ronen (2019) report that analyst optimism is greater when estimates are issued earlier within the forecast period, indicating that analyst estimates decline over time. Chu and Zhai (2021) show that analyst optimism is greater for higher distress risk firms, implying that analysts underestimate the poor performance of such firms. In contrast to the upward-biased results, Gao and Wu (2014) combine the analysts' long-term growth forecast with several other information sources (past earnings growth, forward earnings-to-price ratio, and past returns), and their method leads to unbiased earnings predictions.

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Second, analyst estimates of expected earnings are compared with time-series model estimates. According to Ball and Brown (1968), Collins and Hopwood (1980), and Fried and Givoly (1982), analysts' forecasts outperform time-series model forecasts. However, Conroy and Harris (1987) argue that the primary forecasting advantages of analysts over time series methods appear to occur over short forecast horizons (less than a year). Bradshaw, Drake, Myers, and Myers (2012) state that analysts' superiority over simple time-series model forecasts is limited. They find that random-walk (RW) forecasts of earnings per share (EPS) are more accurate than analysts' forecasts over longer horizons, for smaller or younger firms, and when analysts forecast negative or large changes in EPS.

Third, the accuracy of analyst estimates may be subject to the influence of economic uncertainty. As Chourou, Purda, and Saadi (2021) demonstrate, economic policy uncertainty is significantly and positively associated with absolute forecast error and forecast dispersion, controlling for competing sources of economy-wide uncertainty such as national elections, recessions, equity market and macroeconomic fluctuations, etc.

### **Model-implied estimates of expected earnings for stocks**

First, expected earnings growth affects price-to-earnings (P/E) and price-to-book (P/B) ratios, and thus, it may be derived from these price multiples. Malkiel and Cragg (1970) and Damodaran (2002) establish the P/E and P/B cross-sectional regression models. Specifically, a firm's P/E is affected by its dividend payout ratio (PAYOUT), expected growth rate of EPS, and beta; and the firm's P/B is affected by its PAYOUT, expected growth rate of EPS, beta, and return on equity (ROE). Recently, Sloan and Wang (2025) demonstrate that a significant proportion of the cross-sectional variation in observed earnings-to-price (E/P) ratios is attributable to rationally anticipated variation in future EPS growth rates. In particular, E/P ratios are strongly and negatively related to future EPS growth rates.

Second, expected earnings are estimated by RW and other time-series models. Many studies discover that annual earnings generally approximate a simple RW process, see Ball and Watts (1972), Albrecht, Lookabill, and McKeown (1977), Watts and Leftwich (1977), Conroy and Harris (1987), and Brown (1993). Moreover, as Bradshaw, Drake, Myers, and Myers (2012) illustrate, very little evidence suggests that more sophisticated time-series models are more accurate than simple RW models of annual earnings. The more sophisticated models demand long-time data series to estimate model parameters, whereas the RW models need no parameter estimates. Hence, the RW models avoid costly data requirements that might skew the results to large and mature firms.

Third, expected earnings growth is assessed based on the slope of a log regression of EPS. Wang, Ke, Liao, Chiang, and Hsu (2020) use the log-linear regression model to forecast the earnings growth rate. Then, they use the expected growth rate to calculate PEGR (the ratio of the stock's price/earnings to its estimated earnings growth rate). Their results show that the returns of the lowest PEGR portfolio dominate those of higher PEGR portfolios.

Fourth, expected earnings growth is derived from the Residual Income Valuation (RIV) model of Ohlson (1995). As Lacina and Ro (2013) show, future long-term earnings growth rates can be derived from the RIV model; and the RIV-imputed earnings growth rates are more accurate and less biased than the analyst long-term earnings growth forecasts.

Fifth, expected earnings are estimated by models including macroeconomic variables. Chant (1980) reveals the predictable relationship between some economic lead-indicators and EPS

numbers. Eckel (1982) uses a model that includes forecasts of industry and economic performance to predict annual EPS, and the results are more accurate than those of a naïve model that incorporates only historical EPS. Bansal, Strauss, and Nasseh (2015) utilize a combination forecast method that involves both firm-specific and macroeconomic variables, leading to substantial improvements in predictive power relative to the autoregressive benchmark.

### **Earnings perspectives for equity ETFs**

ETFs have developed rapidly in the past two decades. Like stocks, ETFs are traded on exchanges throughout the day. In addition, they are open-ended funds that can create and redeem shares at any time. According to the Investment Company Fact Book, the US-registered ETFs had \$10.3 trillion total net assets in 2024.

Several papers have explored the earnings perspectives for equity ETFs. Israeli, Lee, and Sridharan (2017) document that a firm's ETF ownership leads to its reduced earnings response coefficient and declined number of analysts covering the firm. Thus, firms that are widely held by ETFs seem to experience a decrease in informational efficiency. Bhojraj, Mohanram, and Zhang (2020) report that firms held by sector ETFs exhibit more efficient pricing of the underlying firms' earnings news than firms held by broad-based ETFs. So, sector ETFs appear to experience an increase in informational efficiency. Glosten, Nallareddy, and Zou (2021) show that ETF ownership increases short-run informational efficiency for stocks with weak information environments, possibly due to the timely integration of systematic earnings. Fu, Huang, and Tang (2022) investigate the effect of ETFs on their underlying American depositary receipts (ADRs). They show that ETF ownership improves ADR firms' information environment by helping include their systematic earnings into stock returns. Antoniou, Li, Liu, Subrahmanyam, and Sun (2023) point out that inclusion of stocks in industry ETFs enhances investment-q sensitivity, implying greater incorporation of earnings information into prices prior to public releases. Jang and Kang (2024) reveal that higher ownership by ETFs is associated with lower penalty to missing an earnings expectation. So, firms with higher ETF ownership engage in earnings management less frequently, and they are less likely to reduce discretionary spending to meet or beat earnings expectations. Lundholm and Zheng (2025) present that an increase in a firm's industry ETF ownership tends to be followed by an increase in the firm's analyst coverage. This evidence is associated with an ETF strategy: long the firm and short the industry. For example, when hedge funds expect a positive earnings surprise on a firm, they go long in the firm and short in the firm's industry ETF.

Given the above findings, however, there seems to be no research on direct earnings forecasts for individual equity ETFs. Analysts do not closely follow ETFs as they follow stocks, and they do not offer direct estimates of expected earnings for ETFs.<sup>2</sup> Thereby, our study utilizes model-implied estimates of earnings growth for ETFs.

#### **I.4. Purpose and summary of this study**

In this paper, we forecast earnings growth for equity ETFs, rather than for stocks. Specifically, we forecast earnings growth in the next 5 years for stock-market-indexed ETFs. Our methods include the P/E and P/B cross-sectional regression-implied (RI) estimates and the earnings growth random-walk (RW) estimates. The U.S. stock-market-indexed ETFs include SPY,

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<sup>2</sup> The growth estimate for an ETF from analysts, if there is any, is usually indirect, based on the weighted average of analyst estimates for the individual stocks in the ETF.

SPTM, and DIA, while the foreign stock-market-indexed ETFs include IEV, EWJ, and EEM.<sup>3</sup> The entire sample period is from 2000 to 2023.

Our results show that compared with the actual earnings growth, both the RI and RW forecasts of earnings growth are unbiased for the U.S. ETFs but biased for the foreign ETFs. In addition, the RI method generates smaller forecast errors than the RW method for the U.S. ETFs but holds no advantage over the RW method for the foreign ETFs. Therefore, the RI forecast may be a useful method for the U.S. ETFs during our sample period of 2000-2023 but may not be so for the foreign ETFs.

## II. Data and Methods

### Data sample

Our entire period for data collection is from January 2000 to December 2023, and our period for empirical tests is from January 2005 to December 2018. The test period (2005-2018) is shorter than the entire period (2000-2023) for two reasons. First, the regression data from Damodaran's website started in January 2005. Second, the inputs of the past 5-year annual average earnings growth need to go back 5 years (such as from 2005 to 2000); and the inputs of the next 5-year annual average earnings growth need to go forward 5 years (such as from 2018 to 2023).

We gather data from various sources. The monthly data of Fama-French three-factor returns are obtained from French's website, and they are provided respectively for the U.S., developed, and emerging markets.<sup>4</sup> The monthly data of each individual ETF adjusted prices are downloaded from Yahoo's website.<sup>5</sup> The quarterly data of each individual ETF's financial variables and ratios are acquired from FactSet, including EPS, PAYOUT, ROE, P/E, P/B, etc.<sup>6</sup> The annual data of P/E and P/B regression coefficients are collected from Damodaran's website, and they are reported separately for the U.S., European, Japanese, and emerging market companies.<sup>7</sup>

We investigated a total of six market-indexed ETFs. They represent the U.S., European, Japanese, and emerging stock markets. Their periods of available historical data are longer than those of many other similar ETFs. The U.S. ETFs include SPY, SPTM, and DIA, and the foreign ETFs include IEV, EWJ, and EEM. The details are as follows.

- SPY is the ticker symbol for SPDR S&P 500 ETF Trust. Its underlying index is the S&P 500.
- SPTM is the ticker symbol for SPDR Portfolio S&P 1500 Composite Stock Market ETF. Its underlying index is the S&P Composite 1500.
- DIA is the ticker symbol for SPDR Dow Jones Industrial Average ETF Trust. Its underlying index is the DJ Industrial Average.
- IEV is the ticker symbol for iShares Europe ETF. Its underlying index is the S&P Europe 350.

<sup>3</sup> The explanations of these ETF ticker symbols are provided in Section 2.1.

<sup>4</sup> French's website: [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html)

<sup>5</sup> Yahoo's website: <http://finance.yahoo.com>

<sup>6</sup> FactSet publishes quarterly EPS for individual ETFs. For the detailed information on ETF-level EPS construction, please consult both FactSet's general definition and individual ETFs' specific explanations.

<sup>7</sup> Damodaran's website: [http://pages.stern.nyu.edu/~adamodar/New\\_Home\\_Page/datafile](http://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile)

- EWJ is the ticker symbol for iShares MSCI Japan ETF. Its underlying index is the MSCI Japan.
- EEM is the ticker symbol for iShares MSCI Emerging Markets ETF. Its underlying index is the MSCI Emerging Markets.

### Regression-Implied (RI) earnings growth forecast

We derive expected earnings growth from the price-multiple regression models of Damodaran (2002). The P/E and P/B cross-sectional regression models are:

$$P/E = a_0 + a_1 * gEPS + a_2 * PAYOUT + a_3 * BETA, \quad (1)$$

$$P/B = b_0 + b_1 * gEPS + b_2 * PAYOUT + b_3 * BETA + b_4 * ROE, \quad (2)$$

where P/E is a firm's price-to-earnings ratio, P/B is the firm's price-to-book ratio, gEPS is the analyst estimate of the firm's annual average earnings growth in the next 5 years, PAYOUT is the firm's annual dividend payout ratio, BETA is the firm's market risk measure (beta), and ROE is the firm's annual return on equity. We obtain the regression coefficients (a<sub>0</sub> to a<sub>3</sub> and b<sub>0</sub> to b<sub>4</sub>) from Damodaran's website (see Footnote 6). We estimate BETA based on several types of returns; moreover, the data sources of Fama-French three-factor returns differ for the six ETFs. Please see Appendix A for the details of BETA estimation.

Employing the P/E model, we estimate the P/E-regression-implied expected earnings growth for an ETF in a quarter. Hence,  $gEPS^{P/E \text{ model}}$  is derived from the P/E regression model as follows, and t denotes a quarter.

$$gEPS_t^{P/E \text{ model}} = [P/E_t - (a_0 + a_2 * PAYOUT_t + a_3 * BETA_t)] / a_1. \quad (3)$$

The inputs to Equation (3) include the ETF's quarterly data of P/E, PAYOUT, and BETA, as well as the regression coefficients a<sub>0</sub>, a<sub>1</sub>, a<sub>2</sub>, and a<sub>3</sub>. Please note that since regression coefficients are reported annually, the four quarters in a year use the same inputs for a<sub>0</sub>, a<sub>1</sub>, a<sub>2</sub>, and a<sub>3</sub>.

Likewise, employing the P/B model, we estimate the P/B-regression-implied expected earnings growth for the same ETF in the same quarter. Thus,  $gEPS^{P/B \text{ model}}$  is derived from the P/B regression model as follows, and t denotes a quarter.

$$gEPS_t^{P/B \text{ model}} = [P/B_t - (b_0 + b_2 * PAYOUT_t + b_3 * BETA_t + b_4 * ROE_t)] / b_1. \quad (4)$$

The inputs to Equation (4) include the ETF's quarterly data of P/B, PAYOUT, BETA, and ROE, as well as the regression coefficients b<sub>0</sub>, b<sub>1</sub>, b<sub>2</sub>, b<sub>3</sub>, and b<sub>4</sub>. Please note that the four quarters in a year use the same inputs for b<sub>0</sub>, b<sub>1</sub>, b<sub>2</sub>, b<sub>3</sub>, and b<sub>4</sub>.

Then, we take the average of the above two regression-implied earnings growth estimates in each quarter to get the quarterly RI\_gEPS. So, RI\_gEPS denotes an ETF's regression-implied expected earnings growth in the next 5 years. The formula is:

$$RI\_gEPS_t = (gEPS_t^{P/E \text{ model}} + gEPS_t^{P/B \text{ model}}) / 2. \quad (5)$$

In terms of the RI\_gEPS estimation, the data sources of P/E and P/B regressions vary for the six ETFs. Please see Appendix B for the details of RI\_gEPS estimation.

### II.3. Random-Walk (RW) earnings growth forecast

Following previous studies, we assume a random-walk process for earnings growth. The RW expected earnings growth is estimated as:

$$RW\_gEPS_t = PAST\_gEPS_t, \quad (6)$$

where RW\_gEPS is the random-walk annual average earnings growth in the next 5 years, PAST\_gEPS is the past 5-year annual average earnings growth, and t denotes a quarter.

Overall, both forecasts (RI\_gEPS and RW\_gEPS) utilize quarterly rolling and point-in-time evaluations. This design is largely in accordance with the patterns of long-term investments.

### Forecast errors

For either RI\_gEPS or RW\_gEPS, we define the forecast error as the absolute difference between the forecasted and the actual annual average earnings growth in the next 5 years. The formula is:

$$\text{Forecast Error}_t = |\text{FORECAST}_t - \text{AT\_gEPS}_t|, \quad (7)$$

where FORECAST can be either RI\_gEPS or RW\_gEPS, AT\_gEPS is the actual annual average earnings growth in the next 5 years, and t denotes a quarter.

## III. Empirical Results

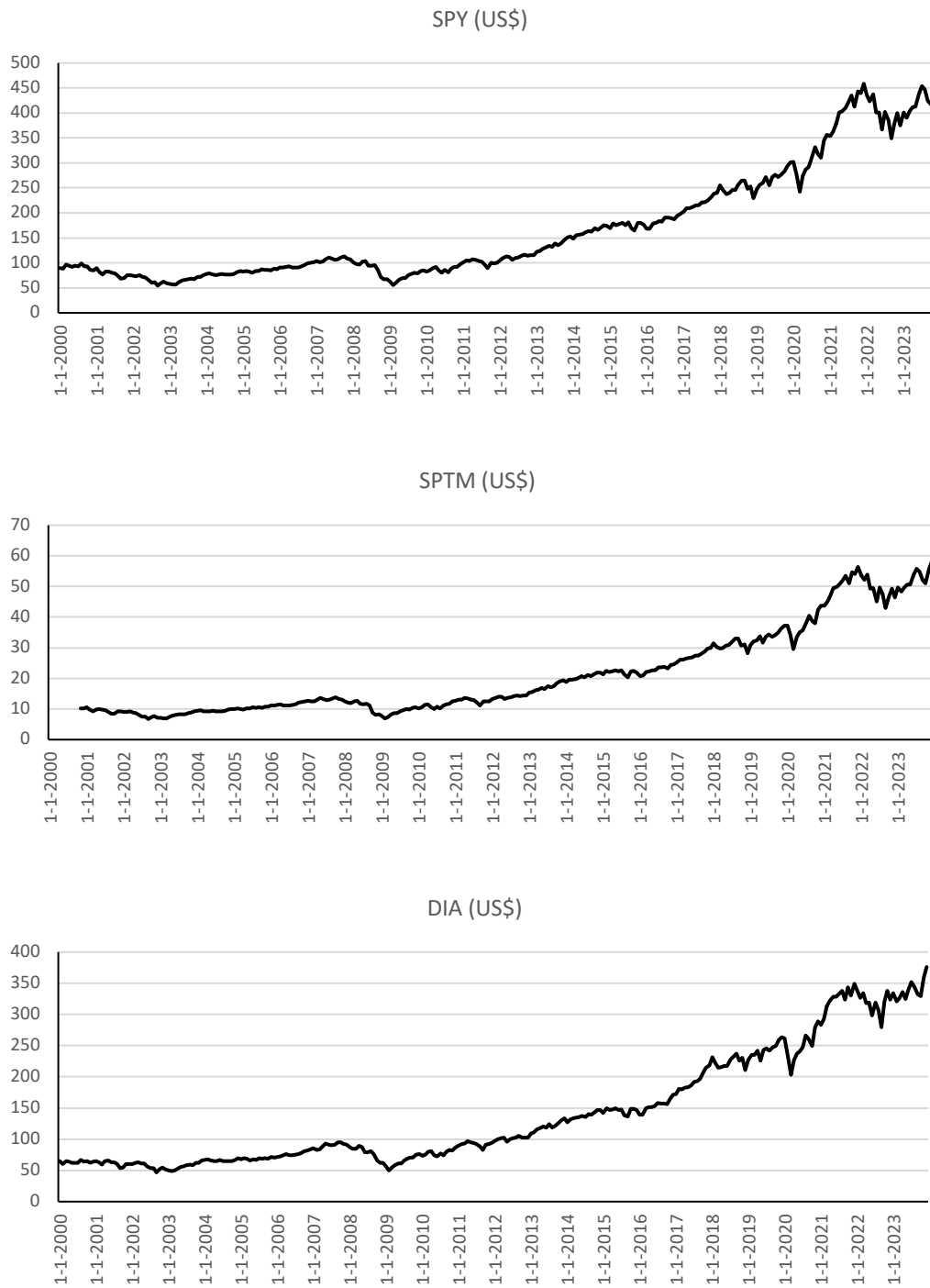
### Preliminary results

Figures 1 and 2 display monthly adjusted prices of the U.S. and foreign stock market ETFs over the entire period. All the prices are denominated in US\$. In Figure 1, the three ETFs (SPY, SPTM, and DIA) represent the three major U.S. stock market indexes (the S&P 500, the S&P Composite 1500, and the DJ Industrial Average). They tend to move together, and their trends are chiefly upward. In Figure 2, the three ETFs (IEV, EWJ, and EEM) represent the three major foreign stock market indexes (the S&P Europe 350, the MSCI Japan, and the MSCI Emerging Markets). They seem volatile, and their trends are largely upward.

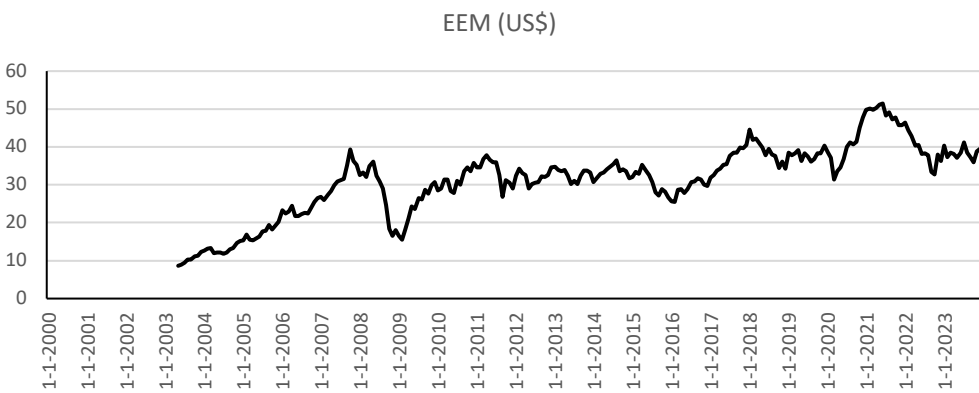
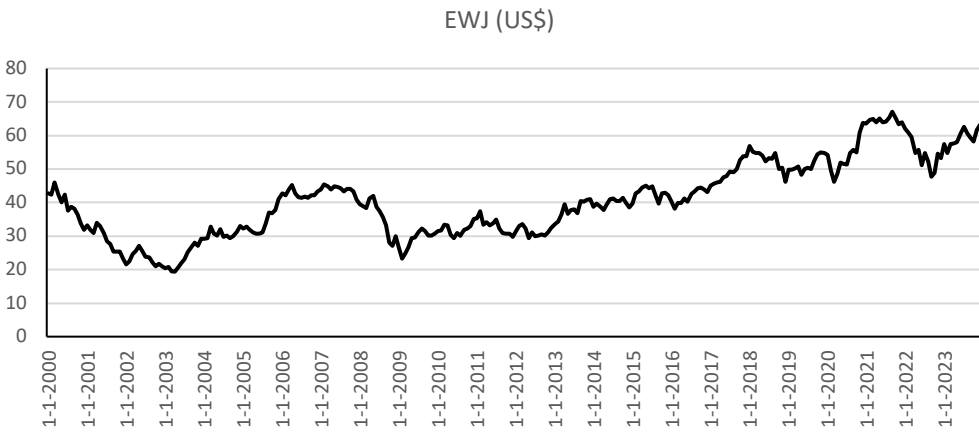
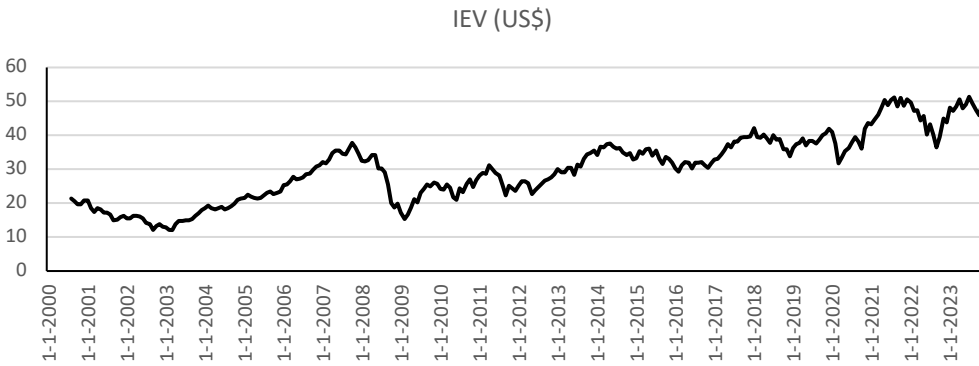
For the RI earnings growth forecast, the inputs contain P/E and P/B ratios as well as other variables. Figures 3 and 4 present quarterly P/E and P/B ratios of the U.S. and foreign stock market ETFs during the test period (2005-2018). In Figure 3, SPY, SPTM, and DIA have similar trends in P/E and P/B, respectively. Both ratios are largely stable, with big volatilities around the great recession of 2008-2009. In Figure 4, IEV, EWJ, and EEM have mildly dissimilar tendencies in P/E and P/B respectively because they are from different foreign regions. Both ratios are mainly steady, with big fluctuations around 2008-2009.

Table 1 summarizes the inputs to the RI earnings growth forecast during the test period (2005-2018). The forecast models are Equations (3), (4), and (5). The inputs to the models include P/E, P/B, BETA, PAYOUT, and ROE. First, we note that the three U.S. ETFs, on average, have higher P/E and P/B ratios than the three foreign ETFs during the test period. Second, among the six ETFs, IEV has the highest BETA (1.18), while EWJ has the lowest

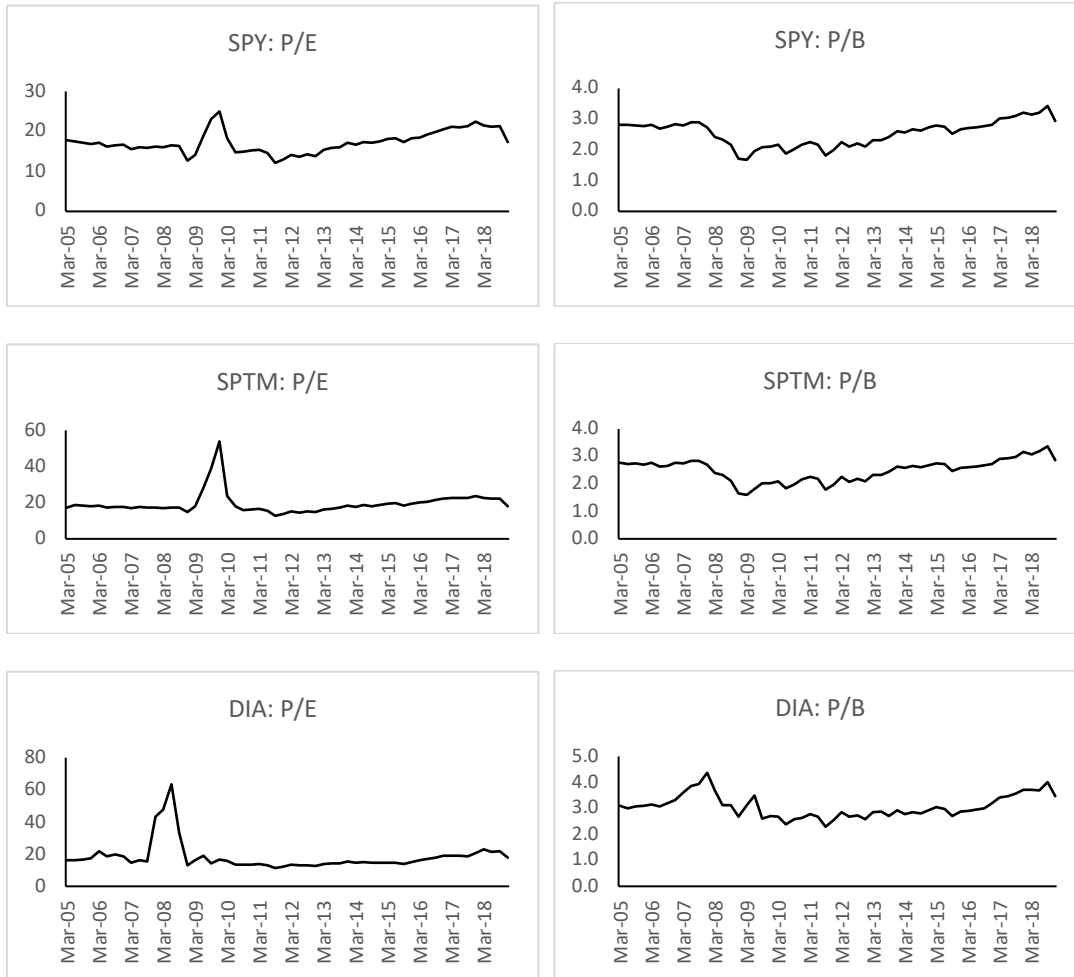
**Figure 1. Monthly Adjusted Prices of U.S. Stock Market ETFs**



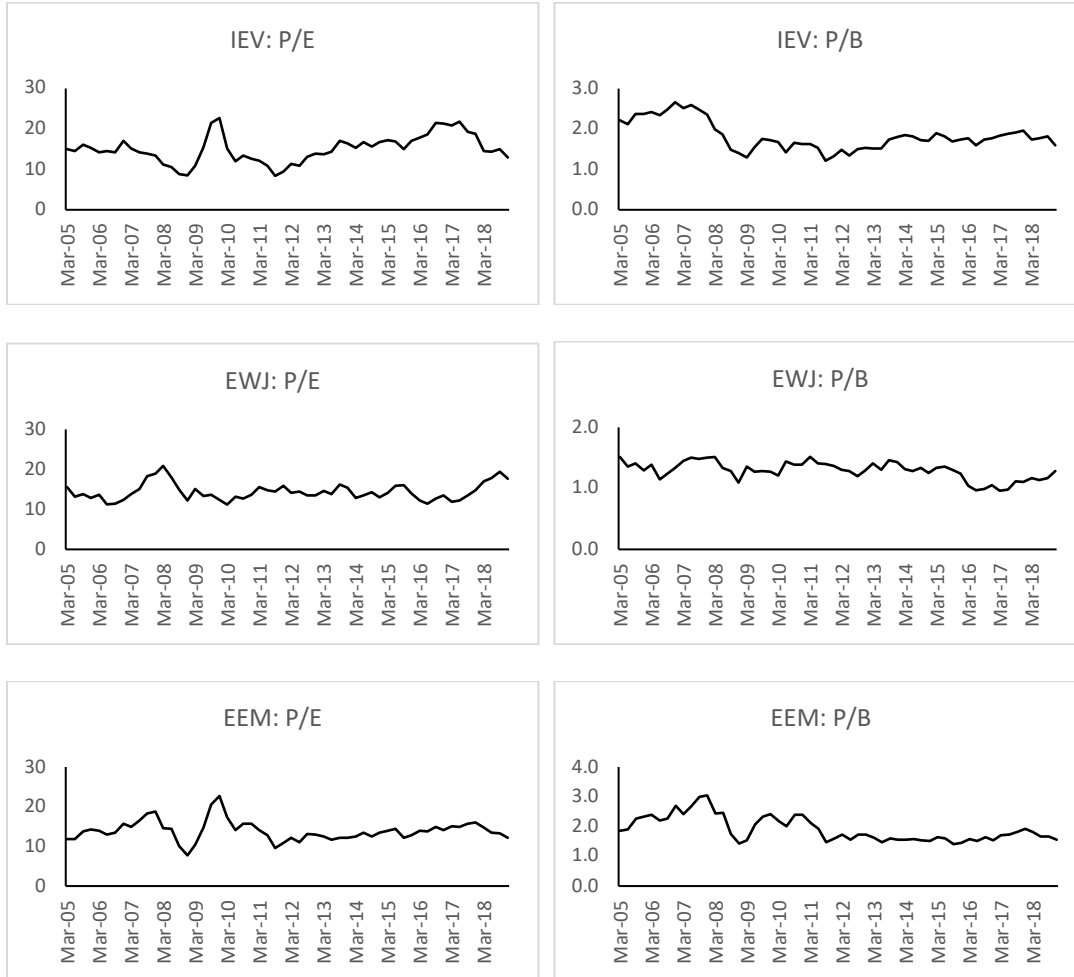
**Figure 2. Monthly Adjusted Prices of Foreign Stock Market ETFs**



**Figure 3. Quarterly P/E and P/B Ratios of U.S. Stock Market ETFs**



**Figure 4. Quarterly P/E and P/B Ratios of Foreign Stock Market ETFs**



**Table 1. Summary of Inputs to the RI Earnings Growth Forecast**

ETF	Statistics	P/E	P/B	BETA	PAYOUT	ROE
SPY	Mean	17.20	2.55	0.96	33.90%	14.94%
	SD	2.74	0.41	0.02	7.00%	1.88%
SPTM	Mean	18.65	2.50	0.99	35.09%	13.65%
	SD	6.12	0.40	0.03	14.32%	2.53%
DIA	Mean	17.10	3.06	0.91	40.76%	18.48%
	SD	8.94	0.44	0.06	24.03%	3.53%
IEV	Mean	14.97	1.82	1.18	47.73%	12.59%
	SD	3.42	0.36	0.10	11.10%	2.96%
EWJ	Mean	14.30	1.29	0.81	33.57%	8.26%
	SD	2.14	0.15	0.13	46.58%	1.74%
EEM	Mean	13.77	1.89	1.05	28.85%	13.82%
	SD	2.51	0.42	0.06	3.19%	2.21%

The table summarizes the inputs to the RI earnings growth forecast. The forecast models are Equations (3), (4), and (5). The inputs to the models include P/E, P/B, BETA, PAYOUT, and ROE. Quarterly data are used. The test period is 2005-2018. SD denotes standard deviation.

BETA (0.81). Third, IEV has the highest PAYOUT (47.73%), while EEM has the lowest PAYOUT (28.85%). Fourth, DIA has the highest ROE (18.48%), while EWJ has the lowest ROE (8.26%).

### U.S. stock market ETFs

Table 2 shows earnings growth statistics for the individual ETFs in the U.S. stock market. Panel A of Table 2 reports annual average earnings growth in the next 5 years. We observe that the three ETFs (SPY, SPTM, and DIA) show similar forecasting outcomes. First, for the ETF of the S&P 500 (SPY), the mean RI\_gEPS is 6.31%, the mean RW\_gEPS is 6.73%, and the mean AT\_gEPS is 6.06%. The t-value on the mean difference between the RI forecast and actual growth is insignificant (0.27), implying that the regression-implied earnings growth forecast is unbiased, compared to the actual earnings growth. The t-value on the mean difference between the RW forecast and actual growth is also insignificant (0.59), implying that the random-walk earnings growth forecast is unbiased, compared to the actual earnings growth. Second, for the ETF of the S&P Composite 1500 (SPTM), the mean RI\_gEPS is 7.95%, the mean RW\_gEPS is 6.38%, and the mean AT\_gEPS is 7.14%. The t-value on the mean difference between the RI forecast and actual growth is insignificant (0.58), indicating that the regression-implied earnings growth is an unbiased forecast. The t-value on the mean difference between the RW forecast and actual growth is also insignificant (-0.45), indicating that the random-walk earnings growth is an unbiased forecast. Third, for the ETF of the DJ Industrial Average (DIA), the mean RI\_gEPS is 6.65%, the mean RW\_gEPS is 6.44%, and the mean AT\_gEPS is 7.72%. The t-value on the mean difference between the RI forecast and actual growth is insignificant (-0.69), suggesting the regression-implied earnings growth as an unbiased forecast. The t-value on the mean difference between the RW forecast and actual growth is also insignificant (-0.69), suggesting the random-walk earnings growth as an unbiased forecast.

**Table 2. Earnings Growth: U.S. Stock Market Individual ETFs****Panel A. Annual average earnings growth in the next 5 years**

ETF	Statistics	Forecast: RI_gEPS	Forecast: RW_gEPS	AT_gEPS
SPY	Mean	6.31%	6.73%	6.06%
	SD	5.00%	6.87%	4.96%
	t value on mean difference: RI (or RW) - AT	0.27	0.59	-
SPTM	Mean	7.95%	6.38%	7.14%
	SD	6.99%	9.67%	7.86%
	t value on mean difference: RI (or RW) - AT	0.58	-0.45	-
DIA	Mean	6.65%	6.44%	7.72%
	SD	7.73%	10.60%	8.33%
	t value on mean difference: RI (or RW) - AT	-0.69	-0.69	-

**Panel B. Forecast errors**

ETF	Statistics	RI_gEPS - AT_gEPS	RW_gEPS - AT_gEPS
SPY	Mean	3.63%	7.31%
	SD	2.48%	6.92%
	t value on mean difference:  RI-AT  -  RW-AT		-3.74*
SPTM	Mean	4.05%	8.89%
	SD	2.85%	12.06%
	t value on mean difference:  RI-AT  -  RW-AT		-2.92*
DIA	Mean	3.73%	11.16%
	SD	2.77%	12.44%
	t value on mean difference:  RI-AT  -  RW-AT		-4.25*

The table shows earnings growth statistics for U.S. stock market individual ETFs. Quarterly data are used. The entire data period is 2000-2023 and the test period is 2005-2018. Panel A reports annual average earnings growth in the next 5 years. Panel B reports forecast errors. SD denotes standard deviation. RI\_gEPS denotes the regression-implied annual average earnings growth in the next 5 years. RW\_gEPS denotes the random-walk annual average earnings growth in the next 5 years. AT\_gEPS denotes the actual annual average earnings growth in the next 5 years. The star (\*) represents statistical significance at the 5% level.

Panel B of Table 2 reports forecast errors for the individual ETFs in the U.S. stock market. As we note, the three ETFs (SPY, SPTM, and DIA) demonstrate similar error patterns. For SPY, the mean regression-implied forecast error is 3.63%; the mean random-walk forecast error is 7.31%; and the t-value on the mean difference (-3.74) is negative and significant at the 5% level, indicating smaller errors of the RI forecast than those of the RW forecast. For SPTM, the mean regression-implied forecast error is 4.05%; the mean random-walk forecast error is 8.89%; and the t-value on the mean difference (-2.92) is negative and significant, implying smaller errors of the RI (vs. RW) forecast. For DIA, the mean regression-implied forecast error is 3.73%; the mean random-walk forecast error is 11.16%; and the t-value on the mean difference (-4.25) is negative and significant, suggesting smaller errors of the RI (vs. RW) forecast.

**Table 3. Earnings Growth: U.S. Stock Market Panel ETFs****Panel A. Annual average earnings growth in the next 5 years**

Statistics	Forecast: RI_gEPS	Forecast: RW_gEPS	AT_gEPS
Mean	6.89%	6.43%	6.73%
SD	6.67%	9.10%	7.19%
Median	6.59%	4.99%	5.72%
Max	40.94%	41.89%	41.89%
Min	-4.82%	-20.52%	-5.56%
t value on mean difference: RI (or RW) - AT	0.21	-0.33	

**Panel B. Forecast errors**

Statistics	RI_gEPS - AT_gEPS	RW_gEPS - AT_gEPS
Mean	3.77%	8.69%
SD	2.69%	10.84%
Median	3.65%	6.95%
Max	12.22%	60.91%
Min	0.02%	0.06%
t value on mean difference:  RI-AT  -  RW-AT		-5.66*

The table shows earnings growth statistics for U.S. stock market panel ETFs. Quarterly data are used. The entire data period is 2000-2023 and the test period is 2005-2018. The U.S. panel contains SPY, SPTM, and DIA during the test period. Panel A reports annual average earnings growth in the next 5 years. Panel B reports forecast errors. SD denotes standard deviation. RI\_gEPS denotes the regression-implied annual average earnings growth in the next 5 years. RW\_gEPS denotes the random-walk annual average earnings growth in the next 5 years. AT\_gEPS denotes the actual annual average earnings growth in the next 5 years. The star (\*) represents statistical significance at the 5% level.

Table 3 shows earnings growth statistics for the panel ETFs in the U.S. stock market. The U.S. panel contains quarterly data of SPY, SPTM, and DIA during the test period. Panel A of Table 3 reports annual average earnings growth in the next 5 years. The results here are in line with those in Table 2. For the U.S. panel of SPY, SPTM, and DIA, the mean RI\_gEPS is 6.89%, the mean RW\_gEPS is 6.43%, and the mean AT\_gEPS is 6.73%. The t-value on the mean difference between the RI forecast and actual growth is insignificant (0.21), implying that the regression-implied earnings growth is unbiased. The t-value on the mean difference between the RW forecast and actual growth is also insignificant (-0.33), implying that the random-walk earnings growth is unbiased. Therefore, both the regression-implied and random-walk forecasts of earnings growth in the next 5 years are unbiased for the U.S. stock market panel ETFs.

Panel B of Table 3 reports forecast errors for the panel ETFs in the U.S. stock market. The outcomes here are consistent with those in Table 2. For the U.S. panel of SPY, SPTM, and DIA, the mean regression-implied forecast error is 3.77%, and the mean random-walk forecast error is 8.69%. The t-value on the mean difference (-5.66) is negative and significant at the 5% level, indicating smaller errors of the RI forecast than those of the RW forecast. Thus, for the U.S. stock market panel ETFs, the regression-implied forecast of earnings growth in the next 5 years tends to be more effective than the random-walk forecast.

## Foreign stock market ETFs

Table 4 presents earnings growth statistics for the individual ETFs in the foreign stock market. Panel A of Table 4 reports annual average earnings growth in the next 5 years. We note that the three ETFs (IEV, EWJ, and EEM) display various forecasting outcomes. First, for the ETF of the S&P Europe 350 (IEV), the mean RI\_gEPS is  $-19.07\%$ , the mean RW\_gEPS is  $10.06\%$ , and the mean AT\_gEPS is  $0.81\%$ . The t-value on the mean difference between the RI forecast and actual growth is significant and negative ( $-4.62$ ), implying that the regression-implied earnings growth is biased downward. The t-value on the mean difference between the RW forecast and actual growth is significant and positive ( $2.06$ ), implying that the random-walk earnings growth is biased upward. Second, for the ETF of the MSCI Japan (EWJ), the mean RI\_gEPS is  $-19.97\%$ , the mean RW\_gEPS is  $19.92\%$ , and the mean AT\_gEPS is  $1.85\%$ . The t-value on the mean difference between the RI forecast and actual growth is significant and negative ( $-5.80$ ), indicating that the regression-implied earnings growth is biased downward. The t-value on the mean difference between the RW forecast and actual growth is significant and positive ( $3.85$ ), indicating that the random-walk earnings growth is biased upward. Third, for the ETF of the MSCI Emerging Markets (EEM), the mean RI\_gEPS is  $-4.26\%$ , the mean RW\_gEPS is  $3.12\%$ , and the mean AT\_gEPS is  $-0.45\%$ . The t-value on the mean difference between the RI forecast and actual growth is insignificant ( $-1.63$ ), suggesting the regression-implied earnings growth as an unbiased forecast. The t-value on the mean difference between the RW forecast and actual growth is significant and positive ( $2.71$ ), suggesting the random-walk earnings growth as an upward-biased forecast.

Panel B of Table 4 reports forecast errors for the individual ETFs in the foreign stock market. We note that the three ETFs (IEV, EWJ, and EEM) demonstrate comparable error patterns. For IEV, the mean RI forecast error is  $22.33\%$ ; the mean RW forecast error is  $18.05\%$ ; and the t-value on the mean difference is insignificant ( $0.79$ ), indicating similar errors of the RI and RW forecasts. For EWJ, the mean RI forecast error is  $24.71\%$ ; the mean RW forecast error is  $18.73\%$ ; and the t-value on the mean difference is insignificant ( $1.48$ ), implying similar errors of the RI and RW forecasts. For EEM, the mean RI forecast error is  $9.02\%$ ; the mean RW forecast error is  $7.16\%$ ; and the t-value on the mean difference is insignificant ( $1.10$ ), suggesting similar errors of the RI and RW forecasts.

Table 5 provides earnings growth statistics for the panel ETFs in the foreign stock market. The foreign panel contains quarterly data of IEV, EWJ, and EEM during the test period. Panel A of Table 5 reports annual average earnings growth in the next 5 years. The results here are analogous to those in Table 4. For the foreign panel of IEV, EWJ, and EEM, the mean RI\_gEPS is  $-14.48\%$ , the mean RW\_gEPS is  $8.51\%$ , and the mean AT\_gEPS is  $0.81\%$ . The t-value on the mean difference between the RI forecast and actual growth is significant and negative ( $-6.87$ ), implying that the regression-implied earnings growth is biased downward. The t-value on the mean difference between the RW forecast and actual growth is significant and positive ( $3.90$ ), implying that the random-walk earnings growth is biased upward. Therefore, both the RI and RW forecasts of earnings growth in the next 5 years are biased for the foreign stock market panel ETFs.

Panel B of Table 5 presents forecast errors for the panel ETFs in the foreign stock market. The results here are comparable to those in Table 4. For the foreign panel of IEV, EWJ, and EEM, the mean RI forecast error is  $18.47\%$ , and the mean RW forecast error is  $14.58\%$ . The t-value on the mean difference ( $1.51$ ) is insignificant, indicating similar errors of the RI and RW forecasts.

Thus, for the foreign stock market panel ETFs, the RI forecast of earnings growth in the next 5 years holds no advantage over its RW counterpart.

**Table 4. Earnings Growth: Foreign Stock Market Individual ETFs**

**Panel A. Annual average earnings growth in the next 5 years**

ETF	Statistics	Forecast: RI_gEPS	Forecast: RW_gEPS	AT_gEPS
IEV	Mean	-19.07%	10.06%	0.81%
	SD	29.44%	30.80%	6.44%
	t value on mean difference: RI (or RW) - AT	-4.62*	2.06*	-
EWJ	Mean	-19.97%	12.92%	1.85%
	SD	25.04%	18.39%	8.08%
	t value on mean difference: RI (or RW) - AT	-5.80*	3.85*	-
EEM	Mean	-4.26%	3.12%	-0.45%
	SD	14.17%	7.44%	3.56%
	t value on mean difference: RI (or RW) - AT	-1.63	2.71*	-

**Panel B. Forecast errors**

ETF	Statistics	RI_gEPS - AT_gEPS	RW_gEPS - AT_gEPS
IEV	Mean	22.33%	18.05%
	SD	26.32%	27.38%
	t value on mean difference:  RI-AT  -  RW-AT		0.79
EWJ	Mean	24.71%	18.73%
	SD	22.78%	16.70%
	t value on mean difference:  RI-AT  -  RW-AT		1.48
EEM	Mean	9.02%	7.16%
	SD	8.72%	6.03%
	t value on mean difference:  RI-AT  -  RW-AT		1.10

The table presents earnings growth statistics for foreign stock market individual ETFs. Quarterly data are used. The entire data period is 2000-2023 and the test period is 2005-2018. Panel A reports annual average earnings growth in the next 5 years. Panel B reports forecast errors. SD denotes standard deviation. RI\_gEPS denotes the regression-implied annual average earnings growth in the next 5 years. RW\_gEPS denotes the random-walk annual average earnings growth in the next 5 years. AT\_gEPS denotes the actual annual average earnings growth in the next 5 years. The star (\*) represents statistical significance at the 5% level.

**Table 5. Earnings Growth: Foreign Stock Market Panel ETFs****Panel A. Annual average earnings growth in the next 5 years**

Statistics	Forecast: RI_gEPS	Forecast: RW_gEPS	AT_gEPS
Mean	-14.48%	8.51%	0.81%
SD	25.24%	22.17%	6.51%
Median	-12.35%	5.14%	0.00%
Max	40.01%	155.89%	38.11%
Min	-111.05%	-20.05%	-20.05%
t value on mean difference: RI (or RW) - AT	-6.87*	3.90*	-

**Panel B. Forecast errors**

Statistics	RI_gEPS - AT_gEPS	RW_gEPS - AT_gEPS
Mean	18.47%	14.58%
SD	22.38%	20.23%
Median	14.19%	10.38%
Max	116.43%	155.89%
Min	0.05%	0.13%
t value on mean difference:  RI-AT  -  RW-AT		1.51

The table presents earnings growth statistics for foreign stock market panel ETFs. Quarterly data are used. The entire data period is 2000-2023 and the test period is 2005-2018. The foreign panel contains IEV, EWJ, and EEM during the test period. Panel A reports annual average earnings growth in the next 5 years. Panel B reports forecast errors. SD denotes standard deviation. RI\_gEPS denotes the regression-implied annual average earnings growth in the next 5 years. RW\_gEPS denotes the random-walk annual average earnings growth in the next 5 years. AT\_gEPS denotes the actual annual average earnings growth in the next 5 years. The star (\*) represents statistical significance at the 5% level.

**IV. Conclusions**

We forecast earnings growth in the next 5 years for six stock-market-indexed ETFs. Our approach is the model-implied estimates of expected earnings growth, including the RI and RW estimates. Our forecasts adopt quarterly rolling and point-in-time evaluations. Our results differ between the U.S. and foreign ETFs.

For the three U.S. stock-market-indexed ETFs (SPY, SPTM, and DIA), both the RI and RW forecasts of earnings growth are unbiased in comparison with the actual earnings growth. Moreover, the RI method generates smaller forecast errors than the RW method. Therefore, the RI forecast seems a better method than the RW forecast for the U.S. ETFs during 2000-2023.

For the three foreign stock-market-indexed ETFs (IEV, EWJ, and EEM), the RI forecast of earnings growth is biased downward and the RW forecast is biased upward when both forecasts are compared with the actual earnings growth. Furthermore, the RI and RW methods generate similar forecast errors. Hence, the RI forecast seems to hold no advantage over the RW forecast for the foreign ETFs during 2000-2023.

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## Appendix A: BETA Estimation

The appendix shows the data sources in the BETA estimation for the six ETFs. BETA is estimated based on monthly data in the past three years, and it is the slope coefficient in the following regression:

$$R_i - R_f = \text{intercept} + \text{slope} * (R_m - R_f),$$

where  $R_i$  is the monthly return of an ETF,  $R_f$  is the monthly return of 1-month Treasury bills, and  $R_m - R_f$  is the monthly excess market return.  $R_f$  and  $R_m - R_f$  are obtained from the Fama-French three-factor data sources.

<i>ETF</i>	<i>Data Sources for <math>R_f</math> and <math>R_m - R_f</math></i>
SPY	U.S. market: Fama-French three-factor returns
SPTM	U.S. market: Fama-French three-factor returns
DIA	U.S. market: Fama-French three-factor returns
IEV	Developed markets: Fama-French three-factor returns
EWJ	Developed markets: Fama-French three-factor returns
EEM	Emerging markets: Fama-French three-factor returns

**Appendix B: RI\_gEPS Estimation**

The appendix shows the data sources used in the RI\_gEPS estimation for the six ETFs. For the RI\_gEPS estimation, see equations (3), (4), and (5) in Section 2.2.

<i>ETF</i>	<i>Data Sources for Regression Coefficients</i>
SPY	U.S. companies: P/E and P/B regressions
SPTM	U.S. companies: P/E and P/B regressions
DIA	U.S. companies: P/E and P/B regressions
IEV	European companies: P/E and P/B regressions
EWJ	Japanese companies: P/E and P/B regressions
EEM	Emerging market companies: P/E and P/B regressions

## CEO Power, CEO Compensation, and Firm Performance

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### Abstract

We investigate the impact of CEO power on the relation between CEO compensation and firm performance to find how CEO incentive compensation affects firm performance by reducing agency conflicts between managers and shareholders. We measure the CEO pay slice (CPS) for CEO power and the pay-performance sensitivity (PPS) for CEO incentive compensation. Employing standard control variables, we run multiple OLS regressions and show that PPS increases firm performance at the high level of CPS, but the impact of PPS decreases at the low level of CPS. To resolve the potential endogeneity concerns, we perform robustness checks by adopting instrumental variables in a two-stage least square (2SLS) estimation. We also consider the year-effect and find that our results remain the same as before. The finding implies that considering stand-alone associations of either PPS or CPS with firm performance—a common practice in the literature—will not be appropriate because there is an interaction effect between CEO power and incentive compensation.

**Keywords:** CEO compensation, Pay-performance sensitivity, CEO pay slice, Firm performance

**JEL Classification:** G30, G35, J33

### I. Introduction

CEO compensation is one of the most debated topics in the literature of corporate finance. In the last few decades, researchers have explored numerous ways to alleviate the agency conflict between the shareholders/owners (principals) and the managers/executives (agents) in modern corporations, and subsequently offered various solutions such as offering equity-based compensation (EBC), long-term incentive plans in the form of stock options, restricted stock grants etc. to CEOs in order to enhance their ownership and attract their commitment to firm performance. However, analyses of such propositions have found mixed results on the efficacy of high compensation packages on firm performance. There exist two viewpoints in general. First, the optimal contracting theory suggests executive incentive contracts to maximize shareholders' value. Second, the rent seeking theory, or the managerial power view of compensation argues that agency conflicts could be worse in lucrative compensation packages, leading to shareholders' value destruction (Bebchuk & Fried, 2006).

A common measure of executive incentives is the pay-performance sensitivity (PPS) - defined as the marginal increase in CEO pay given an incremental effect on firm value - which is

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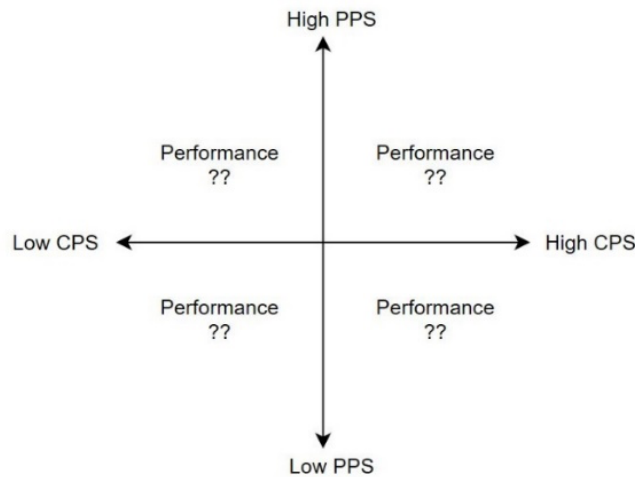
positively associated with firm performance (Jensen & Murphy, 1990). A recent body of literature focuses on executive power, or relative importance of CEOs and supports the rent seeking theory. It considers CEO pay slice (CPS)—the proportion of salary captured by CEO from aggregate compensation of the top-five executives—as a measure of pay-heterogeneity and CEO dominance and finds that it is negatively associated with firm value (Bebchuk et al., 2011). However, the negative association invites concern from researchers, who have found conflicting results based on these instruments. For instance, while studying the association of CEO power to firm value using CEO pay slice as a proxy, some researchers found non-monotonic relations to various agency conflict issues (Chintrakarn et al., 2015; Jiraporn & Chintrakarn, 2013; Lee et al., 2015). In addition, some authors support CEO power in favor of firm riskiness as it improves firm value (Larcker & Tayan, 2012). Mixed results in the literature motivate this study to pursue reasons behind the potential inconsistency with a perspective different from previous studies.

It is important to note that executive power might vary across firms since it connects to the idiosyncratic characteristics of firm factors such as the team structure, compensation practices etc. Incentive compensation appears to be linked with the CEO power as well. In a recent article published in Bloomberg, Melin and Sam (2020) rank top-paid CEOs and segregate their compensation components. They show that incentive compensation packages such as stock options hold a major portion in the CEO compensation contracts. When we study these CEOs to understand their dominance level, we observe that they have a very high level of CPS.<sup>1</sup> The link between powerfulness and incentive payment becomes more prominent when we examine the calculation of the proxies i.e., CPS and PPS. While CPS indicates the pay heterogeneity among top executives, the calculation involves the Black & Scholes value of stock options granted in a given year. On the other hand, Coles et al. (2006) follow Core and Guay (2002) for PPS calculation and consider the sensitivity of stock option portfolios that consist of all the vested and unvested tranches of stock options. Based on (1) the mixed results between CEO dominance and firm performance documented in prior literature, (2) the current industry practice of incentive compensation packages for powerful CEOs, and (3) the nature of the CEO compensation components and their proxies, we posit that PPS being a measure of executive incentives is not separable from CPS, a measure of executive dominance, and thus, argue that their stand-alone associations with firm performance might be inadequate in drawing implications on agency conflicts.

To capture the compensation component, this study simultaneously considers the incentive component measured by PPS and the dominance or the pay-heterogeneity component measured by CPS instead of focusing only on incentives or cash compensation. Ozkan (2011) argues that studies that do not consider the equity-based component of compensation could be criticized and shows that performance-sensitive component of compensation is vital in assessing the association of pay with performance. However, in contrast to our analysis, the author only focuses on UK CEOs and does not consider the executive dominance component that is naturally prevalent in companies. Finally, we also examine the interaction effect between PPS and CPS, which, to the best of our knowledge, has not been considered in prior studies.

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<sup>1</sup>For instance, Tim Cook of Apple Inc. and Tom Rutledge of Charter Communications Inc. have a CPS of 72% as of 2019.

**Figure 1: Prospective interaction between PPS and CPS.**

The figure portrays how this study analyzes the interaction effect by considering each quadrant.

The analysis of interaction effect appears crucially important since it proposes answers to questions such as: (1) Is PPS always positively associated with firm performance? (2) Does the negative association of CPS with firm performance always hold? (3) Is it better to have non-dominant CEOs? etc. To understand the effect, we have analyzed CEO compensation from the Execucomp database for the 1992-2019 period. Firm specific data and stock-related information are obtained from Compustat and CRSP, respectively. The analysis of this study is comprised of two parts, each of which examines six OLS models. While Tobin's  $q$  as a proxy of firm value is taken as the dependent variable in the first part, the second part considers annual stock returns. We first evaluate stand-alone associations, and then further analyze the interaction effect. Figure 1 portrays how this study analyzes the interaction effect by considering each quadrant. Estimates from the stand-alone cases indicate that PPS shows a positive association to firm performance and CPS shows a negative association to firm performance, both of which are aligned with the literature. However, findings from the interaction effect show that at high level of CPS, changing PPS level from low to high increases firm performance (measured by annual stock returns), but at low level of CPS, influence of PPS on firm performance declines. This indicates that stand-alone association of either executive incentives (PPS) or executive dominance (CPS) to firm performance could be inadequate in understanding the firm performance implications - a contribution of this study to the existing literature. This finding is different in contrast to Lee et al. (2015), where they estimate an optimal level of power and find that power has a nonmonotonic relation to the firm value i.e., non-optimal power such as excess or deficient power of CEO is negatively related to firm performance. This study indicates that while inadequate power (or, importance) of CEO might not lead to strong performance for the firm, excess executive dominance can be managed for better firm performance by granting incentive compensation with high PPS. The high caliber CEOs who have gained importance indicated by their high CPS levels<sup>2</sup> might not be that bad for the firm, even though their heterogeneous pay structure appears to be linked to firm's value destruction as shown by Bebchuk et al. (2011). Such dominant CEOs might

<sup>2</sup> For example, Sundar Pichai, chief executive officer (CEO) of Alphabet Inc. and its subsidiary Google LLC, has a CPS of 71% (quite high) as of 2019.

still improve firm performance if provided with an optimal bundle of vested and unvested stock options (which would increase their PPS levels) - especially, may be for extended vesting periods - along with the existing compensation contract. In line with the optimal contracting theory, this study contributes to the existing body of literature that circulates around agency conflict issues by adding new perspectives based on the interaction effect, and opens opportunities for further investigations on the optimal contracts.

We organize the remainder of this study as follows. In Section 2 we review the prior literature and develop hypotheses. Data and methodology are described in Section 3, which is followed by results in Section 4. Section 5 discusses the robustness checks. Finally, Section 6 concludes.

## **II. Literature review and hypotheses development**

### **Measures to minimize agency conflicts**

Firms hire CEOs for the best interest of shareholders: maximizing firm value or shareholders' wealth regardless of size, location, industry, and timeline. However, Jensen and Meckling (1976) argue that these managers do not always follow their mission, but rather seek their own interest by extracting undeserved personal benefits. Deep-rooted in agency conflict, several inefficiencies do not let firms acquire the maximum value that they could potentially achieve. Jensen and Meckling (1976) categorize these inefficiencies into three types of expenditure - bonding costs towards the agents, monitoring expenditure by the principals, and opportunity costs or the residual loss due to the divergence in differential interests between the principal and the agent. To establish an efficient management, shareholders incentivize managers through potentially optimal contracts with equity-based compensation and numerous benefits. These high CEO pays are set on purpose as a motivation to improve firm performance (Edmans & Gabaix, 2009). Dow and Raposo (2005) further show that equity-based CEO compensation is a crucial key to make the interest of managers aligned with that of shareholders. Although it appears that equity-based compensation packages reduce the agency problem, in the last few decades researchers have found mixed results on the impacts of equity-based compensation on better firm performance.

Earlier work on compensation effects focused only on salary and bonus to understand the impact of pay on performance.<sup>3</sup> However, since CEOs do not have strong obligation to the firm due to lack of ownership and could make investment decisions destroying firm values by taking pet projects, building empires through personally preferred merger and acquisitions, etc. to pursue own interest, EBC has evolved to ensure CEOs' incentives in the firm (Hölmstrom, 1979; Rajgopal & Shevlin, 2002). Frydman and Jenter (2010) show dramatic increase in compensation that is highly driven by EBCs during the 1970s. However, several studies (Bulan et al., 2010; Demsetz, 1983; Fama & Jensen, 1983; Smith & Stulz, 1985) suggest that even CEO ownership has mixed influence to firm performance. In the early 1990s, stock options became popular in industry practice as an instrument of equity-based CEO pay (Langsam et al., 1997). However, due to several corporate scandals (e.g., backdating in stock option grants), major reforms in the executive compensation were done in the mid-2000s (Bulan et al., 2010). In addition, restricted stock is granted to encourage long-term performance accompanied with other components in the

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<sup>3</sup> Murphy (1985) criticizes this practice and propose more components of executive compensation such as base salaries, bonuses, total compensation, and deferred compensation.

compensation contracts. Studies based on these several industry practices during this timeline have taken two main directions – the optimal contracting theory and the rent seeking theory.

According to the optimal contracting theory, equity-based compensation would be optimal in incentivizing managers (Fama, 1980; Jensen & Murphy, 1990; Lee et al., 2008; Murphy, 1985). On the other hand, the rent seeking theory suggests that as CEOs influence their own compensation packages, they would pursue their own interest over shareholders' interest (Bebchuk & Fried, 2006). At their own discretion, these CEOs manipulate their compensation packages and look for opportunities to enlarge their undeserved compensation at the expense of other stakeholders such as shareholders and employees. Researchers have explained that overconfidence, powerfulness, dominance, high pay dispersion among top executives, and poor corporate governance might be some of the reasons behind such over-compensation (Bebchuk et al., 2011; Cooper et al., 2016; Hayward & Hambrick, 1997).

### **Pay-performance sensitivity in reducing agency conflict**

Pay-performance sensitivity (PPS) is the dollar change in CEO's wealth associated with a one percent change in the firm's stock price, termed as the compensation delta (Murphy, 2012). Murphy (1985) shows that firm performance measured by shareholders' realized returns has a positive relationship with managerial remuneration. Moreover, considering PPS as a compensation metric and shareholders' return as a measure of firm performance, Jensen and Murphy (1990) show that a CEO's wealth increases \$3.25 for every \$1000 increase in shareholders' wealth, revealing a small, yet positive association between them. Following this finding, the literature shows a big increase in PPS during 1990s in the form of CEO's option holdings (Frydman & Jenter, 2010). The positive association between PPS and firm performance is further supported by several other studies (Pyo & Abedin, 2017; Kaplan, 2012).

On the other hand, the EBC can make agency conflicts worse. For instance, Brick et al. (2012) show that incentive compensation components such as stock option grants make CEOs entrenched and risk averse. Hence, they would put their own interest ahead of shareholders', leading to poor firm performance. To resolve mixed results on EBC, we first consider Tobin's  $q$  and then annual stock returns to measure firm performance and analyzes the effect of PPS in the presence of different executive dominance levels. One inherent component of the first two hypotheses is the anticipated influence of dominance level on the association of PPS with firm value.

**H1a:** The CEO power measured by CPS affects the positive relationship between PPS and firm performance measured by Tobin's  $q$ .

**H1b:** The CEO power measured by CPS affects the positive relationship between PPS and firm performance measured by annual stock returns.

### **CEO pay slice in aggravating agency conflicts**

A recent pool of literature that supports the rent seeking theory focuses on executive power or, relative importance of CEOs. CEO pay slice (CPS), a common measure of pay-heterogeneity and executive dominance, is the proportion of salary captured by CEO from aggregate compensation of the top-five executives (Bebchuk et al., 2011). CPS has been found to have important effects on

firm value controlling for standard control variables.<sup>4</sup> For instance - high level of CPS leads to a lower firm value (measured by Tobin's  $q$ ), poorer accounting profitability, and higher rent extraction (Bebchuk et al., 2011). However, dominance or power might differ for each firm and CEO because of the dependency on the idiosyncratic characteristics of factors such as the compensation contract, top management team structure, CEO ownership etc. However, we speculate that this linear negative impact of CPS on firm performance might not hold always. For instance, when performance is recognized and awarded with more long-term incentives (may be bundled with vested and unvested stock options), a dominant CEO—who had previously perceived to be a value destructor—might lead a firm with more confidence/motivation by taking investment decisions to boost stock price as he/she has stock options that are going to vest and increase their values, which would align high executive dominance with strong firm performance. As an extension to the first hypothesis, this is a more discrete approach to assess the interaction effect and examine the impact of higher executive dominance on the assessment of incentives to firm performance, which leads to the second hypothesis.

**H2:** CEO power positively interacts with PPS for strong firm performance.

### **Mixed reviews on the impact of CEO pay**

The literature also reveals some mixed results on the association of executive pay to firm performance. Gregg et al. (2005) show that there exists a rather weak link between pay and performance of the CEOs in the UK, especially when stock returns are relatively high. Such findings are further supported by Girma et al. (2007). In addition, Cooper et al. (2016) show that incentives offered as excess compensation are negatively related to firm performance. However, it is important to note some vital factors when we examine firm performance relative to CEO pay. For instance, researchers have considered the compensation component in various ways in the past. While some researchers focus on the incentive portions (Cooper et al., 2016; Dow & Raposo, 2005; Rajan & Wulf, 2006), others study the impact of cash, or whole compensation (Chintrakarn et al., 2015; Girma et al., 2007; Gregg et al., 2005). Ozkan (2011) shows a positive association between pay and performance, and emphasizes the importance of including performance sensitive components of compensation such as stock options, stock awards etc. Furthermore, various metrics are used to compute firm performance. For instance, some authors have considered Tobin's  $q$  (Bebchuk et al., 2011), while others have preferred stock return or return on assets (Bulan et al., 2010). Finally, the timeline of the sample firms could be related to the conflicting results for the relation between compensation and performance. Literature shows that the practice of remunerating higher incentive compensation packages to top executives started in the 1950s and escalated during 1990s (Langsam et al., 1997). The instruments of incentive compensation have also changed after the 1950s for various reasons (e.g., stock market volatility, corporate scandals etc.) (Bulan et al., 2010). This study offers to simplify these issues by: (1) controlling for PPS and

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<sup>4</sup> Zagonov and Salganik-Shoshan (2018) argue that CPS is inefficient to discern the diverse variations in payment structure of top executives across various organizations. However, in several studies, researchers use similar metrics compared to CPS in order to capture the varied payment structure. Hayward and Hambrick (1997) introduce CEO relative compensation (CRC) to segregate weak CEOs from strong CEOs (where,  $CRC \leq 1$  being weak). Furthermore, Chang et al. (2010) study the impact of managerial departure announcements on abnormal stock returns by considering relative total pay (RTP) for compensation measure which is calculated by dividing the CEO pay by the next four manager's pay. Overall, we recognize that higher pay-heterogeneity among the top executives is positively associated with agency problems and thus, note that CPS might be a useful tool to study firm value associations.

CPS simultaneously along with product terms to account for the individual effect and the interaction effect, (2) taking both Tobin's  $q$  and stock return to measure firm performance. Additionally, the 1992-2019 time period of this study covers the 1990s introduction of high incentive payment practice as well as the financial crisis of 2008<sup>5</sup>

### III. Data and methodology

#### Data

Compensation data have been collected from the Execucomp database for the 1992-2019 period. Execucomp provides ranked compensation data of the top-level executives. We retain the top five executives' data by eliminating companies (9.5% of the total) which have information on less than five executives. When more than five executives' data are provided, only top five observations are taken for consistency. One integral part of CPS computation is the compensation component. We consider the TDC1 item from the Execucomp database which includes salary, bonus, other annual compensation, stock options (Black & Scholes value) granted that year, total value of restricted stock grant that year, long-term incentive plan, all other total compensation following Bebchuk et al. (2011). Post-2006, Execucomp data format was changed to put up with accounting changes and expanded compensation disclosure requirements imposed by Financial Accounting Standards Board (FASB), and Securities and Exchanges Commission (SEC). We address this issue by applying different methods for PPS calculation for time periods: 1992-2006 and 2006-2019, following Coles et al. (2006). In addition, Execucomp stopped providing volatility and dividend yield information as of 2006 which are needed in the Black-Scholes formula for option valuation. Hence, we calculate the 60-month rolling volatility, and estimate the dividend yield as suggested by Coles et al. (2006). Risk-free rates are retrieved from historical data available at the Federal Reserve for 'treasury constant maturities' selecting the 'annual' series.<sup>6</sup>

Firm specific data and stock-related information are obtained from Compustat and CRSP, respectively, for the same period as discussed above. The final sample size stands at 32,042 CEO-year observations representing 3,518 unique firms (identified by GVKEY in Execucomp) including 7,265 unique CEOs (identified by CO\_PER\_ROL in Execucomp) after controlling for the data requirements. However, the number of observations in some regressions varies due to data limitation. For instance, lagged values are missing for the year 1992 (the beginning year of the sample).<sup>7</sup>

#### Variables

Tobin's  $q$ —defined as equity at the market value plus bond at the book value (total assets minus equity, both at the book value), the sum divided by total assets at the book value—as a proxy of

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<sup>5</sup> Relevant studies, such as Ozkan (2011) only focuses on UK CEOs for the 1999-2005 time period and does not consider the executive dominance that is prevalent in companies. Coles et al. (2006) study the riskiness in firms' decision making for the 1992-2002 time period and do not examine executive dominance. Bebchuk et al. (2011) study the association of CEO dominance—proxied by CEO pay slice—to firm performance for the 1993-2004 time period, however, do not consider the interaction effect between pay-performance sensitivity and CEO dominance.

<sup>6</sup> In the annual series, data for 1, 2, 3, 5, 7, and 10-year treasury securities are available. we interpolate the rates to obtain the risk-free rates for 4, 6, 8, and 9 years. If the option maturity is more than 10 years, we use the 10-year rate.

<sup>7</sup> The Execucomp database starts on Jan 1, 1992.

firm value is used as the dependent variable in the first part of the analysis. We also consider annual stock returns to account for firm performance in the second part of the study.

Pay-performance sensitivity (PPS) or Delta is the dollar increase in the CEO's compensation for a 1% increase in stock price. To estimate PPS, we first calculate stock option values following the method proposed by Core & Guay (2002), which is based on the Black & Scholes (1973) model for European call options modified by Merton (1973) to adjust for the dividend yield. Equation 1 reports the model.

$$\text{Option value} = [Se^{-dt}N(Z) - Xe^{-rT}N(Z - \sigma\sqrt{T})] \quad (1)$$

where  $Z = \left[ \left\{ \ln\left(\frac{S}{X}\right) + T\left(r - d + \frac{\sigma^2}{2}\right) \right\} / \sigma\sqrt{T} \right]$ ,  $N$  = cumulative probability function for the normal distribution,  $S$  = price of the underlying stock,  $X$  = strike price of the option,  $\sigma$  = volatility of stock returns over the maturity of the option,  $r$  = risk-free interest rate,  $T$  = time to maturity of the option in years,  $d$  = dividend yield over the maturity of the option.

For the aggregate PPS, we compute the delta for each CEO-year incorporating all the vested and unvested components of stock options in addition to shares owned following Coles et al. (2006), which is based on the "one-year approximation" (OA) method by Core & Guay (2002). The estimation of PPS is in equation 2.

$$\begin{aligned} & \left[ \frac{\delta(\text{Option value})}{\delta(\text{Stock price})} \right] * \left( \frac{\text{Stock price}}{100} \right) \\ & = [e^{-dT}N(Z) * (\#Option_{\text{Holding}} + \#Shares_{\text{owned}})] * \left( \frac{\text{Stock price}}{100} \right) \end{aligned} \quad (2)$$

where  $Z = \left[ \left\{ \ln\left(\frac{S}{X}\right) + T\left(r - d + \frac{\sigma^2}{2}\right) \right\} / \sigma\sqrt{T} \right]$ ,  $N$  = cumulative probability function for the normal distribution,  $T$  = time to maturity of the option in years,  $d$  = dividend yield over the maturity of the option.

CEO pay slice (CPS) is measured as the fraction of the total compensation (Execucomp item TDC1) of top five executives that is received by the chief executive officer (CEO) (see, Bebchuk et al., 2011). While PPS and CPS are our primary explanatory variables, we include other control variables following the prior literature (Bebchuk et al., 2011; Coles et al., 2006; Core & Guay, 2002; Graefe-Anderson et al., 2018). For instance, market-to-book is defined as the market value of equity divided by its book value. Book leverage is the total debt divided by the book value of total assets.<sup>8</sup> ROA is the return on assets computed as operating income divided by book value of total assets. R&D is expenditure in research and development divided by total assets. Capex is capital expenditure minus sales of property, plant, and equipment, also divided by total assets.

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<sup>8</sup> Book leverage (rather than market leverage) is used since market leverage might change passively simply because of changes in stock price performance and may not be an active managerial choice (Coles et al., 2006).

**Table 1: Summary statistics**

Variable	N	Mean	Std Dev	Minimum	Q1	Median	Q3	Maximum
Tobin's $q$	32,042	1.94	1.50	0.42	1.11	1.46	2.15	17.30
Annual Return	32,042	0.17	0.45	-0.89	-0.09	0.12	0.35	2.74
CPS	32,042	0.38	0.11	0.08	0.31	0.38	0.45	0.71
Delta (PPS) (\$000s)	32,042	1,086	9,666	0.000011	78	208	572	709,830
Lag (CPS)	26,456	0.38	0.11	0.08	0.32	0.39	0.45	0.71
Log (PPS)	32,042	5.35	1.61	-11.40	4.36	5.34	6.35	13.47
Lag {Log(PPS)}	24,441	5.37	1.58	-5.77	4.39	5.35	6.36	13.23
MtB	32,042	3.27	4.48	0.16	1.36	2.12	3.52	52.40
Market cap (\$M)	32,042	6,197	10,173	3	700	1,919	6,047	41,486
Sales (\$M)	32,042	4,889	9,033	28	539	1,464	4,492	47,142
CEO age (years)	32,042	56.12	7.22	28.00	51.00	56.00	61.00	91.00
Leverage	32,042	0.13	0.17	0.00	0.00	0.00	0.25	0.87
ROA	32,042	0.11	0.10	-0.40	0.06	0.12	0.17	0.43
R&D	32,042	0.03	0.06	0.00	0.00	0.00	0.03	1.15
Capex	32,042	0.05	0.05	-0.04	0.01	0.03	0.06	0.44

This table provides descriptive statistics on CEO and firm characteristics for the 1992–2019 sample period. Data on executive compensation, firm specific information and stock prices are obtained from Execucomp, Compustat and CRSP databases respectively. Risk-free rates are obtained from the Federal Reserve. The sample contain data of 32,042 CEO year observations (representing 3,518 unique firms having 7,265 unique CEOs). CEO pay slice (CPS) is the part of the total compensation (TDC1 at Execucomp) of the top-five executives that is collected by CEO. Delta (pay-performance sensitivity) is the dollar change in the executive's wealth for a 1% change in stock price. Tobin's  $q$  is defined as equity at the market value plus bond at the book value (total assets minus equity, both at the book value), the sum divided by total assets at the book value. MtB or, market-to-book is the market value of assets divided by their book value. Annual return is the annual stock return. Leverage is total debt divided by total assets. ROA is the return on assets measured as operating income over book value of assets. R&D is the expenditure in research and development divided by assets. Capex is capital expenditure minus sales of property, plant, and equipment, also scaled by assets. The variables have been winsorized at 1st and 99th percentile levels.

**Table 2: Sample correlations of key variables**

Variables	PPS	CPS	MtB	Market Cap	Sales	CEO age	Leverage	ROA	R&D	Capex
PPS	1									
CPS	<i>0.075</i>	1								
MtB	<i>0.223</i>	0.012	1							
Market Cap	<i>0.537</i>	<i>0.129</i>	<i>0.199</i>	1						
Sales	<i>0.362</i>	<i>0.131</i>	-0.004	<i>0.780</i>	1					
CEO age	<i>0.141</i>	0.008	<i>-0.051</i>	<i>0.070</i>	<i>0.112</i>	1				
Leverage	<i>0.035</i>	<i>0.120</i>	<i>0.130</i>	<i>0.087</i>	<i>0.195</i>	-0.002	1			
ROA	<i>0.173</i>	<i>0.036</i>	<i>0.202</i>	<i>0.134</i>	<i>0.149</i>	-0.005	<i>0.095</i>	1		
R&D	0.003	<i>-0.052</i>	<i>0.212</i>	<i>-0.108</i>	<i>-0.282</i>	<i>-0.108</i>	<i>-0.075</i>	<i>-0.149</i>	1	
Capex	-0.002	<i>-0.043</i>	<i>0.036</i>	<i>-0.062</i>	<i>-0.057</i>	<i>-0.038</i>	<i>0.042</i>	<i>0.352</i>	<i>-0.016</i>	1

This table reports the correlations among sample of main variables adopted in the analysis of the impact and association of executive incentives and executive dominance on firm performance. PPS is the natural log of pay-performance sensitivity; Market Cap is the natural log of market capitalization; Sales is the natural log of annual sales. MtB is market-to-book ratio, which is assets at the market value over those at the book value. Leverage is total debt divided by book value of assets. ROA is the return on assets measured as operating income over assets at the book value. R&D is the expenditure in research and development divided by assets. Capex is capital expenditure net of sales of property, plant, and equipment, also scaled by assets. The sample contains data of 32,042 CEO year observations (representing 3,518 unique firms having 7,265 unique CEOs) for the 1992-2019 time period. Italic numbers indicate 1% level of significance.

Table 1 reports the summary statistics. Overall, the firms in our sample are quite large with a mean (median) market capitalization of \$6,197 (\$1,919) million. The mean (median) values of Tobin's  $q$  and annual stock returns are 1.94 (1.46) and 17% (12%), respectively. Mean (median) value of delta is \$1,086,400 (\$208,024), i.e., average CEO wealth changes by \$1,086,000 per 1% change in their firm's stock price. Coles et al. (2006) study pay-performance sensitivity from 1992 to 2002 and find that mean (median) value of delta is \$600,000 (\$206,000). Since 2002, CEOs have been granted very high levels of equity-based compensation, which explains the difference. Mean (median) value of CPS is 0.38 (0.38), which is in accordance with the prior literature.<sup>9</sup> To avoid scale difference among the variables in the regressions, natural log of delta (i.e., PPS), market capitalization, and sales is taken. All the variables are winsorized at 1<sup>st</sup> and 99<sup>th</sup> percentile levels to minimize the effects of outliers.

Table 2 reports the correlations among samples of main variables adopted in the analyses. Most of the correlations are significant at 1% level. 'Sales' and 'Market capitalization' have a correlation coefficient of 0.780, which is strongly positive. The correlation between 'PPS' and 'Market capitalization', 'Sales' and 'PPS' are 0.537 and 0.362, respectively, indicating a moderate degree of correlation. Other correlation values are smaller than 0.362.

## Methodology

The relation between CEO incentive compensation and firm value is analyzed in two parts, each of which examines six models. In the first part, Tobin's  $q$  as a proxy of firm performance is regressed on PPS, and CPS along with other standard control variables. The second part considers annual stock returns as a measure of firm performance and conducts similar analyses.

### *Firm value measured by Tobin's $q$*

This is the first part of the analyses and comprises six OLS models, where Tobin's  $q$  as a proxy of firm performance is the dependent variable. Other control variables such as market to book ratio, market capitalization, sales, CEO age, leverage, return on asset, research and development, capital expenditure, product term of PPS and CPS, and lagged values of the product term are included along with the two primary explanatory variables: PPS and CPS.

Initially we try to analyze if the prior findings in the literature are held within our framework. Hence, we assess the association of PPS to firm performance. Subsequently, we analyze the two primary explanatory variables together and test if prior findings still stand. Equation 3 reports the model.

$$\begin{aligned} \text{Tobin's } Q_{i,t} = & \beta_0 + \beta_1 \text{PPS}_{i,t} + \beta_2 \text{CPS}_{i,t} + \beta_3 \text{Market to Book}_{i,t} + \beta_4 \text{Market Cap}_{i,t} \\ & + \beta_5 \text{Sale}_{i,t} + \beta_6 \text{CEOAge}_{i,t} + \beta_7 \text{Leverage}_{i,t} + \beta_8 \text{ROA}_{i,t} \\ & + \beta_9 \text{R\&D}_{i,t} + \beta_{10} \text{CAPEX}_{i,t} + \epsilon_{i,t} \end{aligned} \quad (3)$$

<sup>9</sup> Bechuk et al. (2011) studies 12,011 firm-year observations from 1993 to 2004 and finds mean (standard deviation) CPS of 35% (11.4%). When we replicate, we get mean (standard deviation) CPS of 36% (12.4%) by studying 18,053 firm-year observations for the same time period. The reason behind the difference in the number of firm-year observations for the same sample period might be the addition of data by Execucomp after 2011.

where  $PPS_{i,t}$  is the natural log of pay-performance sensitivity and  $CPS_{i,t}$  is CEO pay slice of the CEO of firm  $i$  in year  $t$  respectively; Market to Book $_{i,t}$  is the ratio of market value to book value, Market Cap $_{i,t}$  is the natural log of market capitalization, Sale $_{i,t}$  is the natural log of sales, CEOAge $_{i,t}$  is the age of CEO (in years), Leverage $_{i,t}$  is the book leverage, ROA $_{i,t}$  is the return on asset, R&D $_{i,t}$  is the research and development expenditure, CAPEX $_{i,t}$  is the capital expenditure minus sales of property, plant, and equipment of the firm  $i$  in year  $t$ ;  $\epsilon_{i,t}$  is the error term.

As we add more control variables, we further explore firm value characteristics. We speculate that there is any interaction effect between PPS and CPS and includes the product terms (current and lagged) along with prior control variables. Equation 4 denotes the model.

$$\begin{aligned} \text{Tobin's } Q_{i,t} = & \beta_0 + \beta_1 PPS_{i,t} + \beta_2 CPS_{i,t} + \beta_3 (PPS * CPS)_{i,t} + \beta_4 PPS_{i,t-1} \\ & + \beta_5 CPS_{i,t-1} + \beta_6 (PPS * CPS)_{i,t-1} + \beta_7 \text{Market to Book}_{i,t} \\ & + \beta_8 \text{Market Cap}_{i,t} + \beta_9 \text{Sale}_{i,t} + \beta_{10} \text{CEOAge}_{i,t} + \beta_{11} \text{Leverage}_{i,t} \\ & + \beta_{12} \text{ROA}_{i,t} + \beta_{13} \text{R\&D}_{i,t} + \beta_{14} \text{CAPEX}_{i,t} + \epsilon_{i,t}, \end{aligned} \quad (4)$$

where  $(PPS * CPS)_{i,t}$  and  $(PPS * CPS)_{i,t-1}$  are the current and lagged values of the product term of pay-performance sensitivity and CEO pay slice of the CEO of firm  $i$  in year  $t$ . Other variables have the same definitions as above.

### ***Firm performance measured by annual stock returns***

We run six OLS regressions to examine the impacts on firm performance considering annual stock returns as the dependent variable. We initially conduct tests if prior findings are held. We add their product terms along with other control variables to examine the potential interaction effects between PPS and CPS. We expect that  $\gamma_1$  and  $\gamma_2$  would be statistically significant, and their signs would be positive and negative, respectively. We do not expect any sign regarding  $\gamma_3$ ; however, we expect the product term to be statistically significant so that we can investigate how they interact with each other. Equation 5 shows the model.

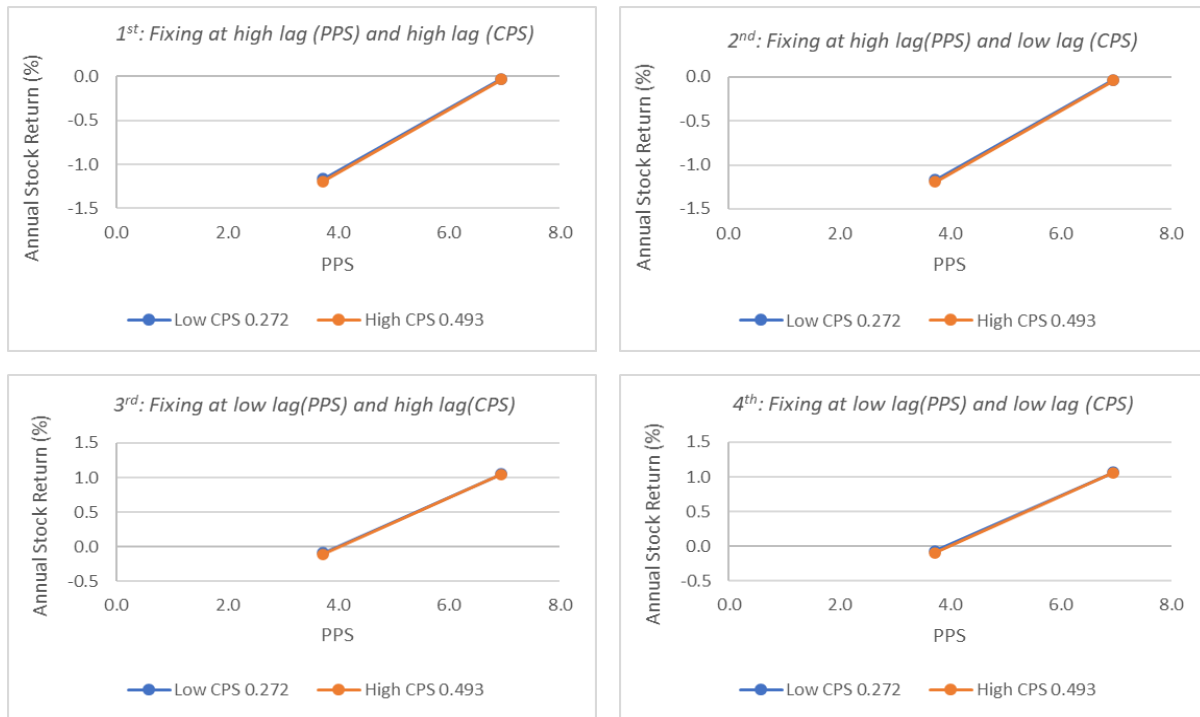
$$\begin{aligned} \text{Stock Return}_{i,t} = & \gamma_0 + \gamma_1 PPS_{i,t} + \gamma_2 CPS_{i,t} + \gamma_3 (PPS * CPS)_{i,t} + \gamma_4 PPS_{i,t-1} \\ & + \gamma_5 CPS_{i,t-1} + \gamma_6 (PPS * CPS)_{i,t-1} + \gamma_7 \text{Market to Book}_{i,t} \\ & + \gamma_8 \text{Market Cap}_{i,t} + \gamma_9 \text{Sale}_{i,t} + \gamma_{10} \text{CEOAge}_{i,t} + \gamma_{11} \text{Leverage}_{i,t} \\ & + \gamma_{12} \text{ROA}_{i,t} + \gamma_{13} \text{R\&D}_{i,t} + \gamma_{14} \text{CAPEX}_{i,t} + \epsilon_{i,t}, \end{aligned} \quad (5)$$

where, the subscript  $i, t$  denotes the firm  $i$  in year  $t$ . All the variables have the same definitions as above.

### ***Main effects vs Interaction effects***

Six of the twelve regressions from the two parts of the analyses (column 1, 2, 4 in both Table 3 and 4) show the stand-alone association of PPS and CPS to firm performance measured by Tobin's  $q$  and annual stock return. As for the interaction effect, it is important to have both statistically significant product term (i.e.,  $PPS \times CPS$ , in our study) and individual terms (i.e., PPS and CPS) as documented by Balli and Sørensen (2013).

**Figure 2: Analyzing the association of PPS with Firm performance.**



Portrayed by the four panels in the figure above, the analysis on equation 5 (estimates in column 6 of Table 4) shows that the association of PPS to firm performance in the presence of different current year CPS levels (high and low) remains the same when controlling for certain lagged values of PPS and CPS. X-axis represents PPS which is the natural log of pay-performance sensitivity, and Y-axis represents annual stock returns (in %). Calculations are shown in the table below. This table calculates four different stock returns by fixing the lag terms of high and low PPS, CPS at four different scenarios: (1) high lagged PPS and high lagged CPS, (2) high lagged PPS and low lagged CPS, (3) low lagged PPS and high lagged CPS, (4) low lagged PPS and low lagged CPS.

We examine the impacts of PPS on firm performance in the presence of CPS when PPS and CPS are controlled for the previous year. We first confirm that the estimates (in column 6 of Table 4) satisfy the conditions for the interaction effect suggested by Balli and Sørensen (2013). Hence, we conduct the analysis on interaction effects in equation 5 by fixing the lag terms of PPS, CPS at four different situations: (1) high lagged PPS and high lagged CPS, (2) high lagged PPS and low lagged CPS, (3) low lagged PPS and high lagged CPS, (4) low lagged PPS and low lagged CPS. Calculation is shown in Figure 2. High PPS and CPS (both current and lagged) levels are defined as one standard deviation higher than their median terms while low levels mean one standard deviation lower than their median terms. This assessment evaluates if the relationship of PPS to firm performance remains the same in the presence of different current year CPS levels when controlling for the fixed lagged values of PPS and CPS levels. If the impacts of PPS on firm performance is not affected by the different CPS levels, it would indicate that there is no interaction effect because all of lagged values are controlled. The lines all appear to be the same because there is no interaction effects between CPS and PPS in their lag terms.

We continue investigating the impacts of PPS on firm performance in the presence of CEO dominance when we do not control for fixed levels (high or, low) of lagged values of PPS and CPS, but control for the current levels (high or, low) of PPS, CPS. That is, we examine the impacts of PPS on firm performance in the presence of different CPS levels (high and low) when the PPS

and the CPS is high/low/high-low/low/high for two consecutive years. We start by taking high and low levels of PPS and CPS (and their lagged values) that are one standard deviation away from their median terms and plug them into equation 5 along with the mean values of statistically significant control variables to find four different stock returns. Calculation is shown in Figure 3. To check the marginal change, we further increase the boundary of PPS and CPS (and lagged PPS and CPS) levels by taking them two (also, three) standard deviation away from their median terms, and conduct similar analyses.

#### IV. Results

Table 3 reports parameter estimates of the six OLS models (column 1-6) that comprise the first part of the analyses. Estimates from the first row in column 1-6 show that PPS is significantly positive to firm performance at the 1% level, being consistent with the literature. In addition, we note that there is no significant relationship between Tobin's  $q$  and the lagged values of PPS. We also find that CPS in column 2 and 4 is significant, which shows a negative association to firm value, being consistent with the literature. In addition, estimates from column 4 and 5 (significant at 1%) indicate that lagged value of CPS also has a negative relation to firm value. Overall we note that PPS holds a positive relationship with Tobin's  $q$ , while CPS has an inverse relationship, showing consistency with the prior literature.

Regarding the interaction effect, Table 3 shows that the product terms are statistically significant at the 1% level in column 3 and 5, and at the 10% level in column 6. However, CPS is statistically insignificant. Hence, we do not find any significant result to support the first hypothesis ( $H1a$ ) at this point.

Estimates from the second part of the analyses are reported in Table 4. Except for CPS in the second column, we find that while PPS is positively related to stock return performance, CPS is negatively related to it in all cases. The findings are consistent with those in the literature that illustrates such stand-alone relations (e.g., Bebchuk et al., 2011; Graefe-Anderson et al., 2018; Pyo & Abedin, 2017).

Estimates from Table 4 also show that the product terms are statistically significant at the 5%, 1%, and 10% levels in columns 3, 5, and 6, respectively. Confirming an indication for the interaction effect, individual components i.e., PPS and CPS also show significant associations (positive and negative respectively) in column 5 and 6. Analysis of column 6 (that satisfies the conditions for the interaction effect) is conducted to investigate the impacts of PPS levels on firm performance in the presence of different current year CPS levels by fixing the lag terms of high and low PPS, CPS at four different scenarios. The analysis shows that the impacts of PPS on firm performance in the presence of CPS levels (high and low) remain the same (positive, in particular), portrayed by the four panels in Figure 2, which is a strong indication and support for the interaction effect.

Further assessment of column 6 reveals significant interaction effects when fixing the lagged values of (high or low) PPS, CPS similar (i.e., uncontrolled) at their current levels (high or low). Left panel of Figure 3 shows how different current year CPS levels affect the impacts of PPS on firm performance, supporting the second part of the first hypothesis ( $H1b$ ). The graph shows that when CPS is 1 standard deviation (0.11) higher than its median term (0.38), annual stock returns increase from -10.70% to -3.60%, indicating a 66.56% increase because of a rise in PPS (log) from 3.73 to 6.95. However, the graph also indicates that when CPS is 1 standard deviation

lower than its median, annual return changes from -6.60% to -3.30%, which is a 50.62% increase for the given change in PPS levels, confirming the impact of CPS levels and supporting *H1b*.

**Table 3: Dependent variable: Tobin's  $q$** 

	Tobin's $q$					
	(1)	(2)	(3)	(4)	(5)	(6)
PPS	0.1291*** (13.40)	0.1161*** (30.35)	0.1471*** (14.75)	0.13063*** (13.39)	0.1582*** (10.55)	0.1521*** (9.52)
CPS		-0.4149*** (-8.97)	0.0905 (0.58)	-0.3161*** (-5.31)	0.1393 (0.71)	0.0529 (0.25)
PPS X CPS			-0.0879*** (-3.37)		-0.0765*** (-2.42)	-0.0607* (-1.75)
Lag (PPS)	-0.0093 (-1.03)			-0.0117 (-1.27)	-0.0116 (-1.27)	0.0023 (0.15)
Lag (CPS)				-0.2843*** (-4.87)	-0.2754*** (-4.71)	-0.0664 (-0.34)
Lag (PPS) X Lag (CPS)						-0.0373 (-1.12)
MtB	0.1532*** (106.45)	0.1553*** (122.77)	0.1554*** (122.83)	0.1509*** (104.26)	0.1511*** (104.30)	0.1509*** (104.29)
Market Cap	0.3345*** (49.18)	0.3492*** (57.44)	0.3495*** (57.49)	0.3393*** (49.55)	0.3392*** (49.54)	0.3393*** (49.54)
Sales	-0.4047*** (-61.89)	-0.4205*** (-72.14)	-0.4203*** (-72.09)	-0.4017*** (-61.00)	-0.4014*** (-60.96)	-0.4014*** (-60.95)
CEO Age	-0.0058*** (-7.17)	-0.0073*** (-10.31)	-0.0073*** (-10.27)	-0.0061*** (-7.56)	-0.0061*** (-7.55)	-0.0061*** (-7.55)
Leverage	-0.8578*** (-25.40)	-0.8485*** (-27.60)	-0.8484*** (-27.60)	-0.8082*** (-23.63)	-0.8091*** (-23.65)	-0.8092*** (-23.65)
ROA	4.3267*** (66.70)	4.0641*** (70.90)	4.0631*** (70.89)	4.3587*** (66.62)	4.3566*** (66.59)	4.3566*** (66.59)
R&D	5.0349*** (49.62)	4.8328*** (53.02)	4.8371*** (53.07)	4.9915*** (48.20)	4.9943*** (48.23)	4.9948*** (48.24)
Capex	-0.5118*** (-4.38)	-0.3536*** (-3.42)	-0.3548*** (-3.44)	-0.5585*** (-4.71)	-0.5584*** (-4.71)	-0.5592*** (-4.72)
Industry fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	24,441	32,042	32,042	23,975	23,975	23,975
$R^2$	64%	63%	63%	64%	64%	64%

This table provides parameter estimates from regressing Tobin's  $q$  on the explanatory variables for the six OLS models. Presented in parentheses are the  $t$ -statistics. Intercepts are not reported. PPS is the natural log of pay-performance sensitivity; Market Cap is the natural log of market capitalization; Sales is the natural log of annual sales. Control variables are explained in data section and summary statistics has been provided in Table 1. Number of observations in the regressions varies due to data limitation. For instance, data are missing for the lagged values of the variables for the year 1992 (the beginning year of the sample). \*\*\*, \*\*, \* indicate 1%, 5%, 10% level of significance respectively.

**Table 4: Dependent variable: Annual stock return**

	Annual Return					
	(1)	(2)	(3)	(4)	(5)	(6)
PPS	0.3532*** (91.77)	0.0304*** (16.93)	0.0217*** (4.64)	0.3548*** (90.81)	0.3411*** (56.80)	0.3456*** (54.04)
CPS		0.0973*** (4.48)	-0.0437 (-0.59)	-0.0448** (-1.88)	-0.2722*** (-3.45)	-0.2077** (-2.45)
PPS X CPS			0.0245** (2.00)		0.0382*** (3.02)	0.0264* (1.90)
Lag (PPS)	-0.3429*** (-94.76)			-0.3449*** (-93.49)	-0.3449*** (-93.51)	-0.3554*** (-57.08)
Lag (CPS)				-0.0224 (-0.96)	-0.0268 (-1.15)	-0.1828** (-2.33)
Lag (PPS) X Lag (CPS)						0.0277** (2.08)
MtB	0.0120*** (20.92)	0.0152*** (25.63)	0.0152*** (25.60)	0.0119*** (20.52)	0.0118*** (20.43)	0.0119*** (20.44)
Market Cap	0.0535*** (19.69)	0.0759*** (26.58)	0.0758*** (26.56)	0.0532*** (19.39)	0.0532*** (19.41)	0.0532*** (19.40)
Sales	-0.0603*** (-23.08)	-0.0901*** (-32.93)	-0.0902*** (-32.96)	-0.0595*** (-22.55)	-0.0596*** (-22.60)	-0.0596*** (-22.61)
CEO Age	0.0005 (1.49)	-0.0019*** (-5.77)	-0.0019*** (-5.80)	0.0005* (1.66)	0.0005* (1.65)	0.0005* (1.66)
Leverage	-0.0705*** (-5.23)	-0.1344*** (-9.31)	-0.1344*** (-9.31)	-0.0656*** (-4.79)	-0.0652*** (-4.76)	-0.0651*** (-4.75)
ROA	0.2196*** (8.47)	0.3797*** (14.11)	0.3799*** (14.12)	0.2148*** (8.20)	0.2158*** (8.24)	0.2159*** (8.24)
R&D	-0.1329*** (-3.28)	-0.3804*** (-8.89)	-0.3816*** (-8.92)	-0.1279*** (-3.09)	-0.1294** (-3.12)	-0.1297*** (-3.13)
Capex	-0.2780*** (-5.96)	-0.5751*** (-11.86)	-0.5748*** (-11.86)	-0.2793*** (-5.88)	-0.2793*** (-5.88)	-0.2787*** (-5.87)
Industry fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	24,441	32,042	32,042	23,975	23,975	23,975
R <sup>2</sup>	34%	11%	11%	34%	34%	34%

	Annual Stock Returns (%)							
	1 <sup>st</sup>		2 <sup>nd</sup>		3 <sup>rd</sup>		4 <sup>th</sup>	
	Low CPS (0.272)	High CPS (0.493)	Low CPS (0.272)	High CPS (0.493)	Low CPS (0.272)	High CPS (0.493)	Low CPS (0.272)	High CPS (0.493)
Low PPS (3.729)	<b>-1.202*</b>	<b>-1.226</b>	<b>-1.204</b>	<b>-1.228</b>	<b>-0.119</b>	<b>-0.143</b>	<b>-0.102</b>	<b>-0.126</b>
High PPS (6.947)	<b>-0.066</b>	<b>-0.072</b>	<b>-0.069</b>	<b>-0.074</b>	<b>1.017</b>	<b>1.011</b>	<b>1.033</b>	<b>1.028*</b>

Bold face numbers are used to draw Figures 2.

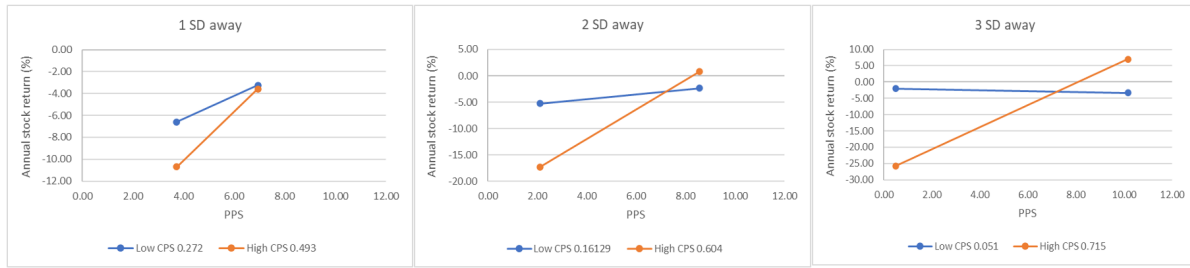
This table reports parameter estimates from regressing annual stock return on the explanatory variables for the six OLS models. Presented in parentheses are the *t*-statistics. Intercepts are not reported. PPS is the natural log of pay-performance sensitivity; Market Cap is the natural log of market capitalization; Sales is the natural log of annual sales. Control variables are explained in data section and summary statistics has been provided in Table 1. Number of observations in the regressions varies due to data limitation. For instance, data is missing for the lagged values of the variables for the year 1992 (the beginning year of the sample). \*\*\*, \*\*, \* indicate 1%, 5%, 10% level of significance respectively.

We also conduct investigation on column 6 by increasing the ranges of PPS and CPS (and lagged PPS and CPS) levels. That is, taking them two (also, three for extreme cases) standard deviations away from their median levels exposes the interaction effects to be more pronounced. The center panel of Figure 3 portrays that while CPS is fixed at the high level, changing PPS level from low to high increases stock return performance, supporting the second hypothesis (*H2*). On the other hand, CPS is fixed at the low level, the effect of PPS on firm performance diminishes (and opposite in extreme cases). More specifically, for the same increase in PPS (log) from 2.12 to 8.56, we observe two different cases. First, when CPS is two standard deviations higher than its median term (defined as high CPS), stock returns change from -17.3% to 0.80%, showing a 104.45% increase. Second, when CPS is two standard deviations lower than the median (defined as low CPS), stock returns increase from -5.30% to -2.40%, indicating a 54.94% increase. Influenced by CPS levels, the variation in firm performance for the same changes in PPS levels indicates a strong interaction effect. For a more extreme case, when we look into high and low PPS, CPS that are 3 standard deviation away from the medians (right panel of Figure 3), we find that for the same changes in PPS levels, stock returns rather decrease by 65.61% when CPS is low compared to an increase by 127.25% when CPS is high, showing a strong and opposite impact of CPS levels. However, when PPS, CPS are taken 3 standard deviations away from medians to be defined as high (low) PPS, CPS levels, their values stand at 10.16 (0.51) and 0.72 (0.05), respectively, while their maximum (minimum) values are 13.47 (-11.40) and 0.71 (0.08), respectively, in our sample (summary statistics in Table 1). We consider this case to be extreme as the ‘low CPS’ (0.05) is lower than the minimum CPS (0.08) from our sample, and are cautious in suggesting that the extreme CPS levels have opposite effects on the impacts of PPS on firm performance.

In summary, estimates from the stand-alone cases (i.e., without considering the product terms for the interaction effect) from columns 1, 2, 4 in both Table 3 and 4 indicate that PPS shows positive impacts, but CPS shows negative impacts on firm performance, both of which are aligned with the literature. However, considering the potential interaction effects, we need to conduct more analyses as below.

To obtain economic implications from coefficients above, we compute the impacts of incentive compensation in dollar terms on firm performance for two groups: more powerful CEOs and less powerful CEOs. More precisely, an increase of \$966,558 (one-tenth of standard deviation) in PPS for a CEO whose CPS is 0.49 (more powerful CEO) is related to an increase in annual stock returns by 9.50%, whereas an identical increase in incentive amount for a CEO whose CPS is 0.27 (less reflected in 9.50% CEO) is related to an increase in annual stock returns by 6.56%. As the study finds positive impacts of CEO incentive compensation on firm performance, it supports the optimal contracting theory and advocates CEO incentive contracts in maximizing shareholders’ value. However, it illustrates that stand-alone effects of either pay-performance sensitivity or CEO pay slice to firm performance might not be enough in understanding their impacts on firm performance.

**Figure 3:**



X-axis represents PPS which is the natural log of pay-performance sensitivity, and Y-axis represents annual stock returns (in %). Calculation is shown in the table below. High PPS and CPS (also true for lagged PPS and CPS) levels are defined as one standard deviation higher than their median terms while low levels mean one standard deviation lower than their median terms. To check the marginal effect, we further increase the boundary of PPS and CPS (and lagged PPS and CPS) levels by taking them two (and three) standard deviation away from their median terms. Mean values of other control variables (which are statistically significant) are considered to examine the effect.

**Panel B (Figure 3): With current terms and St. Deviations**

Current		1 St. Deviation		2 St. Deviation		3 St. Deviation				
		CPS	Low	High	PPS	Low	High	CPS	Low	High
PPS	Low	(3.729)	<b>-6.596</b>	<b>-10.703</b>	(2.120)	<b>-5.265*</b>	<b>-17.265</b>	(0.511)	<b>-2.042</b>	<b>-25.720</b>
	High	(6.947)	<b>-3.257</b>	<b>-3.579</b>	(8.556)	<b>-2.373</b>	<b>0.769</b>	(10.165)	<b>-3.381</b>	<b>7.009</b>

Bold face numbers are used to draw Figures 3.

Putting those values in equation 5 (estimates in column 6 of Table 4), this table calculates stock returns at high and low PPS, CPS levels (which are 2SD away from their median terms). Center panel of Figure 3 portrays that while high level of CPS is present, changing PPS level from low to high improves stock return performance, and at low level of CPS, stock return performance rather starts to decline (declines in the right panel) for the same changes in PPS levels.

## V. Robustness

We conduct robustness tests by addressing the potential endogeneity issues in equation 5 that includes the interaction effects. These endogeneity issues could be present due to several factors. First, when we examine firm values affected by changes in CPS, it is important to understand that CPS itself might be affected by components that are also relevant to firm values (Bebchuk et al., 2011). Second, several studies (Gomes & Schmid, 2010; Hu & Gong, 2018) suggest that there a close relation between leverage and stock returns. The stock market also reacts when firms undertake new projects (Bajo et al., 1998; Burton et al., 1999). Jensen and Meckling (1976) exhibit how managers could forego positive NPV projects, provided a limited or no ownership in the firm. Numerous studies (Jensen & Murphy, 1990; Lee et al., 2008; Murphy, 1985) argue that this problem is attenuated when equity-based compensation is granted to these managers. While positive NPV projects increase firm performance represented by positive stock returns, they also increase the leverage of firms (Graefe-Anderson et al., 2018). These issues raise potential concerns on the correlation between leverage, an explanatory variable, and the error term.

Following the suggestion by Wooldridge (2002), we control for the above endogeneity concerns by employing instrumental variables (IV) with a two-stage least square (2SLS) estimation procedure. In the first stage, we introduce IVs which are related to the potential endogenous variables: CPS and leverage, but unrelated to the stock returns, a dependent variable.

Equation 6 and 7 display the first stage of the estimation where CPS and leverage are the dependent variables in two separate OLS models. To account for the endogeneity of CPS, we add industry median CPS (InMedCPS) i.e., median CPS by the four-digit SIC (standard industrial classification) codes as an IV following Bebchuk et al. (2011). To address the endogeneity of leverage, we add Marginal tax rate before interest expense (MTBI) as an IV following Givoly et al. (1992) as employed in Graefe-Anderson et al. (2018). Other control variables follow equation 5.

$$\begin{aligned} \text{CPS}_{i,t} = & \gamma_0 + \gamma_1 \text{PPS}_{i,t} + \gamma_2 (\text{PPS} * \text{CPS})_{i,t} + \gamma_3 \text{PPS}_{i,t-1} + \gamma_4 \text{CPS}_{i,t-1} \\ & + \gamma_5 (\text{PPS} * \text{CPS})_{i,t-1} + \gamma_6 \text{Market to Book}_{i,t} + \gamma_7 \text{Market Cap}_{i,t} \\ & + \gamma_8 \text{Sale}_{i,t} + \gamma_9 \text{CEOAge}_{i,t} + \gamma_{10} \text{ROA}_{i,t} + \gamma_{11} \text{R\&D}_{i,t} \\ & + \gamma_{12} \text{CAPEX}_{i,t} + \gamma_{13} \text{InMedCPS}_t + \gamma_{14} \text{MTBI}_{i,t} + \epsilon_{i,t}, \end{aligned} \quad (6)$$

$$\begin{aligned} \text{Leverage}_{i,t} = & \gamma_0 + \gamma_1 \text{PPS}_{i,t} + \gamma_2 (\text{PPS} * \text{CPS})_{i,t} + \gamma_3 \text{PPS}_{i,t-1} + \gamma_4 \text{CPS}_{i,t-1} \\ & + \gamma_5 (\text{PPS} * \text{CPS})_{i,t-1} + \gamma_6 \text{Market to Book}_{i,t} + \gamma_7 \text{Market Cap}_{i,t} \\ & + \gamma_8 \text{Sale}_{i,t} + \gamma_9 \text{CEOAge}_{i,t} + \gamma_{10} \text{ROA}_{i,t} + \gamma_{11} \text{R\&D}_{i,t} \\ & + \gamma_{12} \text{CAPEX}_{i,t} + \gamma_{13} \text{InMedCPS}_t + \gamma_{14} \text{MTBI}_{i,t} + \epsilon_{i,t}, \end{aligned} \quad (7)$$

where  $\text{InMedCPS}_t$  is the industry median CPS in the four-digit SIC group in time  $t$ ;  $\text{MTBI}_{i,t}$  is the marginal tax rate before interest expense of firm  $i$  in time  $t$ . Other variables have the same definitions as before.

In the second stage, we regress annual stock return on the fitted value of CPS and leverage derived in the first stage. Equation 8 shows the model.

$$\begin{aligned} \text{Stock Return}_{i,t} = & \gamma_0 + \gamma_1 \text{PPS}_{i,t} + \gamma_2 \widehat{\text{CPS}}_{i,t} + \gamma_3 (\text{PPS} * \text{CPS})_{i,t} + \gamma_4 \text{PPS}_{i,t-1} \\ & + \gamma_5 \text{CPS}_{i,t-1} + \gamma_6 (\text{PPS} * \text{CPS})_{i,t-1} + \gamma_7 \text{Market to Book}_{i,t} \\ & + \gamma_8 \text{Market Cap}_{i,t} + \gamma_9 \text{Sale}_{i,t} + \gamma_{10} \text{CEOAge}_{i,t} + \gamma_{11} \widehat{\text{Leverage}}_{i,t} \\ & + \gamma_{12} \text{ROA}_{i,t} + \gamma_{13} \text{R\&D}_{i,t} + \gamma_{14} \text{CAPEX}_{i,t} + \epsilon_{i,t}, \end{aligned} \quad (8)$$

where  $\widehat{\text{CPS}}_{i,t}$  is the fitted value of CEO pay slice of the CEO of firm  $i$  in time  $t$ ;  $\widehat{\text{Leverage}}_{i,t}$  is the fitted value of leverage of firm  $i$  in year  $t$ . Other variables are defined similarly as before.

Table 5 reports parameter estimates from the first stage of the 2SLS estimation of stock return performance. Statistical significance and the signs of the estimates of the IVs confirm that they are indeed strong instruments. For instance, coefficient of InMedCPS is positive and statistically significant at the 1% level in both OLS models. Although MTBI is statistically insignificant when CPS is the dependent variable, it is significantly negative at the 1% level while leverage is the dependent variable. The findings are not unexpected because the purpose of introducing MTBI is to account for leverage, not for CPS.

Estimates from the second stage in Table 6 indicate that prior findings in Table 3 and 4 stay with strong statistical significance. The results show that PPS is positively related to firm performance while the association of CPS is negatively. Both the relations are statistically significant at the 1% level. Most importantly, the product term i.e., the interaction effect, is also significant at the 1% level. Hence, this study suggests that stand-alone relations of either pay-performance sensitivity or CEO power to firm performance is inadequate in understanding the firm value implications. As an additional robustness check, we account for the year-effect and find that these results hold to the same level of significance.

**Table 5: Two-stage least square estimation: First stage**

	<b>CPS</b>	<b>Leverage</b>
PPS	-0.0550*** (-141.81)	-0.0192*** (-8.15)
PPS X CPS	0.1551*** (453.59)	0.0157*** (7.56)
Lag (PPS)	0.0194*** (36.84)	0.0189*** (5.94)
Lag (CPS)	0.3256*** (50.63)	0.1385*** (3.55)
Lag (PPS) X Lag (CPS)	-0.0550*** (-50.88)	-0.0074 (-1.13)
MtB	-0.0002*** (-3.88)	0.0067*** (21.54)
Market Cap	0.0004* (1.66)	-0.0201*** (-13.04)
Sales	-0.0001 (-0.42)	0.0294*** (19.96)
CEO Age	-0.0001*** (-2.81)	-0.0004** (-2.43)
ROA	0.0091*** (3.55)	-0.0711*** (-4.55)
R&D	-0.0055 (-1.42)	-0.3336*** (-14.29)
Capex	-0.0051 (-1.28)	-0.0412* (-1.69)
<i>InMedCPS</i>	0.0520*** (8.52)	0.4327*** (11.69)
<i>MTBI</i>	-0.0063 (-1.30)	-0.2975*** (-10.09)
Observations	17,901	17,901
$R^2$	93%	91%

This table reports parameter estimates from the first stage of the two-stage least square estimation of stock return performance. Industry median CPS (*InMedCPS*) i.e., median CPS in the four-digit SIC (standard industrial classification) codes and marginal tax rate before interest deductions (*MTBI*) are used as the instrumental variables (IV) to account for the two potential endogenous variables: CPS and leverage. Presented in parentheses are the t-statistics. Intercepts are not reported. PPS is the natural log of pay-performance sensitivity; Market Cap is the natural log of market capitalization; Sales is the natural log of annual sales. Control variables are explained in data section and summary statistics has been provided in Table 1. \*\*\*, \*\*, \* indicate 1%, 5%, 10% level of significance respectively.

**Table 6: Two-stage least square estimation: Second stage**

	<b>Annual Return</b>
PPS	0.4036*** (59.92)
CPS	-5.5913*** (-2.90)
PPS X CPS	0.7362*** (2.85)
Lag (PPS)	-0.5588*** (-10.32)
Lag (CPS)	1.2694** (2.42)
Lag (PPS) X Lag (CPS)	-0.1886*** (-2.48)
MtB	-0.0008 (-0.19)
Market Cap	0.0454*** (4.61)
Sales	-0.0816*** (-11.40)
CEO Age	0.0005 (0.94)
Leverage	1.2751*** (3.47)
ROA	0.2507*** (4.78)
R&D	0.1751** (1.27)
Capex	-0.1599* (-1.90)
Observations	17,901
$R^2$	26%

This table reports parameter estimates from the second stage of the two-stage least square estimation of stock return performance. CPS and leverage have been instrumented by industry median CPS (InMedCPS) i.e., median CPS in the four-digit SIC (standard industrial classification) codes and marginal tax rate before interest deductions (MTBI). That is, fitted values of CPS and leverage derived in the first stage are plugged into the second stage along with other control variables. Presented in parentheses are the t-statistics. Intercepts are not reported. PPS is the natural log of pay-performance sensitivity; Market Cap is the natural log of market capitalization; Sales is the natural log of annual sales. Control variables are explained in data section and summary statistics has been provided in Table 1. Number of observations in the regression (compared to equation 5 i.e., column 6 of Table 4) varies due to data limitation (missing values of instrumental variables). \*\*\*, \*\*, \* indicate 1%, 5%, 10% level of significance respectively.

**Table 7: Stock Returns Behavior over Potential Interaction Effects in Figure 2 and 3**

Annual Stock Returns (%)										
Panel A (Figure 2): With lag terms and 1 standard deviation										
Lag		1 <sup>st</sup> Interaction		2 <sup>nd</sup> Interaction		3 <sup>rd</sup> Interaction		4 <sup>th</sup> Interaction		
		Low CPS (0.272)	High CPS (0.493)	Low CPS (0.272)	High CPS (0.493)	Low CPS (0.272)	High CPS (0.493)	Low CPS (0.272)	High CPS (0.493)	
Low PPS	(3.729)	<b>-1.202*</b>	<b>-1.226</b>	<b>-1.204</b>	<b>-1.228</b>	<b>-0.119</b>	<b>-0.143</b>	<b>-0.102</b>	<b>-0.126</b>	
High PPS	(6.947)	<b>-0.066</b>	<b>-0.072</b>	<b>-0.069</b>	<b>-0.074</b>	<b>1.017</b>	<b>1.011</b>	<b>1.033</b>	<b>1.028*</b>	
Panel B (Figure 3): With current terms and various standard deviations										
Current		1 St. Deviation			2 St. Deviation			3 St. Deviation		
		CPS	Low	High	PPS	Low	High	CPS	Low	High
PPS	Low	(3.729)	<b>-6.596</b>	<b>-10.703</b>	(2.120)	<b>-5.265*</b>	<b>-17.265</b>	(0.511)	<b>-2.042</b>	<b>-25.720</b>
	High	(6.947)	<b>-3.257</b>	<b>-3.579</b>	(8.556)	<b>-2.373</b>	<b>0.769</b>	(10.165)	<b>-3.381</b>	<b>7.009</b>
Lag terms with corresponding deviations										
Lag		1 St. Deviation			2 St. Deviation			3 St. Deviation		
		LagCPS	Low	High	LagCPS	Low	High	LagCPS	Low	High
Lag PPS	Low	(3.769)	-0.066	-0.107	(2.184)	-0.053	-0.173	(0.599)	-0.020	-0.257
	High	(6.938)	-0.033	-0.036	(8.523)	-0.024	0.008	(10.108)	-0.034	0.070

Bold face numbers are used to draw Figures 2 and 3. Numbers in parentheses and with star (\*) are computed as below.

**Panel A:** Using the summary statistics in Table 1, we compute high and low quantities as below.

1st interaction: Fixing at high lag (log(PPS)) and high lag (CPS): 6.938 and 0.494

2nd interaction: Fixing at high lag (log(PPS)) and low lag (CPS): 6.938 and 0.278

3rd interaction: Fixing at low lag (log(PPS)) and high lag (CPS): 3.769 and 0.494

4th interaction: Fixing at low lag (log(PPS)) and low lag (CPS): 3.769 and 0.278

Current terms: Low log(PPS) = Median log(PPS) – St.Dev(log(PPS)) = 5.3377 – 1.6090 = 3.729

High log(PPS) = Median log(PPS) + St.Dev(log(PPS)) = 5.3377 + 1.6090 = 6.947

Low (CPS) = Median (CPS) – St.Dev(CPS) = 0.3827 – 0.1107 = 0.272

High (CPS) = Median (CPS) + St.Dev(CPS) = 0.3827 + 0.1107 = 0.493

Lag terms: Low lag(log(PPS)) = Median Lag(log(PPS) – St.Dev(lag(log(PPS)))) = 5.3534 – 1.5849 = 3.769

High lag(log(PPS)) = Median Lag(log(PPS) + St.Dev(lag(log(PPS)))) = 5.3534 + 1.5849 = 6.938

Low lag(CPS) = Median Lag(CPS) – St.Dev(lag(CPS)) = 0.3860 – 0.1081 = 0.278

High lag(CPS) = Median Lag(CPS) + St.Dev(lag(CPS)) = 0.3860 + 0.1081 = 0.494

A short form of Eq.(5) is used to illustrate interaction effects

$$\text{Stock Return}_{i,t} = \gamma_1 \text{PPS}_{i,t} + \gamma_2 \text{CPS}_{i,t} + \gamma_3 (\text{PPS} * \text{CPS})_{i,t} + \gamma_4 \text{PPS}_{i,t-1} + \gamma_5 \text{CPS}_{i,t-1} + \gamma_6 (\text{PPS} * \text{CPS})_{i,t-1}$$

Applying coefficients in Column 6, Table 4

$$\text{Stock Return}_{i,t} = 0.3456 \text{PPS}_{i,t} - 0.2077 \text{CPS}_{i,t} + 0.0264 (\text{PPS} * \text{CPS})_{i,t} - 0.3554 \text{PPS}_{i,t-1} - 0.1828 \text{CPS}_{i,t-1} + 0.0278 (\text{PPS} * \text{CPS})_{i,t-1}$$

We compute the stock return for low log(PPS) and low CPS on the 1st, fixing at high lag (log(PPS)) and high lag (CPS): 6.938 and 0.494. The first cell at the 1<sup>st</sup> interaction: 0.3456 × 3.729 – 0.2077 × 0.272 +

$$0.0264(3.729 \times 0.272) - 0.3554 \times 6.938 - 0.1828 \times 0.494 + 0.0278(6.938 \times 0.494) = -1.202$$

We compute the stock return for high log(PPS) and high CPS on the 4th, fixing at high lag (log(PPS)) and high lag (CPS): 3.769 and 0.278. The last cell at the 4<sup>th</sup> interaction: 0.3456 × 6.947 – 0.2077 × 0.493 +

$$0.0264(6.947 \times 0.493) - 0.3554 \times 3.769 - 0.1828 \times 0.278 + 0.0278(3.769 \times 0.278) = 1.028$$

**Panel B:** Using the summary statistics in Table 1, we compute high and low quantities as below.

**With 1 Standard Deviation:**

Current terms: Low log(PPS) = Median log(PPS) – St.Dev(log(PPS)) = 5.3377 – 1.6090 = 3.729

High log(PPS) = Median log(PPS) + St.Dev(log(PPS)) = 5.3377 + 1.6090 = 6.947

Low (CPS) = Median (CPS) – St.Dev(CPS) = 0.3827 – 0.1107 = 0.272

High (CPS) = Median (CPS) + St.Dev(CPS) = 0.3827 + 0.1107 = 0.493

Lag terms: Low lag(log(PPS)) = Median Lag(log(PPS)) – St.Dev(lag(log(PPS))) = 5.3534 – 1.5849 = 3.769  
 High lag(log(PPS)) = Median Lag(log(PPS)) + St.Dev(lag(log(PPS))) = 5.3534 + 1.5849 = 6.938  
 Low lag(CPS) = Median Lag(CPS) – St.Dev(lag(CPS)) = 0.3860 – 0.1081 = 0.278  
 High lag(CPS) = Median Lag(CPS) + St.Dev(lag(CPS)) = 0.3860 + 0.1081 = 0.494

**With 3 Standard Deviation:**

Current terms: Low log(PPS) = Median log(PPS) – 3\*St.Dev(log(PPS)) = 5.3377 – 3\*1.6090 = 0.511  
 High log(PPS) = Median log(PPS) + 3\*St.Dev(log(PPS)) = 5.3377 + 3\*1.6090 = 10.165  
 Low (CPS) = Median (CPS) – 3\*St.Dev(CPS) = 0.3827 – 3\*0.1107 = 0.051  
 High (CPS) = Median (CPS) + 3\*St.Dev(CPS) = 0.3827 + 3\*0.1107 = 0.715  
 Lag terms: Low lag(log(PPS)) = Median Lag(log(PPS)) – 3\*St.Dev(lag(log(PPS))) = 5.3534 – 3\*1.5849 = 0.599  
 High lag(log(PPS)) = Median Lag(log(PPS)) + 3\*St.Dev(lag(log(PPS))) = 5.3534 + 3\*1.5849 = 10.108  
 Low lag(CPS) = Median Lag(CPS) – 3\*St.Dev(lag(CPS)) = 0.3860 – 3\*0.1081 = 0.062  
 High lag(CPS) = Median Lag(CPS) + 3\*St.Dev(lag(CPS)) = 0.3860 + 3\*0.1081 = 0.710

**With 2 Standard Deviation:**

Current terms: Low log(PPS) = Median log(PPS) – 2\*St.Dev(log(PPS)) = 5.3377 – 2\*1.6090 = 2.120  
 High log(PPS) = Median log(PPS) + 2\*St.Dev(log(PPS)) = 5.3377 + 2\*1.6090 = 8.556  
 Low (CPS) = Median (CPS) – 2\*St.Dev(CPS) = 0.3827 – 2\*0.1107 = 0.161  
 High (CPS) = Median (CPS) + 2\*St.Dev(CPS) = 0.3827 + 2\*0.1107 = 0.604  
 Lag terms: Low lag(log(PPS)) = Median Lag(log(PPS)) – 2\*St.Dev(lag(log(PPS))) = 5.3534 – 2\*1.5849 = 2.184  
 High lag(log(PPS)) = Median Lag(log(PPS)) + 2\*St.Dev(lag(log(PPS))) = 5.3534 + 2\*1.5849 = 8.523  
 Low lag(CPS) = Median Lag(CPS) – 2\*St.Dev(lag(CPS)) = 0.3860 – 2\*0.1081 = 0.170  
 High lag(CPS) = Median Lag(CPS) + 2\*St.Dev(lag(CPS)) = 0.3860 + 2\*0.1081 = 0.602

Omitting the intercept and error term in Eq.(5), we estimate -5.265%\* in Panel B fixing low CPS and low PPS as below.

$$\begin{aligned} \text{Stock Return}_{i,t} &= \gamma_1 \text{PPS}_{i,t} + \gamma_2 \text{CPS}_{i,t} + \gamma_3 (\text{PPS} * \text{CPS})_{i,t} + \gamma_4 \text{PPS}_{i,t-1} + \gamma_5 \text{CPS}_{i,t-1} \\ &\quad + \gamma_6 (\text{PPS} * \text{CPS})_{i,t-1} + \gamma_7 \text{Market to Book}_{i,t} + \gamma_8 \text{Market Cap}_{i,t} + \gamma_9 \text{Sale}_{i,t} \\ &\quad + \gamma_{10} \text{CEOAge}_{i,t} + \gamma_{11} \text{Leverage}_{i,t} + \gamma_{12} \text{ROA}_{i,t} + \gamma_{13} \text{R\&D}_{i,t} + \gamma_{14} \text{CAPEX}_{i,t} \\ \text{Stock Return} &= \gamma_1 \times 2.11961 + \gamma_2 \times 0.16129 + \gamma_3 \times (2.11961 \times 0.16129) + \gamma_4 \times 2.18367 \\ &\quad + \gamma_5 \times 0.16977 + \gamma_6 \times (2.18367 \times 0.16977) + \gamma_7 \times 3.27154 + \gamma_8 \times 7.63634 \\ &\quad + \gamma_9 \times 7.36355 + \gamma_{10} \times 56.12324 + \gamma_{11} \times 0.13166 + \gamma_{12} \times 0.11446 \\ &\quad + \gamma_{13} \times 0.02644 + \gamma_{14} \times 0.04572 \\ -0.05265 &= 0.34564 \times 2.11961 - 0.20773 \times 0.16129 + 0.0264 \times (2.11961 \times 0.16129) \\ &\quad - 0.35539 \times 2.18367 - 0.18278 \times 0.16977 + 0.02779 \times (2.18367 \times 0.16977) \\ &\quad + 0.01185 \times 3.27154 + 0.0532 \times 7.63634 - 0.05964 \times 7.36355 + 0.000537 \times 56.12324 \\ &\quad - 0.06508 \times 0.13166 + 0.21585 \times 0.11446 - 0.12969 \times 0.02644 - 0.27874 \times 0.04572 \end{aligned}$$

**VI. Conclusion**

Prior literature suggests that pay-performance sensitivity (PPS) as a measure of CEO incentive compensation is positively related to firm performance, while CEO pay slice (CPS) as a measure of CEO power is negatively related to firm value. This study conducts an empirical investigation on CEO pay-for-performance in the presence of CEO power and shows that determining such stand-alone implications might be insufficient in maximizing firm values because of an unexplored interaction effect between CEO pay and CEO power.

This study considers multiple factors from a different perspective compared to prior studies. First, to capture the compensation component it simultaneously considers CEO incentive compensation measured by PPS and CEO power measured by CPS instead of focusing only on incentives or cash compensation. Second, the timeline 1992-2019 is selected for this study to

include the accelerated CEO incentive pay in 1990s and to assess firm performance in terms of stock returns during the 2008 financial crisis. Third, this study analyzes the product term of PPS and CPS to investigate the interaction effects, which have not been considered in prior studies.

Collecting compensation and firm related data from Execucomp and Compustat, respectively, and stock price information from CRSP, this study conducts the analysis in two parts. First, Tobin's  $q$  as a proxy of firm performance is regressed on PPS and CPS along with other standard control variables. Second, stock return as a measure of firm performance is considered and similar analyses are performed. Analysis from multiple OLS models suggests that while the high level of CPS is present, changing the PPS level from low to high increases firm value. However, the impact of PPS declines at the low level of CPS. To address the potential endogeneity issues, we conduct robustness tests by employing instrumental variables (IV) with a two-stage least square (2SLS) estimation procedure. Estimates indicate that original findings stay with strong statistical significance. As an additional robustness check, we account for the year-effect and find that the results still stand at the same level of significance. Consistent with the optimal contracting theory, these results contribute to the literature with the interaction effects between CEO incentive pay measured by pay-performance sensitivity and CEO power measured by CEO pay slice, and opens opportunity for further investigations on the characteristics on CEO pay.

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## Corporate Social Responsibility (CSR) and Firm Features in the Immediate Aftermath of the COVID-19 Pandemic

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### Abstract

We examine the relationships between key firm features and their Corporate Social Responsibility (CSR) scores in the context of the COVID-19 pandemic and its aftermath. The pandemic created a unique environment that might have altered investor expectations, CSR priorities, and corporate strategies. We examine scores for (a) energy and water; (b) ethics, customer service, and labor; and (c) governance, as well as the (d) aggregate CSR score, and find that large market-cap and profitable firms are significantly associated with higher CSR energy and water management scores, CSR governance scores, and with higher CSR overall scores. This implies that these firms make these CSR investments, perhaps because of public scrutiny. However, these investments do not appear to lead to significant changes in key financial features such as market capitalization or Market Value of Equity (MVE), profitability as measured by Return on Assets (ROA), or growth (or the lack thereof) as measured by Book-to-Market ratio (BTM), at least in the short run suggesting that CSR benefits may be more strategic.

**Keywords:** Corporate Social Responsibility (CSR); CSR energy and water score; CSR ethics, customer service and labor score; CSR governance score; Firm features; Market Value of Equity (MVE); Return on Assets (ROA); Book-to-Market ratio (BTM); CSR investments; COVID shock.

**JEL Classification:** G32

### I. Introduction

Using comprehensive data over the years 2021 through 2023, we examine the relationships between key firm features and Corporate Social Responsibility (CSR) investments they make, measured by various CSR scores. We examine the separate CSR scores for (a) energy and water, (b) ethics, customer service, and labor, and (c) governance, as well as the aggregate CSR score - CSR Overall defined as the sum of the three CSR energy and water, CSR ethics, customer, and labor, and CSR governance. Our objective is to understand which firms make CSR investments, and what are the outcomes for the firms.

Several papers have examined different aspects of CSR investments made by firms. Markidou, Doumpos, and Lemonakis (2021) found that energy management practices are increasingly valued by stakeholders, with positive impacts on firms' long-term valuation, despite initial costs. The study emphasizes the value of sustainable practices in building resilience in volatile markets. Eccles et al. (2014) found that firms with high energy-management scores tend to have lower operational costs and improved long-term financial performance, linking energy efficiency to competitive advantage and sustainability in resource-intensive industries. Delmas and Pekovic (2013) studied the relationship between environmental practices, including energy management, and productivity. Their results indicate that firms prioritizing energy management experience greater operational efficiency and enhanced reputations, which can translate into higher profitability.

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Edmans (2011) found that companies with high employee satisfaction outperformed others in stock returns, underscoring the value of fair labor practices in enhancing shareholder wealth. The study emphasizes labor practices as a strategic asset for corporate success. Lins, Servaes, and Tamayo (2017) observe that ethical governance and compliance practices bolster firm performance, especially during economic downturns. Ethical behavior is associated with greater investor trust and can be a safeguard during periods of financial instability.

Cavusoglu et al. (2004) investigated the impact of data breaches on firm market value, revealing a significant negative impact when customer data becomes compromised. This underscores the financial risks tied to inadequate data security practices. Acquisti, Brandimarte, and Loewenstein (2015) analyzed consumer reactions to privacy concerns, showing that firms with higher data privacy standards retain customer trust and loyalty. This highlights data security as essential for maintaining a competitive edge in customer satisfaction. Gordon, Loeb, and Zhou (2011) examined the impact of cybersecurity investments on firm value, finding that firms with high data security standards face lower costs from breaches and maintain customer trust, enhancing firm reputation and loyalty.

Acquisti, Taylor, and Wagman (2016) explored how strong data privacy practices influence consumer behavior, showing that companies with high data security scores retain better customer loyalty and suffer fewer financial impacts from breaches. Romanosky, Hoffman, and Acquisti (2014) analyzed cyber event disclosures, concluding that firms with proactive data security measures are less susceptible to legal penalties and market backlash, linking strong data security practices to firm value protection. Edmans (2011) finds that companies prioritizing employee satisfaction see improved stock performance, suggesting that fair labor practices positively correlate with long-term financial success.

Bae, Kang, and Wang (2011) analyzed the effects of labor practices on firm performance, showing that strong employment practices correlate with higher employee satisfaction and lower turnover, which positively impact productivity and profitability. Edmans (2011) demonstrated that companies with positive labor practices tend to outperform peers in long-term stock returns, supporting the idea that labor scores are valuable indicators of firm resilience and growth potential.

Carter et al. (2010) demonstrate that diverse boards improve decision-making, which can lead to better ESG outcomes and enhance overall governance quality. The presence of a diverse board composition is associated with stronger firm performance. Hartzell and Starks (2003) suggest that linking executive compensation to ESG metrics can incentivize sustainable practices. Bebchuk's (2004) seminal work investigates how executive compensation is often not tied to firm performance, focusing on the misalignment between executive pay and shareholder value. The authors argue that excessive executive compensation may be driven by weak governance structures and the influence of executives on the compensation-setting process. Frydman (2010) study uses historical data to examine trends in executive compensation over a period of 70 years and finds that while the pay-performance relationship has strengthened over time, compensation growth has outpaced firm performance, highlighting that increasing compensation is not always aligned with firm outcomes.

Bergstresser (2006) explores the relationship between executive compensation and earnings management. It finds that executives with compensation structures heavily reliant on stock options are more likely to engage in earnings manipulation to meet performance targets, highlighting potential negative effects of poorly designed compensation packages. Bebchuk et al. (2009) show that stronger shareholder rights are correlated with better corporate governance, which supports higher firm value. Bebchuk, Cohen, and Ferrell (2009) analyzed the influence of shareholder rights on firm value, showing that stronger shareholder protections, reflected in high shareholder rights scores, often correlate with better financial performance and reduced agency conflicts. Gompers, Ishii, and Metrick (2003) examined the "Governance Index" and

found that companies with more robust shareholder rights tend to have higher market valuations and profitability, linking strong governance practices with firm stability.

Aggarwal, Erel, Stulz, and Williamson (2009) found that firms in countries with strong shareholder rights structures often benefit from lower capital costs, as shareholders' influence over key decisions aligns corporate strategies with long-term growth goals. Simunic and Stein (1996) find that strong auditing reduces the likelihood of financial misreporting, thereby building investor confidence and promoting sustainable firm performance. Carcello et al. (2011) examined the relationship between audit quality and corporate governance, finding that firms with strong governance and high audit scores are less likely to face financial misreporting issues, thus bolstering investor confidence and firm valuation. DeFond and Zhang (2014) reviewed the significance of audit quality, noting that high audit scores reduce agency costs and improve transparency, particularly in large, public firms where governance plays a crucial role. Cheng, Dhaliwal, and Zhang (2013) explored audit scores in the context of firm risk, concluding that stronger audit practices, measured by audit scores, mitigate risk and align financial reporting with regulatory standards.

Overall, Friede, Busch, and Bassen (2015) aggregate findings from over 2,000 studies and conclude that CSR investments positively impact firm valuation, especially in large-cap firms, due to their greater visibility and regulatory pressure. Khan, Serafeim, and Yoon (2016) report that CSR activities related to material issues, specific to industry sectors, have a positive impact on profitability, particularly in firms where CSR activities are aligned with core business operations. Dhaliwal et al. (2011) found that firms with high CSR disclosure practices tend to have a lower book to market ratio, suggesting that investors may assign a premium to companies with strong CSR initiatives due to perceived future growth and lower risk profiles. Lins, Servaes, and Tamayo (2017) explored the financial value of CSR during crisis periods and observed that firms with high CSR scores had higher valuation multiples (and lower book to market ratios) during financial downturns, indicating CSR's role in enhancing firm resilience. Ameer and Othman (2012) found that CSR investments are linked to higher profitability, especially in sectors sensitive to public perception. Firms with larger market capitalization (market cap) were more likely to invest in CSR, suggesting an alignment between CSR activities and firm resources. Choi, Kwak, and Choe (2010) analyzed the book-to-market ratio and CSR scores, finding that firms with lower book-to-market ratios are more engaged in CSR, suggesting that high market valuation firms often prioritize sustainability. In a recent paper, Bhagawan and Mukhopadhyay (2023) examine the impact of mandatory CSR spending on firm value (Tobin's Q) in the Indian context and find that mandatory CSR spending has a positive and statistically significant impact on firm value, especially for firms with higher information asymmetry problem and lower institutional holdings.

Thus, several papers in the extant literature have examined firm investments in energy and water; in ethics, customer service, and labor; and in governance, and have generally found positive effects of these investments on firm market share or profitability or other features. In this paper, we examine the most recent data (2021-2023) to provide an updated perspective on CSR investments' relationship with firm characteristics in the context of the COVID-19 pandemic and its aftermath. The pandemic created a unique environment that might have altered investor expectations, CSR priorities, and corporate strategies.

The COVID-19 pandemic disrupted financial markets, influencing how investors assess companies' financial health and long-term prospects. It heightened societal and investor focus on corporate social responsibility (CSR), as issues such as employee well-being, environmental sustainability, and governance became more critical in shaping corporate strategies. Extant literature has generally argued that firms with higher CSR scores might maintain investor confidence even during economic crises, supporting the idea that CSR activities contribute to continued higher market cap. Firms with higher CSR scores could attract more investor interest

due to their perceived resilience and responsible business practices during the crisis. This heightened scrutiny and investor preference may contribute to a positive relationship between CSR and market cap, as well as profitability. We review the literature below.

Xia et al. (2021) explore how firms with strong CSR practices can be more resilient to stock market shocks during the COVID-19 crisis. Companies with better CSR performance tended to have more stable stock returns, supporting the notion that CSR can be tied to investor trust, especially in times of economic uncertainty. Kotsantonis et al. (2020) examine whether CSR activities act as a protective shield during the COVID-19 pandemic, and find that high CSR scores were associated with a smaller decline in firm value during the market downturn. Wang et al. (2020) examine the relationship between CSR and financial performance during the COVID-19 crisis, noting that while CSR investments may not immediately impact financial metrics, they can contribute to firms' long-term resilience and helped maintain investor confidence. Bolton and Kacperczyk (2021) demonstrate that during the COVID-19 pandemic, investor concern over carbon risk increased, leading to greater integration of environmental factors into investment decisions. Firms with higher carbon emissions faced higher expected returns, reflecting a risk premium demanded by investors. Environmental responsibility became more central to ESG evaluations and capital allocation, with carbon risk likely a key factor in assessing firm resilience and long-term value. Cheema-Fox et al (2021) analyze how corporate responses to the COVID-19 crisis influenced stock performance. After examining 3,023 global companies, they argue that firms demonstrating positive actions such as supporting employees, maintaining supply chains, and repurposing operations to meet urgent needs, experienced less severe stock declines during the market downturn.

Al Amosh et al (2023) conducted a study analyzing 12,325 firm-year observations from 2016 to 2021 to assess how ESG performance evolved during the COVID-19 pandemic. Their findings indicate that the pandemic positively influenced overall ESG performance, particularly enhancing environmental and social dimensions, while governance performance declined. The study suggests that during crises, firms prioritize ethical behavior and societal expectations, leading to increased ESG compliance. Qing et al (2023) examine 359 mobile business platform firms to assess how ESG activities and AI-driven digital transformation affect corporate sustainability, with green innovation as a moderating factor. Findings show ESG efforts significantly improve sustainability, especially post-pandemic. Gipper et al (2024) provides a large-scale evidence on ESG assurance practices among U.S. firms, focusing on S&P 500 companies from 2010 to 2020. Their study reveals a significant increase in both the number of firms issuing ESG reports and those obtaining third-party assurance—rising from 38% to 76% and from 16% to 46%, respectively, over the decade. This growth is primarily driven by the adoption of ESG reporting frameworks and peer influence, rather than firm-specific characteristics. The authors find that ESG assurance is associated with improved disclosure quality, higher ESG ratings, and increased institutional investor interest, suggesting that assurance enhances the credibility of ESG reporting and may positively influence firm performance.

Lu et al (2024) analyzed the impact of pre-pandemic corporate social responsibility (CSR) on stock resilience during the COVID-19 crisis, utilizing contract theory to understand stakeholder dynamics. Their study reveals that firms with strong CSR practices prior to the pandemic experienced moderate stock gains, whereas those with weaker CSR faced significant losses, particularly around the lockdown. This suggests that robust CSR initiatives can enhance stakeholder trust and operational stability, thereby mitigating financial uncertainties during crises. The findings underscore the strategic value of CSR as a mechanism for risk management and resilience in the face of systemic shocks. Mohy-ud-Din (2024) investigates the impact of ESG reporting on corporate green innovation in U.S. firms, highlighting the moderating role of board diversity. Analyzing data from 334 non-financial companies listed in the S&P 1500 Index between 2010 and 2021, the study finds that higher ESG ratings and robust CSR strategies

are significantly associated with improved environmental performance and innovation outcomes. Notably, board diversity enhances the positive relationship between ESG efforts and green innovation, suggesting that diverse boards contribute to increased green revenues, product innovation, and the adoption of eco-friendly technologies.

Some studies were less sanguine. Bae et al (2021) analyzed 1,750 U.S. firms during the COVID-19 pandemic and found that pre-crisis CSR investments did not significantly influence stock returns during the market crash or the subsequent recovery period. This suggests that CSR activities prior to the pandemic were not effective in shielding shareholder wealth from the adverse effects of the crisis. The authors also observed that firms with explicit commitments to stakeholder interests, such as Business Roundtable signatories, did not outperform others during the crisis. Lööf et al (2022) examined whether ESG investing provided a "free lunch" for investors during the COVID-19 crisis by analyzing 5,073 stocks across ten countries. Their study finds that higher ESG ratings are associated with lower downside tail risk, meaning these stocks were less susceptible to extreme losses during market turmoil. However, this reduced risk comes at the cost of lower upside potential, indicating a trade-off between risk mitigation and return. Cardillo et al (2005) conducted a bibliometric and systematic review to examine the moderating variables influencing the relationship between ESG/CSR activities and financial performance. Their analysis identifies key factors such as firm size, industry sector, geographic region, and governance structures that affect how ESG and CSR initiatives impact financial outcomes. The study emphasizes that these moderating variables can either strengthen or weaken the link between sustainability efforts and financial performance, highlighting the importance of context in evaluating ESG/CSR effectiveness. Narula et al (2025) analyzed 198 studies published between 2010 and 2022. They found that ESG activities generally have a positive impact on firm performance. However, the strength of this relationship varies depending on factors such as ESG rating methodologies (e.g., Bloomberg vs. Thomson Reuters), industry sectors, and geographic regions.

Therefore, examining how the relationship between CSR investments and key firm features holds up in the post-pandemic context offers new insights into the effects of CSR investments after a significant global disruption, help validate previous findings using recent data, and show whether these relationships can be generalized across different market conditions.

We examine the relationships between key firm features and CSR scores for energy and water; ethics, customer service, and labor; and governance (proxying for investments in these areas), as well as the aggregate CSR score, and find that large market-cap and profitable firms are significantly associated with higher CSR energy and water management scores, CSR governance scores, and with higher CSR overall scores. However, we find these investments do not appear to lead to significant changes in key financial features such as market cap or Market Value of Equity (MVE), profitability as measured by Return on Assets (ROA), or (the inverse of) growth options as measured by Book-to-Market ratio (BTM), at least in the short run (over our time period 2021-2023) suggesting that CSR benefits may be long-term and strategic.

## **II. Data, Variables, and Methodology**

We obtain comprehensive data for the different (annual) CSR scores from Bloomberg and MSCI (Morgan Stanley Capital International) database for as many firms as we can, for the years 2021 through 2023. MSCI is a leading provider of global equity indices, widely used in the investment and financial sectors to measure and manage risk, as well as to assess the sustainability and governance practices of companies across different industries. The CSR scores are based on companies' reporting and disclosures, with MSCI incorporating sector-specific risks and opportunities into its scoring model. We matched these data to firm data from S&P's Compustat database for 2021 through 2023 also, to examine trends in firm characteristics and

relate them to changing CSR scores over the same period, resulting in a final dataset of 255 companies based in the United States, with all the information we need (detailed below) for the years 2021, 2022 and 2023. This timeline allows us to analyze the relationship between CSR investments and key firm features immediately following the COVID-19 pandemic. The same companies are present in our final database every year; so it is a balanced sample. This list of companies is dominated by Software and Tech Services, or the HiTech industry (as classified in Kenneth French data library site, also called “Fama-French industry classification”), because these companies are often at the forefront of CSR and ESG activities due to their visibility, global impact, and regulatory environment, in the recent years.

Prior literature has found that the market cap, the book to market ratio, and the profitability as measured by the return on assets are important firm characteristics that can reflect managerial strategies’ outcomes. Therefore, we used four firm characteristics:

- (a) industry, the 4-digit SIC code;
- (b) market value of equity (MVE), the market cap, a measure of presence in the market that could influence its revenue streams, financing options, and brand recognition;
- (c) the book-to-market ratio (BTM), a proxy for the (inverse of) growth prospects or real option value of the firm (Tobin’s Q); and
- (d) return on assets (ROA), an accounting-based measure of operating performance that focuses on profitability per dollar of assets, which is widely used in long-run performance studies.

MVE (market value of equity) is calculated by multiplying CSHO (shares outstanding) from annual Compustat, the net number of all common shares outstanding at year-end excluding treasury shares and scrip, with PRC (price) from CRSP, which is the closing price of the same day as the report date of CSHO. Book value of equity (BVE), which equals shareholders’ equity + deferred taxes and investment tax credit - preferred/preference stock (capital) from annual Compustat. The book to market ratio (BTM), which is the BVE divided by the MVE as computed above. ROA (return on assets), every year, is calculated by dividing NI (net income) by AT (total assets), from annual Compustat. Industry is a 4-digit SIC code from Compustat.

Our CSR data from MSCI database, where the definitions are all coded by MSCI, contains all relevant firm scores on the following metrics. MSCI evaluates CSR components based on sector-specific and region-specific standards, and it adjusts the weight and scale accordingly. For instance, environmental scores (energy, water) may be quantitatively measured, while governance factors (board composition, executive compensation) are often rated on qualitative criteria like board diversity or structure.

### **CSR Energy and Water Score**

We define this as the sum of:

- EMI (Energy Management Issue Score): Reflects a company’s effectiveness in managing energy use, focusing on energy conservation, renewable energy adoption, and carbon emissions reduction.
- H2O (Water Management Issue Score): Assesses a company’s policies and actions related to water management, including water conservation efforts, efficiency improvements, and handling of water-related risks.

### **CSR Ethics, Customer, and Labor Score**

We define this as the sum of:

- **ETH (Ethics and Compliance Score):** Evaluates a company's ethical standards, anti-corruption measures, and data privacy protocols, showing how robustly it upholds compliance, privacy, and integrity in its operations.
- **CUST (Data Security and Customer Score):** Measures a company's data security practices, with a focus on protecting customer information, cybersecurity protocols, and response strategies for data breaches.
- **LABOR (Labor and Employment Practices Score):** Assesses a company's labor policies, including diversity and inclusion, employee well-being, fair employment practices, and adherence to labor standards.

## **CSR Governance**

We define this as the sum of:

- **BOARD (Board Composition Score):** Evaluates the diversity, independence, and expertise of board members, reflecting governance quality and potential effectiveness of corporate oversight.
- **EXEC (Executive Compensation Score):** Measures the alignment of executive compensation with company performance, considering incentives, equity in pay, and support for long-term value creation.
- **SHARE (Shareholder Rights Score):** Examines the governance framework protecting shareholder rights, such as voting mechanisms, board accountability, and shareholder engagement policies.
- **AUDIT (Audit Score):** Assesses the strength of a company's audit practices and internal controls, indicating the quality of oversight and transparency in financial reporting.

## **Descriptive Statistics**

We start by examining the descriptive statistics of the various CSR scores that we collected, and the various firm features associated with these scores year by year for 2021, 2022 and 2023. Table 1 reports the descriptive statistics (the mean, median, and standard deviation (SD)) of the overall CSR score and its components.

The table shows that the CSR energy and water scores are stable year after year, with low variability, while the CSR governance score increases and remains relatively stable. There are more fluctuations in the CSR ethics, customer and labor score. The overall mean and median CSR scores shows a steady increase over the years.

Table 2 reports the descriptive statistics (the mean, median, and standard deviation (SD)) of the characteristics for the firms for which we have the CSR scores.

The mean market cap or MVE is higher than the median, indicating a few large market cap firms in our sample. The book-to-market ratio is also increasing over time, indicating more "value orientation" over time in our sample for which we have the CSR scores. The average ROA remains slightly negative over time, although the median has moved towards being positive in 2023.

**Table 1: Descriptive Statistics of CSR scores****Panel A: Descriptive statistics of 2023 MSCI CSR scores.**

	<b>N</b>	<b>Mean</b>	<b>SD</b>	<b>Median</b>
CSR energy and water	255	4.33	4.72	3.00
CSR ethics, customer, and labor	255	6.94	4.89	5.09
CSR governance	255	27.57	3.50	27.26
CSR Overall	255	38.81	10.53	36.90

**Panel B: Descriptive statistics of 2022 MSCI CSR scores.**

	<b>N</b>	<b>Mean</b>	<b>SD</b>	<b>Median</b>
CSR energy and water	255	4.31	4.72	3.00
CSR ethics, customer, and labor	255	4.36	3.85	3.35
CSR governance	255	27.28	3.87	27.16
CSR Overall	255	35.96	8.02	35.57

**Panel C: Descriptive statistics of 2021 MSCI CSR scores.**

	<b>N</b>	<b>Mean</b>	<b>SD</b>	<b>Median</b>
CSR energy and water	255	4.33	4.72	3.00
CSR ethics, customer, and labor	255	9.07	4.99	8.70
CSR governance	255	21.35	5.55	22.24
CSR Overall	255	34.75	8.91	34.87

This table shows the descriptive statistics (the mean, median, and standard deviation (SD)) of the overall CSR score, as well as the three components of the CSR score – CSR energy and water, CSR ethics, customer, and labor, and CSR governance. The three panels show these statistics year by year: Panel A for 2023, Panel B for 2022 and Panel C for 2021.

**Table 2: Descriptive Statistics of Firm Features****Panel A: Descriptive statistics of 2023 firm characteristics.**

	<b>N</b>	<b>Mean</b>	<b>SD</b>	<b>Median</b>
MVE	255	28.6 billion	16.5 billion	3.8 billion
BTM	255	0.271	0.39	0.19
ROA	255	-0.029	0.15	0.005

**Panel B: Descriptive statistics of 2022 firm characteristics.**

	<b>N</b>	<b>Mean</b>	<b>SD</b>	<b>Median</b>
MVE	255	23.3 billion	12.6 billion	3.4 billion
BTM	255	0.273	0.71	0.22
ROA	255	-0.048	0.18	-0.011

**Panel C: Descriptive statistics of 2021 firm characteristics.**

	<b>N</b>	<b>Mean</b>	<b>SD</b>	<b>Median</b>
MVE	255	29.1 billion	13.6 billion	4.6 billion
BTM	255	0.175	0.65	0.13
ROA	255	-0.031	0.21	-0.003

This table shows the descriptive statistics (the mean, median, and standard deviation (SD)) of the market value of equity (MVE), the book-to-market ratio (BTM), and the return on assets (ROA). The three panels show these statistics year by year: Panel A for 2023, Panel B for 2022 and Panel C for 2021.

We now state our main hypotheses linking CSR scores with key firm features that we test in this paper.

## Hypothesis

Investments in CSR can be costly and may be made by large market cap firms likely because they face increased investor and institutional scrutiny as well as media attention. Further, based on the theory of “doing-well-by-doing-good”, a positive view perceives CSR as a business strategy to improve firm performance and hence, shareholders’ wealth (see Galema et al., 2008; Gillan et al., 2010; and Bhagwan and Mukhopadhyay, 2024). However, the COVID-19 pandemic disrupted financial markets, and heightened societal and investor focus on corporate social responsibility (CSR), as issues such as employee well-being, environmental sustainability, and governance arguably became more critical in shaping corporate strategies. Therefore, examining how the relationship between CSR investments and key firm features holds up in the post-pandemic context can offer new insights, help validate previous findings using recent data, and show whether previously uncovered relationships can be generalized across different market conditions.

This leads to our two hypotheses:

**Hypothesis 1.** *Larger market cap and/or more profitable firms are significantly associated with higher CSR scores, for reasons that include higher investor interest and scrutiny.*

**Hypothesis 2.** *In the aftermath of global disruption, changes in CSR scores are significantly associated with changes in market cap, the book-to-market ratio and return on assets.*

## III. Results

We first examine the univariate results for the associations between CSR scores and key firm features.

### *Firm Features and CSR Scores*

To analyze the relation between firm features and the various CSR scores, we start with univariate results, segregating the MVE, BTM and ROA for the firms in our sample as above median or below or equal to the median for each of the 3 years in our sample: 2021, 2022, and 2023.

Table 3 shows the result. The three panels show that CSR Overall score and the CSR energy and water scores are significantly higher for the larger market cap firms than those for the smaller market cap firms across all years, while the CSR ethics, customer, and labor and CSR governance scores are significantly higher for the larger market cap firms than those for the smaller market cap firms, in the later years. The CSR Overall, the CSR energy and water scores and CSR governance scores are significantly higher for the more profitable firms than those for the less profitable firms in 2022 and 2023. There are no significant differences in any CSR score (except for CSR energy and water, which is lower for the higher BTM firms) when firms are segregated by BTM. Thus, overall, larger and more profitable firms are associated with higher CSR investments and scores.

**Table 3: Univariate Results of CSR Scores and Firm Features****Panel A: 2023 CSR measures by firm features in 2023**

	MVE			ROA			BTM		
	Above Median	Below or Equal to Median	Difference between group Means	Above Median	Below or Equal to Median	Difference between group Means	Above Median	Below or Equal to Median	Difference between group Means
CSR overall	42.42	35.24	7.18***	40.83	36.78	4.05***	38.23	39.39	-1.16
CSR energy and water	6.11	2.56	3.55***	5.08	3.57	1.51***	4.07	4.59	-0.52
CSR ethics, customer, and labor	8.14	5.75	2.39***	7.22	6.66	0.56	6.76	7.13	-0.37
CSR governance	28.15	26.93	1.22***	28.53	26.54	1.99***	27.40	27.67	-0.27

**Panel B: 2022 CSR measures by firm features in 2022**

	MVE			ROA			BTM		
	Above Median	Below or Equal to Median	Difference between group Means	Above Median	Below or Equal to Median	Difference between group Means	Above Median	Below or Equal to Median	Difference between group Means
CSR overall	39.44	32.51	6.93***	39.36	32.54	6.82***	36.27	35.66	0.61
CSR energy and water	6.99	1.66	5.33***	5.90	2.73	3.17***	3.65	4.97	-1.32*
CSR ethics, customer, and labor	4.60	4.12	0.48	4.65	4.07	0.58	4.72	4.00	0.72
CSR governance	28.33	26.25	2.08***	28.82	25.75	3.07***	27.28	27.29	-0.01

**Panel C: 2021 CSR measures by firm features in 2021**

	MVE			ROA			BTM		
	Above Median	Below or Equal to Median	Difference between group Means	Above Median	Below or Equal to Median	Difference between group Means	Above Median	Below or Equal to Median	Difference between group Means
CSR overall	35.80	33.72	2.08*	35.53	33.98	1.55	35.06	34.46	0.6
CSR energy and water	5.73	2.94	2.79***	5.09	3.55	1.54***	4.18	4.48	-0.3
CSR ethics, customer, and labor	9.14	9.00	0.14	9.24	8.90	0.34	9.66	8.48	1.18
CSR governance	21.08	21.63	-0.55	21.19	21.52	-0.33	21.22	21.49	-0.27

This table shows means, the difference of means, and the significance of the difference in means of (a) CSR overall score, (b) CSR energy and water score, (c) CSR ethics, customer, and labor score, and (d) CSR governance, between firms that are grouped by MVE, ROA and BTM by firms that are above the median in each of these features and those that are below and equal to median, from the 2023 MSCI ESG scores database (Panel A), 2022 MSCI ESG scores database (Panel B), and 2021 MSCI ESG scores database (Panel C).

\*\*\*, \*\*, \* denote significantly different from the other cohort at the 1%, 5%, and 10% significance levels.

We examine these associations next in a multivariate regression setting that controlled for industry fixed effects that includes indicator variables for the consumer, tech, manufacturing, health and other industries (the 5 “Fama-French Industry classifications”, as well as year dummy variables for 2023, 2022 and 2021, to control for unspecified year and industry-related fixed effects, using the following specification:

$$Y_i = \beta_1 MVE_i + \beta_2 BTM_i + \beta_3 ROA_i + \text{Year Fixed Effects} + \text{Industry Fixed Effects} + \varepsilon,$$

where  $Y_i$  is the CSR overall score, or one of the component CSR scores.

**Table 4: Multivariate Results of CSR Scores and Firm Features**

	CSR overall	CSR energy and water	CSR ethics, customer, and labor	CSR governance
	N = 765	N = 765	N = 765	N = 765
MVE <sub>t</sub>	0.0005 ** (2.16)	0.0004 *** (3.02)	0.0001 (0.07)	0.0002 (1.21)
ROA <sub>t</sub>	10.50 *** (5.19)	5.97 *** (5.79)	0.57 (0.55)	3.96 *** (4.06)
BTM <sub>t</sub>	-1.63 *** (-2.76)	-0.779 ** (-2.59)	-0.435 (-1.43)	-0.420 (-1.47)
Industry Effects (Fama-French 5)	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes

Regression coefficients and robust t-statistics when firm features are regressed on (a) CSR Overall score, (b) CSR energy and water score, (c) CSR ethics, customer, and labor score, (d) CSR governance score from the 2023 MSCI database. Indicator variables for the 5 Fama-French Industry classifications, and for year, are included.

\*\*\*, \*\*, \* denote coefficients that are significantly different from 0 at the 1%, 5%, and 10% significance levels.

The table shows that the firm’s market value of equity or market cap, and profitability, measured by ROA, are significantly and positively associated with CSR overall score and CSR energy and water scores. Profitability is also significantly and positively associated with CSR governance score. In line with the associations with market cap, BTM (in which the market cap comes in the denominator) is significantly and negatively associated with CSR overall and CSR energy and water scores.

Tables 3 and 4 show larger and more profitable firms are associated with higher CSR investments and scores. Thus, we find support for Hypothesis 1.

We tackle Hypothesis 2 next.

### Change in CSR Scores and Change in Firm Features

Are changes in CSR scores are significantly associated with changes in market cap, the book-to-market ratio and return on assets. after controlling for industry fixed effects? Table 5 shows this result using the following regression specification:

$$\Delta Z_i = \beta_1 \Delta \text{CSR overall} + \beta_2 \Delta \text{CSR energy and water} + \beta_3 \Delta \text{CSR governance} + \text{Industry Fixed Effects} + \varepsilon,$$

where  $\Delta Z_i$  is  $\Delta MVE_i$  the change in market cap from the change from 2021 to 2023 or  $\Delta BTM_i$  the change in the book-to-market ratio from 2021 to 2023 or  $\Delta ROA_i$  the change in the return on assets from 2021 to 2023. We drop  $\Delta CSR$  ethics, customer, and labor score, from the explanatory variables, in this specification, since the CSR overall score is the sum of the component scores.

**Table 5. Whether changes in CSR Scores Affect Changes in Firm Features.**

	$\Delta MVE_{t-2,t}$	$\Delta ROA_{t-2,t}$	$\Delta BTM_{t-2,t}$
	N = 255	N = 255	N = 255
$\Delta CSR$ overall	146.67 (0.46)	-.0009 (-0.64)	-.0077 (-1.56)
$\Delta CSR$ energy and water	544.49 (0.93)	0.0003 (0.14)	.0080 (0.87)
$\Delta CSR$ governance	169.48 (0.33)	0.0010 (0.45)	.0051 (0.63)
Industry Effects (Fama-French 5)	Yes	Yes	Yes

Regression coefficients and robust t-statistics when changes in (a) CSR Overall score, (b) CSR energy and water score, (c) CSR ethics, customer, and labor score, (d) CSR governance score from 2021 to 2023 are regressed on changes in firm features over the same period. \*\*\*, \*\*, \* denote coefficients that are significantly different from 0 at the 1%, 5%, and 10% significance levels.

Table 5 shows that when the CSR score changes (or when the component CSR score changes) for a firm, there is no significant contemporaneous impact on firm market cap, or profitability, or BTM, on average, after controlling for industry-fixed effects. Therefore, we find no support for our Hypothesis 2, at least contemporaneously and in the immediate aftermath of the Covid-19 disruption and renewed scrutiny.

#### IV. Conclusions and Discussion

Our findings imply that large market-cap and profitable firms may allocate significant resources toward CSR initiatives, particularly in CSR energy and water management, and CSR governance that lead to higher CSR overall scores. However, despite these efforts, these investments do not appear to lead to significant contemporaneous changes in key financial features such as Market Value of Equity (MVE), Return on Assets (ROA), or Book-to-Market ratio (BTM). Bai, and Kim (2024), for example, discuss how CSR practices improve corporate social performance, aligning with stakeholder interests. Their work highlights the nuanced role of CSR in fostering goodwill but notes it is often limited effect on immediate financial outcomes. Hart and Ahuja (1996) argue that while environmental initiatives may improve operational efficiency, their financial impact is often more visible in competitive or resource-intensive industries, whereas firms in dominant market positions may not experience substantial changes in financial metrics. Beekes, & Verhoeven (2011) study the link between corporate governance and firm performance, highlighting that governance improvements strengthen investor confidence and reduce risks but exhibit financial benefits only in the long term. Bear, Rahman, & Post (2010) examine the influence of governance practices, particularly board diversity and independence, on CSR and firm perception. They find that such investments can build stakeholder trust without altering immediate financial performance metrics. We also conclude that CSR investments by large, profitable firms may primarily serve as tools for compliance and reputation enhancement rather than drivers of immediate financial performance, in the immediate aftermath of a pandemic. The financial markets likely perceive CSR efforts as

complementary to core operations, leading to the notion that CSR benefits may be long-term and strategic rather than short-term and transactional.

In this study, we are limited by the time series of MSCI (Morgan Stanley Capital International) data available to us (2021 through 2023), but this allows us to examine the relationships between CSR investments and key firm features in the aftermath of the COVID economic recession. Future studies can examine a longer time series of data to examine effects of different economic crises. Further, other datasets, when available or constructed, may also be examined to corroborate results. For example, Chava, Du, and Malakar (2021) argue that discrepancies in ESG ratings can contribute to the weak or inconsistent relationships observed in financial studies; Serafeim and Yoon (2023) suggest that any inconsistencies of ESG assessments could explain why CSR investments do not always lead to significant shifts in financial performance, as investors may have difficulty interpreting the signals from varying ratings; and Wang, Jiao, Ge, and Sun (2024) examine the relationship between corporate ESG rating divergence and stock returns, showing that the variation in ESG ratings across different agencies can lead to different stock return outcomes.

In summary our findings suggest that while CSR investments may not immediately lead to changes in key financial features, post-pandemic, CSR investments may be seen as a driver of stability, which could help explain the association of these investments in well-followed large cap firms even in this period. While the pandemic had an immediate disruptive effect on markets, CSR practices have the potential to provide long-term strategic benefits, aligning with growing investor demand for companies that are socially responsible and sustainable.

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## **The Relationship Between Firm Growth, Ownership Structure, and Performance: An analysis of U.S. property-liability insurers**

Byeongyong Paul Choi and Eunpyo Hong\*

### **Abstract**

This study investigates the relationship between firm growth and profitability in the U.S. insurance industry, with a focus on differences between stock and mutual insurers. While both forms address stakeholder incentive conflicts, they differ in ownership structure, capital access, and risk preferences. Mutual insurers, owned by policyholders, prioritize long-term stability and operate with limited financial flexibility. In contrast, stock insurers, driven by shareholder interests, have greater access to capital markets and may pursue riskier strategies to boost profitability. Although prior research on profit persistence (POP) in banking and other industries has explored the role of competition, information asymmetry, and firm-specific factors, limited attention has been given to the insurance sector—particularly regarding ownership structure. This study addresses this gap by examining how financial performance and strategic priorities vary between stock and mutual insurers, offering new insights into the determinants of profitability and growth in the insurance industry.

**Keywords:** Firm Growth, Ownership Structure, Profitability, Insurance Industry, Property-Liability Insurers

**JEL Classification:** G22, L25, G32

### **I. Introduction and Literature Review**

Organizational forms in the insurance industry, namely mutuals and stock companies, differ in fundamental ways that influence firm behavior, incentives, and ultimately, the persistence of profit (POP). These structural differences, particularly in terms of ownership, access to capital, and managerial incentives, shape how firms generate and sustain profitability over time. Mutual insurers eliminate the conflict between owners and policyholders by merging these roles, but this can increase the agency problem between managers and owners due to weaker monitoring mechanisms (Mayers and Smith, 1988, 1994; Baranoff and Sager, 2003). Stock insurers, by contrast, face a more distinct separation of ownership and policyholder interests, with greater scrutiny from shareholders and more direct incentives to maximize financial performance.

One critical distinction lies in access to capital markets. Stock insurers, with the ability to issue equity, have more flexibility and resources to pursue profit-generating opportunities, potentially reinforcing persistent profit outcomes through reinvestment, expansion, or acquisition strategies (Cummins and Zi, 1998). Mutuals, restricted from issuing stock, often rely more on retained earnings and customer-financed borrowing, which may constrain long-term profitability and limit their ability to adapt rapidly to market opportunities.

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These financial constraints and incentive differences also affect risk-taking behavior. Stock firms, with stronger incentives to enhance shareholder value, may adopt riskier but potentially more profitable strategies. Mutuals, in contrast, often prioritize long-term stability and policyholder interests, which may lead to more conservative behavior and reduced profit volatility, but potentially also less profit persistence over time.

The implications of organizational form extend to firm strategy and performance metrics. Stock companies, driven by shareholder expectations, may place a greater emphasis on sustaining high levels of profitability to maintain market valuation and investor confidence. Mutuals may tolerate fluctuating profits in exchange for long-term solvency and service to policyholders.

Approximately 30% of the U.S. property and liability insurance market is comprised of mutual insurers, despite the predominance of stock companies both domestically and globally. This underscores the relevance of understanding how mutual structures persist in a competitive environment, especially in light of demutualization trends seen in the early 2000s (Viswanathan and Cummins, 2003). These trends reflect market pressures favoring structures with greater capital access and profit potential.

Persistence of profit (POP) has been the subject of robust academic inquiry. Theoretical frameworks diverge: one view asserts that competitive markets erode abnormal profits over time, while another suggests that firm-specific advantages, e.g., proprietary knowledge, reputational capital, or market power, can lead to sustained profitability. Empirical studies in banking support the latter perspective, showing meaningful levels of POP (Berger et al., 2000; Goddard, Molyneux, and Wilson, 2004; Hirsch, 2018).

Berger et al. (2000) found increased POP in the U.S. banking sector, attributing this persistence to market dominance and information asymmetries. Goddard et al. (2004) demonstrated that ownership structure significantly influences short-run profit and growth persistence across different EU nations. These findings support the idea that firm characteristics, including ownership type, play a pivotal role in shaping POP.

Hirsch (2018) further emphasized the methodological nuances affecting POP studies. Variations in testing procedures, sample duration, and industry sectors can all significantly influence results. His work suggests that rigorous, standardized methods tend to show lower POP, highlighting the need for careful empirical design.

In the insurance industry, Choi and Jeong (2022) find that past profits significantly impact future profits, indicating a strong persistence of profitability within the Property and Liability (P-L) insurance industry. This suggests that firms with higher historical profits are likely to maintain superior performance over time. They also find that lagged growth positively affects current profitability. Conversely, previous profitability is positively associated with current growth, highlighting a bidirectional relationship between these two variables.

This study contributes to the literature by examining the relationship between firm growth and profitability in the U.S. insurance industry, with a focus on how organizational form influences profit persistence. Stock and mutual insurers differ significantly in terms of capital access, incentive structures, and risk preferences, which may lead to divergent paths in sustaining profitability. By highlighting these distinctions, the study addresses a gap in existing research and provides insights into the factors that promote or constrain long-term profit persistence in the insurance sector.

## Data and Methodology

This study focuses on analyzing the POP in the P-L insurance sector for two ownership structures (mutuals and stocks) and identifying the factors that influence firm growth and firm profits for each group. The use of dynamic panel regressions with Generalized Method of Moments (GMM) estimation techniques allows for controlling for endogeneity and unobserved heterogeneity (Hirsch, 2019). The sample period ranges from 1999 to 2014 and includes only insurers with positive values of surplus, assets, premiums, inputs, or outputs.

The regression models include firm-specific variables such as asset size, investment ratio, leverage, reinsurance utilization, proportion of personal lines, market concentration, business diversification, and geographic diversification, as well as industry cycle variables such as Gross Domestic Product (GDP) changes, annualized Economic Policy Uncertainty, membership in an insurance group, and hard market vs. soft market dummies. The results of the study provide insights into the behavior of mutuals and stocks in terms of profitability and growth, highlighting the differences in their responses to various firm characteristics. GMM dynamic panel regression models are:

$$Profit_{i,t} = \alpha + \beta_{1,1}Growth_{i,t-1} + \beta_{1,2}Profit_{i,t-1} + \beta_{1,3}X_{i,t} + \ln(\varepsilon)_{i,t} \quad (1)$$

$$Growth_{i,t} = \alpha + \beta_{2,1}Profit_{i,t-1} + \beta_{2,2}Size_{i,t-1} + \beta_{2,3}X_{i,t} + \ln(\varepsilon)_{i,t} \quad (2)$$

In the testing model, subscript  $i$  represents the  $i^{\text{th}}$  insurance company,  $t$  is a time index, and  $\varepsilon_{it}$  is a random error term with zero mean and a constant variance (see Fier and Pooser, 2016). In Equation (1), the coefficient,  $\beta_{1,1}$ , tests the impact of past growth on current profit and  $\beta_{1,2}$  reflects the abnormal profits and shows the persistence of profit. In Equation (2),  $\beta_{2,1}$ , tests the past profit's impact on current growth, while,  $\beta_{2,2}$  shows the size-growth relationship.

To measure an insurer's profitability (*Profit*), a form of the underwriting profit margin (Profit Margin) is used in addition to the conventional accounting profit rate of return on equity (ROE). To normalize the profitability of a company in a given year ( $t$ ), the average profitability of the industry in the same year is deducted, as outlined in previous studies (Goddard et al., 2004; Hirsch, 2018). The Profit Margin is calculated as one less the adjusted loss ratio, which is the excess of premiums collected over losses and loss adjustment costs. This profitability indicator is commonly used in the insurance industry (e.g., Ma and Pope, 2003).

Previous research has shown an inconsistent relationship between profitability and company expansion (e.g., Santomero and Babbel, 1997; Hardwick and Adams, 2002). Both the current and lagged values of *Profit* are used in this study. Growth is measured using changes in the logarithm form of total assets, specifically  $\ln(Size)_t - \ln(Size)_{t-1}$ . To normalize the Growth variable, the average growth rate of the industry in the same year is deducted, as described in previous research (Goddard et al., 2004; Barth and Eckles, 2009), in order to represent the interactive relationship between growth and profitability.

To measure insurer growth, the annual percentage change in direct written premiums is used. Direct written premiums are the total premiums collected by insurers without taking into account the amount ceded to reinsurers or intermediaries. Growth rate is a commonly used measure of firm performance in the insurance industry (e.g., Cummins et al., 1999 and Jin et al., 2022).

In addition to the primary variables of interest, the study incorporates a set of firm-specific and industry-level control variables ( $X_{i,t}$ ) that have been found to be relevant in the existing literature. Market Share is defined as an insurer's proportion of total direct premiums written in

the market at a given time. This metric is widely used to assess an insurer's relative market position and size.

Advertising Intensity, measured as the ratio of advertising expenses to direct premiums written, captures the strategic role of advertising in influencing profitability and growth. Prior research (e.g., Chen & Waters, 2017; Choi, 2019) indicates that increased advertising can strengthen brand visibility, attract and retain customers, and enhance perceived service quality, thereby affecting market performance.

Investment Ratio reflects net investment income relative to premiums written. Since investment returns contribute significantly to insurers' overall income, this variable serves as a key indicator of financial performance and risk appetite, accounting for differences in asset management strategies across firms.

Leverage, calculated as the ratio of liabilities to assets, indicates the degree of debt financing. While higher leverage may enhance returns, it also raises financial risk. Reinsurance Utilization, the ratio of reinsurance ceded to the sum of reinsurance assumed and direct premiums written, captures how much risk is transferred externally. Greater use of reinsurance can reduce underwriting risk and increase underwriting capacity, but it also affects costs and profit margins.

The model also includes variables related to business lines. Proportion of Personal Lines measures the share of direct written premiums from personal lines (e.g., auto, home) relative to total premiums, indicating specialization in typically higher-risk segments. Market Concentration is captured using the Herfindahl-Hirschman Index (HHI)<sup>1</sup>, where higher values suggest less competition and potentially higher profits. Business Diversification is measured as the inverse HHI across lines of business, with lower values indicating broader diversification<sup>2</sup>. Greater diversification may reduce risk and enhance profitability but can also increase operational complexity.

The model also includes a group membership dummy to distinguish between standalone insurers and those affiliated with larger insurance groups, which may benefit from resource sharing and financial backing.

At the macroeconomic level, GDP changes are included to control fluctuations in overall economic conditions during the sample period. Additionally, we incorporate Economic Policy Uncertainty (EPU<sup>3</sup>), measured using a U.S. time-series index, to examine its impact on insurer profitability and growth. EPU serves as a proxy for external policy-related shocks that may influence firm performance. Given the growing academic and policy interest in the effects of uncertainty on economic behavior, this variable provides valuable insight into its influence on insurer performance and growth dynamics.

## II. Results

This study analyzes a balanced panel of 19,924 surviving U.S. property-liability insurers from 1999 to 2014, excluding firms with negative surplus, assets, or premiums. A detailed summary

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<sup>1</sup> Stigler (1964) argues that the Herfindahl index is superior to the concentration ratio (e.g., four-firm concentration ratio) for measuring concentration to assess the likelihood of effective collusion.

<sup>2</sup> We use the data in the NAIC annual statement – Underwriting and Investment Exhibit, Part 1B-Premiums Written.

<sup>3</sup> Baker, Bloom, and Davis (2016) construct the U.S. EPU index and demonstrate its predictive power for a range of economic outcomes including investment, employment, and stock market volatility.

statistics are provided in Table 1, and t-tests (Table 2) reveal significant differences between mutual and stock insurers across several dimensions.

The U.S. P-L insurance market is generally competitive and non-concentrated (Market Concentration HHI = 0.0084), with average growth rates of 5.8% for mutuals and 5.9% for stock insurers. Stock companies, however, show higher average ROE (5.05% vs. 2.78%), indicating superior accounting profitability. They also hold larger asset bases, invest more heavily in advertising, and transfer more risk to reinsurers (30.28% on average). Stock insurers tend to be more geographically and operationally diversified and are more likely to belong to a group (83.7% vs. 44.4%) or use an independent agency system (82.8% vs. 67.9%). Of the 15 firm-level variables analyzed, 9 show statistically significant differences at the 1% level.

**Table 1: Summary Statistics**

Variable	Mean	Median	Lower Quartile	Upper Quartile	Standard Deviation
Growth	0.059	0.039	0	0.106	0.197
Growth (t-1)	0.068	0.043	0	0.114	0.218
Firm Size	18.556	18.491	17.22	19.736	1.901
Firm Size (t-1)	18.497	18.433	17.162	19.689	1.905
ROE	0.044	0.057	0.006	0.111	0.167
ROE (t-1)	0.045	0.056	0.004	0.11	0.162
Profit Margin	0.309	0.687	0.587	0.781	0.244
Profit Margin (t-1)	0.305	0.692	0.59	0.788	0.242
Market Share	0.001	0	0	0	0.002
Advertising Intensity	0.008	0.001	0	0.005	0.036
Investment Ratio	0.034	0.031	0.021	0.04	0.052
Leverage	1.007	0.866	0.475	1.38	0.793
Reinsurance Utilization	0.427	0.383	0.164	0.672	0.303
Proportion of Personal Lines	0.384	0.301	0	0.739	0.374
Market Concentration	0.008	0.008	0.008	0.009	0.001
Business Diversification	0.465	0.374	0.212	0.655	0.297
Geographic Diversification	0.538	0.478	0.141	1	0.385
Group Dummy	0.723	1	0	1	0.448
Agent Dummy	0.785	1	1	1	0.411
GDP Change	0.216	1.019	1.016	1.029	0.016
Economic Policy Uncertainty	1.225	1.29	91	1.48	0.316
Observation	19,924				

*Growth* is defined as total assets growth from last year.

*Profit Margin* is defined as 1 minus adjusted loss ratio.

*Market Concentration* is defined as the sum of the squared market share of each insurer in the US market.

*Economic Policy Uncertainty* values are annualized and expressed as a percentage by dividing the raw scores by 100.

The empirical models (Equations 1 and 2) assess profit persistence and growth-profit dynamics using GMM to address endogeneity and firm heterogeneity. No multicollinearity issues were found.

Tables 3 and 4 reveal distinct differences between stock and mutual insurers regarding the impact of various firm characteristics. Table 3 confirms profit persistence (positive and significant lagged profit) for both ownership types, consistent with earlier findings (e.g., Berger et al., 2000 and Choi and Jeong, 2020). However, only stock insurers show a positive relationship between lagged growth and current profit. Mutuals exhibit a negative and significant relationship at the 10 percent level. These findings suggest that profit-growth dynamics differ by ownership structure.

**Table 2: T-Test: Mutual Company vs. Stock Company**

	<u>Mutual Company</u>		<u>Stock Company</u>		T-test
	Mean	Stan. Dev.	Mean	Stan. Dev.	
Growth	0.0581	0.1247	0.0594	0.2198	
Firm Size	18.0668	2.1145	18.7568	1.7681	***
ROE	0.0278	0.1547	0.0505	0.1719	***
Profit Margin	0.6874	0.2211	0.6932	0.2525	
Market Share	0.0006	0.0032	0.0005	0.0016	
Advertising Intensity	0.0053	0.0131	0.0090	0.0414	***
Investment Ratio	0.0296	0.0188	0.0352	0.0611	***
Leverage	0.9947	0.8130	1.0125	0.7844	
Reinsurance Utilization	0.3161	0.2424	0.4726	0.3132	***
Proportion of Personal Lines	0.3772	0.3443	0.3864	0.3851	
Business Diversification	0.4985	0.3000	0.4510	0.2949	***
Geographic Diversification	0.6966	0.3629	0.4731	0.3750	***
Group Dummy	0.4442	0.4969	0.8370	0.3693	***
Agent Dummy	0.6791	0.4668	0.8278	0.3775	***
Observation	5,797		14,127		

*Growth* is defined as total assets growth from last year.

*Profit Margin* is defined as 1 minus adjusted loss ratio.

Herfindahl Index is defined as the sum of the squared market share of each insurer in the US market.

In Table 3, we find that mutual insurers with larger market shares tend to generate lower profits, whereas no significant relationship between market share and profitability is observed for stock insurers. While advertising intensity is not a significant determinant of profitability for mutual insurers, it does have a significant positive effect for stock companies. This reflects structural and strategic differences: stock insurers, driven by shareholder value maximization, often compete aggressively in markets, particularly in personal lines, where advertising is crucial for customer acquisition and brand visibility. In contrast, mutual insurers typically prioritize policyholder interests and rely more on relationship-based distribution models, such as agent networks, diminishing the role of advertising in profitability. Furthermore, stock insurers operating in personal lines tend to be less profitable, a relationship not observed among mutual companies. We also find that economic uncertainty, as measured by the Economic Policy Uncertainty index,

which serves as a proxy for policy-related economic uncertainty, is negatively associated with firm profits for both groups.

Table 4 shows that lagged profit significantly drives current growth for both types of firms. However, firm size negatively affects growth only for stock insurers, indicating that smaller stock firms grow faster, whereas this pattern is not observed for mutuals. These differences align with prior literature on firm age and profitability (e.g., Cohen, 2001; D'Arcy & Gorvett, 2004), which posits that firms become more selective and profitable over time.

**Table 3: Profit (Profit Margin) Model**

Independent Variable	<u>Mutual Company</u>			<u>Stock Company</u>		
	Coeff.	Std. Err.		Coeff.	Std. Err.	
Intercept	-0.4142	0.1725	**	-0.2588	0.1022	***
Lagged Profit Margin	0.6233	0.0265	***	0.8093	0.0156	***
Lagged Growth	-0.0447	0.0258	*	0.0241	0.0083	***
Market Share	-1.4504	0.3253	***	0.1009	0.6932	
Advertising Intensity	-0.1021	0.1562		-0.0558	0.0240	**
Investment Ratio	-1.0509	0.1862	***	-0.0573	0.0209	***
Leverage	-0.0210	0.0037	***	-0.0163	0.0022	***
Reinsurance Utilization	-0.0360	0.0105	***	-0.0377	0.0058	***
Proportion of Personal Lines	0.0039	0.0088		-0.0121	0.0044	***
Market Concentration	-6.0722	4.8643		-4.7286	3.2600	
Business Diversification	0.0151	0.0109		0.0151	0.0075	**
Geographic Diversification	-0.0184	0.0076	**	0.0068	0.0047	
Group Dummy	-0.0232	0.0050	***	-0.0099	0.0045	**
Agent Dummy	0.0107	0.0055	**	0.0005	0.0045	
GDP Growth	0.5701	0.1658	***	0.3627	0.0965	***
Economic Policy Uncertainty	-0.0475	0.0087	***	-0.0411	0.0049	***
Observations	5,797			14,127		
Adjusted R <sup>2</sup>	0.3768			0.5191		

Dependent Variable: Profit Margin

\*\*\* significant at 1% level, \*\* significant at 5% level, and \* significant at 10% level.

Further analysis reveals ownership-specific effects. For example, stock firms operating in personal lines are less profitable, while this is not the case for mutuals. Market share negatively affects mutual profitability but has no impact on stock insurers.

Geographic diversification enhances profits for mutuals, while group affiliation and agency system usage also positively affect their performance. In contrast, stock insurers benefit more in terms of growth from larger market shares and broader geographic operations. Mutuals, on the other hand, grow more slowly when operating across multiple business lines, and advertising has

no significant growth effect. We also find that mutual insurers are negatively affected by policy-related economic uncertainty, whereas stock insurers show no significant response.

In summary, the results underscore important structural differences between stock and mutual insurers in terms of profitability drivers and growth patterns. These findings contribute to the literature by identifying ownership-specific determinants and reinforcing the significance of organizational form in the performance of insurers.

**Table 4: Growth Model with Lagged Profit Margin**

Independent Variable	<u>Mutual Company</u>			<u>Stock Company</u>		
	Coeff.	Std. Err.		Coeff.	Std. Err.	
Intercept	-0.7917	0.1790	***	-1.0797	0.2090	***
Lagged Profit Margin	0.0501	0.0087	***	0.0503	0.0082	***
Lagged Firm Size	-0.0003	0.0014		-0.0142	0.0016	***
Market Share	0.6284	0.3759	*	8.5531	1.1899	***
Advertising Intensity	-0.3397	0.1204	***	0.1280	0.0918	
Investment Ratio	-0.8325	0.1482	***	-0.2182	0.0962	**
Leverage	0.0178	0.0040	***	0.0450	0.0038	***
Reinsurance Utilization	-0.0228	0.0088	***	0.0011	0.0093	
Proportion of Personal Lines	-0.0125	0.0062	**	-0.0532	0.0062	***
Market Concentration	33.7281	4.5982	***	37.4317	4.7531	***
Business Diversification	0.0273	0.0075	***	-0.0044	0.0078	
Geographic Diversification	-0.0058	0.0060		-0.0156	0.0057	***
Group Dummy	0.0034	0.0045		-0.0031	0.0058	
Agent Dummy	-0.0075	0.0039	*	-0.0066	0.0048	
GDP Growth	0.6257	0.1717	***	0.9950	0.2011	***
Economic Policy Uncertainty	-0.0760	0.0189	***	0.0233	0.0230	
Observations	5,797			14,127		
Adjusted R <sup>2</sup>	0.0903			0.0654		

Dependent Variable: Growth

\*\*\* significant at 1% level, \*\* significant at 5% level, and \* significant at 10% level.

### III. Conclusions

This study is the first to examine the impact of ownership structure on business growth and profitability in the U.S. insurance industry. The findings reveal that prior profitability significantly influences subsequent profits for both stock and mutual insurers. These results are robust across two alternative profit measures.

However, the effects of growth on profitability appear to differ by ownership type. Specifically, lagged growth positively impacts current profits for stock companies only. In the

growth model, a positive correlation is observed between lagged profit and current growth. While the analysis confirms that smaller stock insurers tend to grow faster, no such relationship is found for mutual companies.

The results also show that both ownership types are affected by investment performance, business risk, reinsurance usage, business concentration, and macroeconomic conditions. However, several firm-specific characteristics have ownership-specific effects. For mutual insurers, market share, geographic diversification, and group and agent distribution systems significantly influence profitability and growth. In contrast, for stock insurers, advertising intensity, the proportion of personal lines, and market concentration are more influential.

In terms of growth determinants, stock insurers with higher market share grow more rapidly, while this relationship does not hold for mutual firms. Evidence of economies of scope is also found among stock insurers, as those with greater geographic diversification tend to grow faster. Conversely, mutual insurers benefit more from advertising intensity, reinsurance usage, and business line concentration, all of which positively affect their growth—effects not observed among stock insurers.

This study contributes to the literature by analyzing profit persistence and identifying key growth and profitability drivers across ownership structures. In a competitive market where, new entrants are growing rapidly, understanding these dynamics is critical. Given that rapid growth is a leading cause of failure among property–liability insurers, and that impairment rates are closely linked to underwriting performance, these findings carry important implications for consumers, insurers, investors, and regulators alike. The choice of ownership structure emerges as a pivotal strategic decision in ensuring sustainable growth and profitability in the insurance industry.

One limitation of this study is its focus on short-term effects, as it relies on a one-year lagged variable. As a result, it does not capture longer-term trends or persistence in the relationship under investigation. To better assess long-term profit persistence, future research should incorporate multi-year lagged variables or employ alternative methodologies capable of capturing extended temporal dynamics.

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